

# MY TALK

## STATISTICAL MECHANICS FOR COMPLEXITY

A CELEBRATION OF THE 80TH BIRTHDAY OF CONSTANTINO TSALLIS

1<sup>st</sup> part

about **PHYSICS**

RIO DE JANEIRO, 6 TO 10 NOVEMBER 2023

2<sup>nd</sup> part

about **HISTORY**



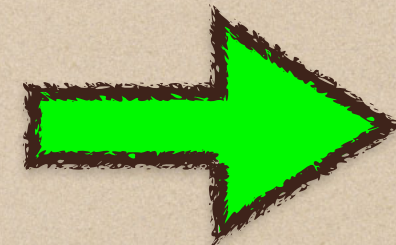
# **Signature of Nonextensive Statistical Mechanics in Asymptotically Scale-free Random Networks**

**Ugur TIRNAKLI**

Izmir University of Economics  
Physics Department, İzmir-Turkey

# EPISODE 1

The story begins with this



EUROPHYSICS LETTERS

1 April 2005

*Europhys. Lett.*, **70** (1), pp. 70–76 (2005)

DOI: 10.1209/epl/i2004-10467-y

## Preferential attachment growth model and nonextensive statistical mechanics

D. J. B. SOARES<sup>1,2</sup>, C. TSALLIS<sup>3,4</sup>, A. M. MARIZ<sup>1</sup> and L. R. DA SILVA<sup>1</sup>

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*22290-180 Rio de Janeiro-RJ, Brazil*

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**Aim:** to explore the connection between scale-free networks and  $q$ -statistics.

**Method:** to propose growth models to explore this connection.

# GROWTH MODEL

➔ Construct a single connected network of sites (or nodes or vertices) and links (or bonds or edges) by gradually (sequentially) making it grow.

➔ First fix one site ( $i = 1$ ) at some arbitrary origin of the plane.

➔ Second site ( $i = 2$ ) is randomly and isotropically chosen at a distance  $r$  distributed according to the probability law

$$P_G(r) \propto \frac{1}{r^{2+\alpha_G}}$$

➔ This second site is then linked to the first one.

# GROWTH MODEL

- ➔ Now locate the new origin as the barycenter of the two first sites.
- ➔ Apply again the distribution  $P_G(r)$  from this new origin and locate the third site ( $i = 3$ ).
- ➔ This third site is now going to be linked to only one of the pre-existing two sites.

?? HOW ??

# GROWTH MODEL

→ To do this, use an attachment probability

$$p_A \propto \frac{k_i}{r_{ij}^{\alpha_G}} \quad (\alpha_A \geq 0)$$

where  $r_{ij}$  is the distance of the newly arrived site to the  $i^{\text{th}}$  site of the pre-existing cluster.

$k_i$  is the connectivity defined as the number of links already arriving to the same site (at the present stage,  $k_1 = k_2 = 1$ ).

# GROWTH MODEL

→ This process is sequentially repeated as many as we like.

If  $N$  is the total number of sites of the cluster, then the linking of the newly arrived site ( $i = N$ ) is done with the probability

$$p_A = \frac{k_i r_{ij}^{-\alpha_A}}{\sum_{j=1}^{N-1} k_j r_{ij}^{-\alpha_A}}$$

The dynamics of the model makes the arriving sites to have preferential attachment to the previous sites that already have many links (hubs), as long as they are not too far.

