Non-linear fractional q-Gaussian diffusion model of stock market

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Summary

- Examined the stylized facts of Bitcoin and compared with conventional stock markets.
- Presented specific solutions for the variable order (VO) nonlinear fractional Fokker-Planck equations, formulated using VO q-Gaussian functions.
- The VO q-Gaussian diffusion model demonstrated effective application in stock markets.

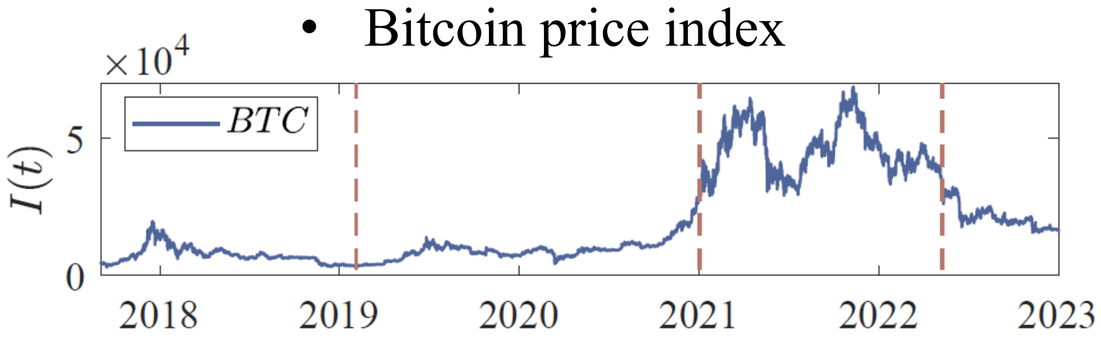
Model

- Variable order porous media equation (VO-PME): $\frac{\partial^{\xi(t)}}{\partial t^{\xi(t)}} P(x,t) = D(t) \frac{\partial^2}{\partial x^2} P^{\nu(t)}(x,t)$
- VO q-Gaussian probability distribution function (PDF) as the solution:

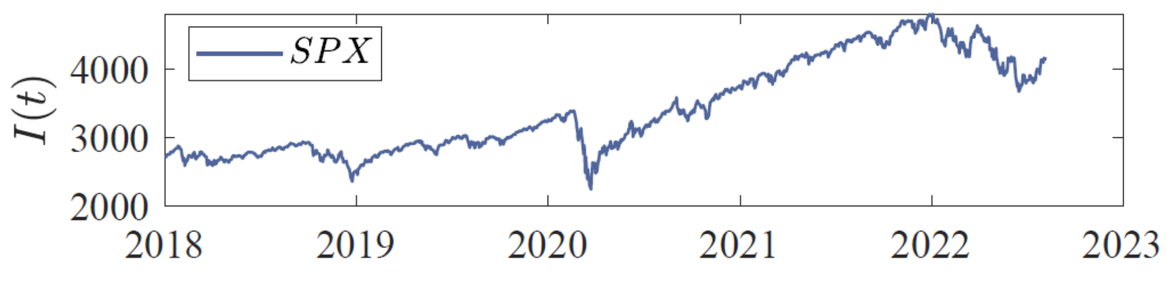
$$P(x,t) = \frac{1}{C_q(t)\Phi(t)} e_{q_t}\left[-\left(\frac{x}{\Phi(t)}\right)^2\right] \qquad C_q(t) = \sqrt{\eta_{q_t}} A_q^{-1}(t) = \frac{\sqrt{\pi}\Gamma\left(\frac{3-q_t}{2(q_t-1)}\right)}{\sqrt{(q_t-1)}\Gamma\left(\frac{1}{q_t-1}\right)} \qquad e_q[x] \equiv [1+(1-q)x]^{\frac{1}{1-q}}$$