# GROWTH MODEL

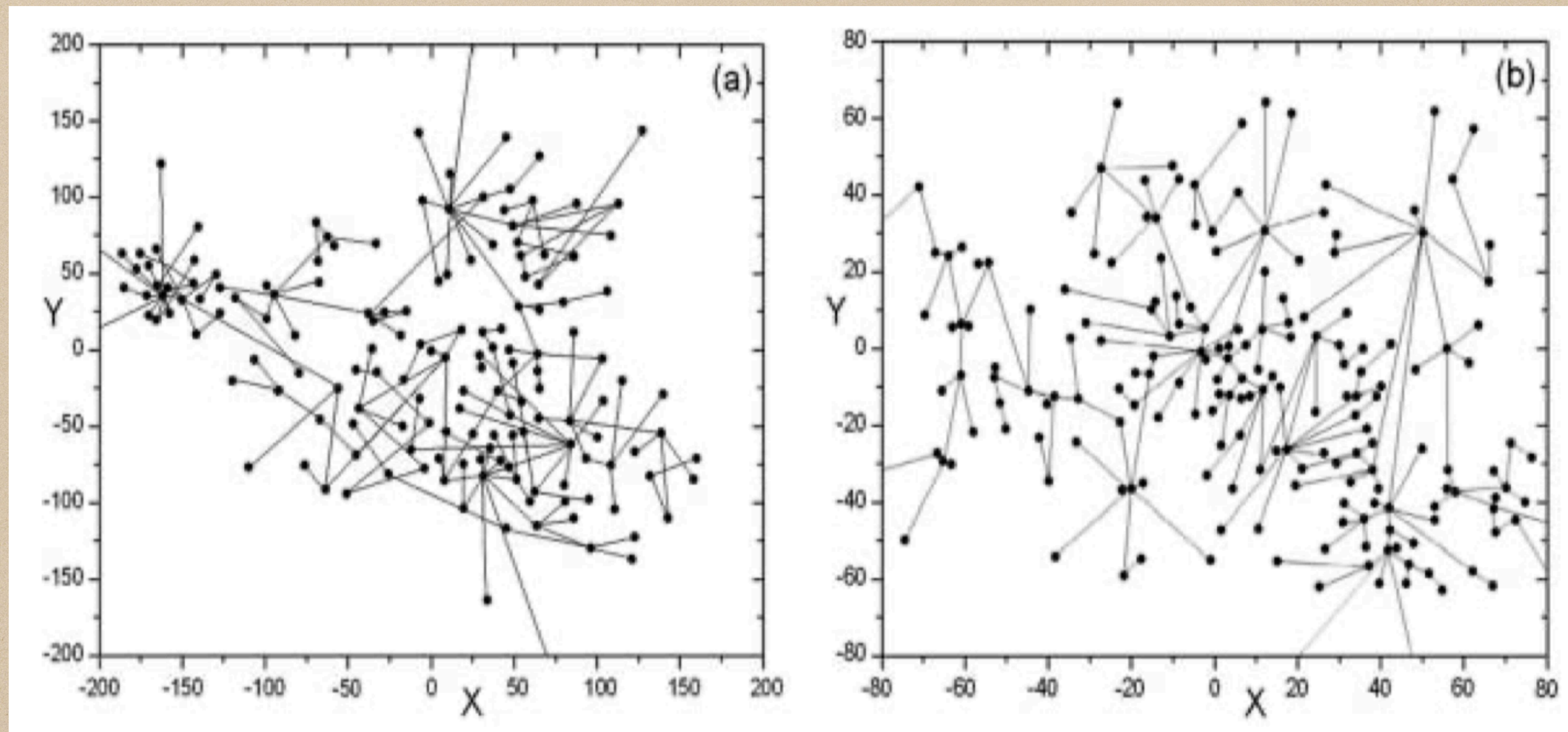
$$p_A = \frac{k_i r_{ij}^{-\alpha_A}}{\sum_{j=1}^{N-1} k_j r_{ij}^{-\alpha_A}}$$

➔ This competition between connectivity and Euclidean proximity is less pronounced when  $\alpha_A$  is close to zero and completely disappears only at  $\alpha_A = 0$ .

For this particular case, one expects behavior consistent with the Barabasi-Albert model, which has topology but no metrics.



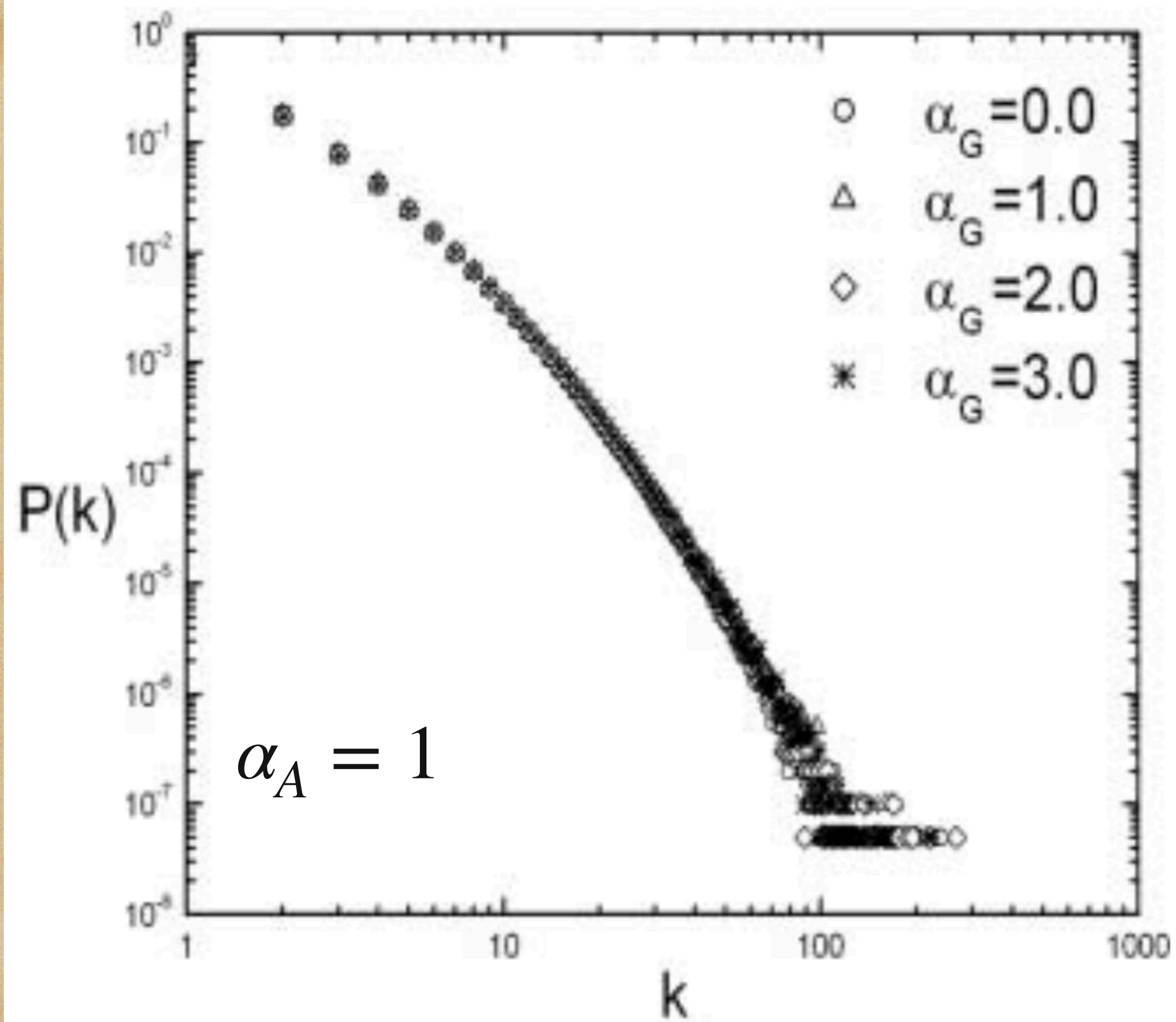
# Typical networks obtained from the model