Data

- Bitcoin price index

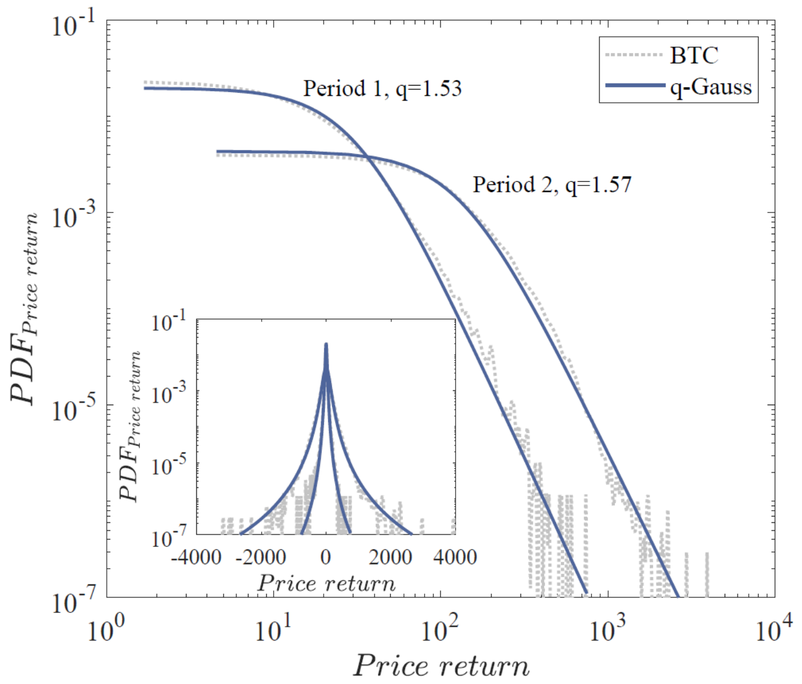


- S&P500 price index

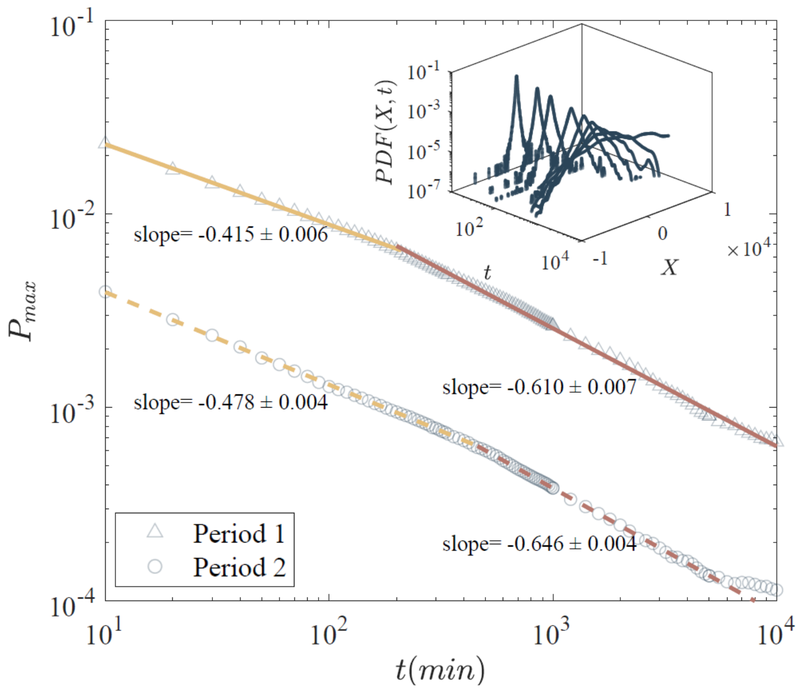


Stylized facts of Bitcoin price return

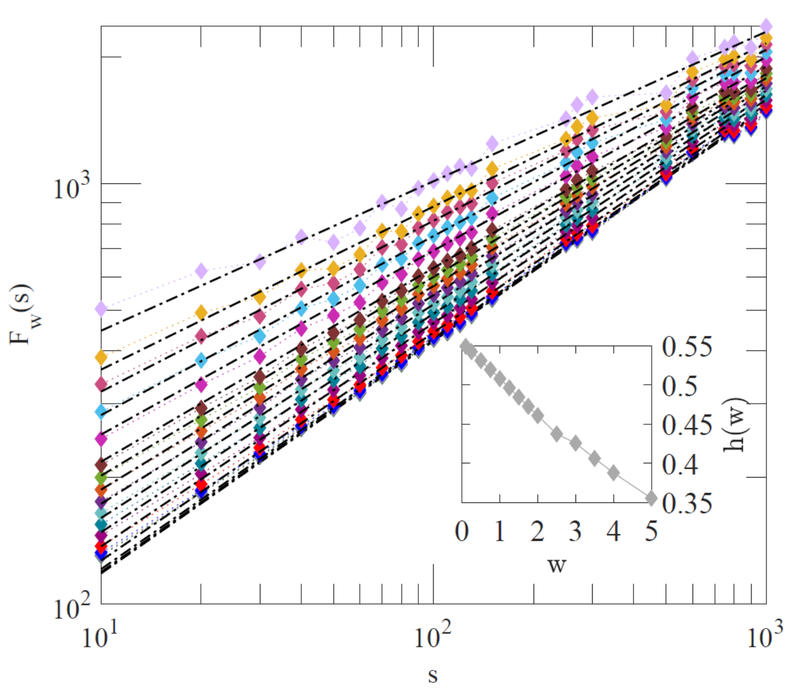
- Heavy tailed distribution



- Anomalous diffusion



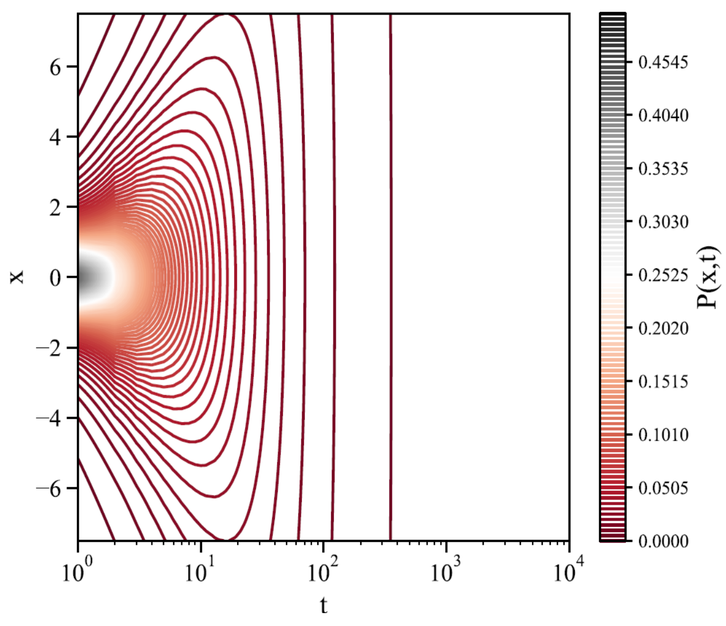
- Multi-fractality



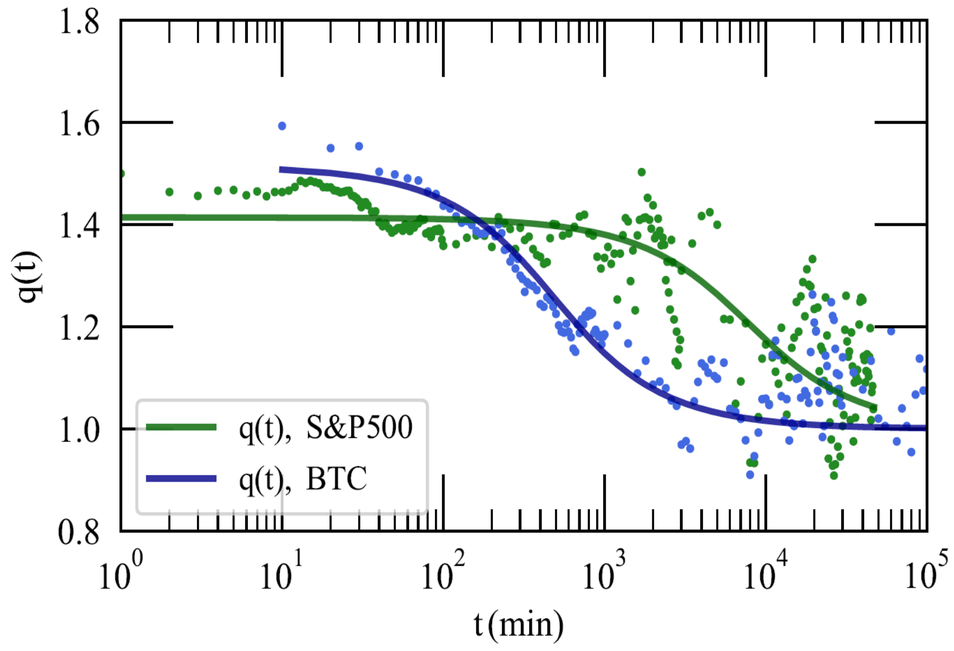
Apply VO q-Gaussian diffusion model to stock market

Time evolution of:

- VO q-Gaussian PDF



- q(t) for PDF



- Bitcoin price return presents stylized facts including heavy tails, anomalous diffusion, short-time autocorrelation, self-similarity.
- Despite the market formation for Bitcoin being quite different from that of conventional markets, the stylized facts presented here are quite similar to those observed for the S&P500.
- The anomalous diffusion for both Bitcoin and S&P500 can be described using a q-Gaussian diffusion process.
- The diffusion parameters show temporal changes in the model, and present convergence to a Gaussian distribution over extended time.

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Variable order porous media equations: Application on modeling the S&P500 and Bitcoin price return.