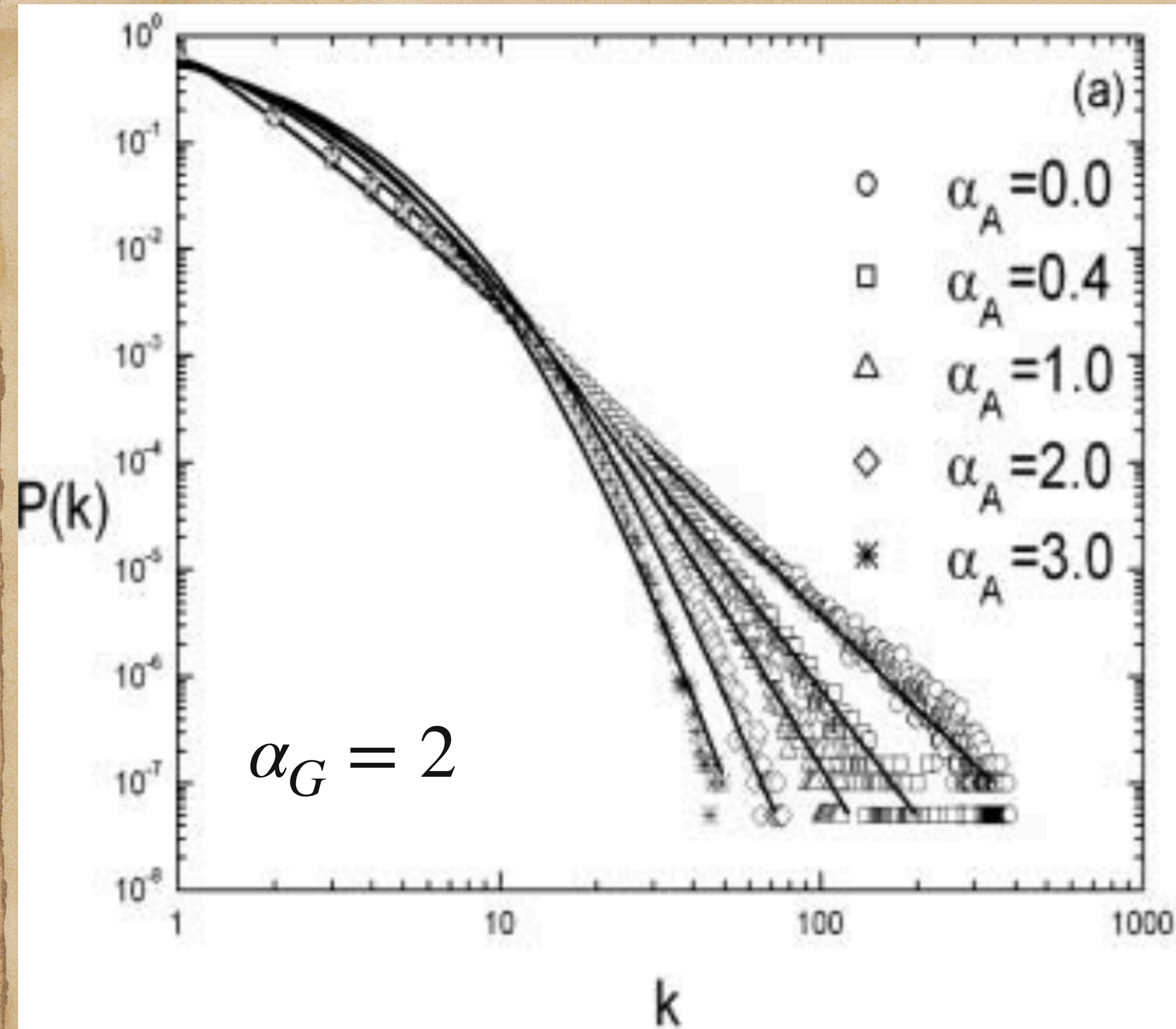
$$(\alpha_G, \alpha_A) = (1, 0)$$

$$(\alpha_G, \alpha_A) = (1, 4)$$

Soares et al., EPL 70 (2005) 70.

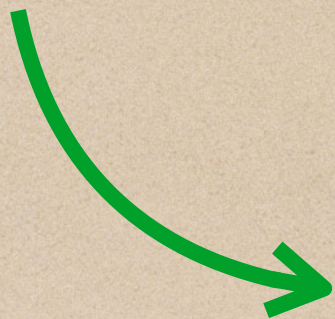


Parameter  $\alpha_G$  controls the metrics but has **no influence** on the connectivity distribution.



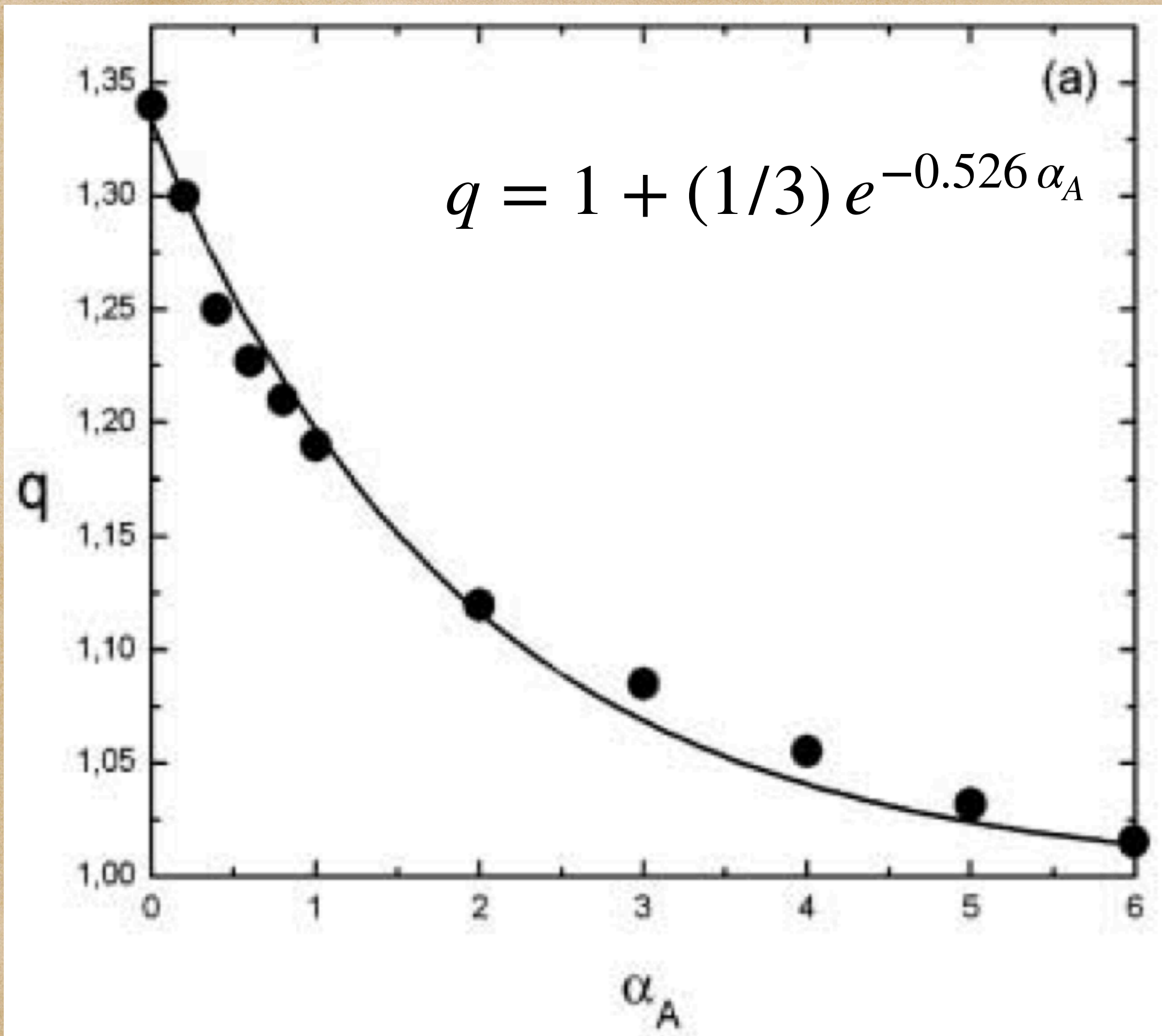
On the other hand, parameter  $\alpha_A$  has a **big influence** on the connectivity distribution.

Try  **$q$ -exponentials** to fit the results.!!



$$P(k) = P(0) e_q^{-k/\kappa}$$

$$e_q^x = [1 + (1 - q)x]^{1/(1-q)} \quad (e_1^x = e^x)$$

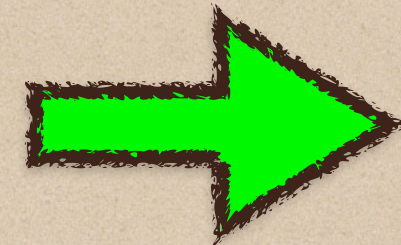


$$P(k) = P(0) e_q^{-k/\kappa}$$

$$e_q^x = [1 + (1 - q)x]^{1/(1-q)} \quad (e_1^x = e^x)$$

## EPISODE 2

Then comes the role of  
dimension



# SCIENTIFIC REPORTS

OPEN

## Role of dimensionality in complex networks

Samurá Brito<sup>1</sup>, L. R. da Silva<sup>1,2</sup> & Constantino Tsallis<sup>2,3</sup>

**Aim:** to explore  $d$ -dimensional geographically located networks which grow with preferential attachment involving Euclidean distances through

$$r_{ij}^{-\alpha_A} \quad (\alpha_A \geq 0).$$

# GROWTH MODEL

$$\rightarrow P_G(r) \propto \frac{1}{r^{2+\alpha_G}} \quad \rightarrow \quad P_G(r) \propto \frac{1}{r^{d+\alpha_G}}$$

$\rightarrow r \geq 1$  is the Euclidean distance from the newly arrived site to the center of mass of the pre-existing system, namely,

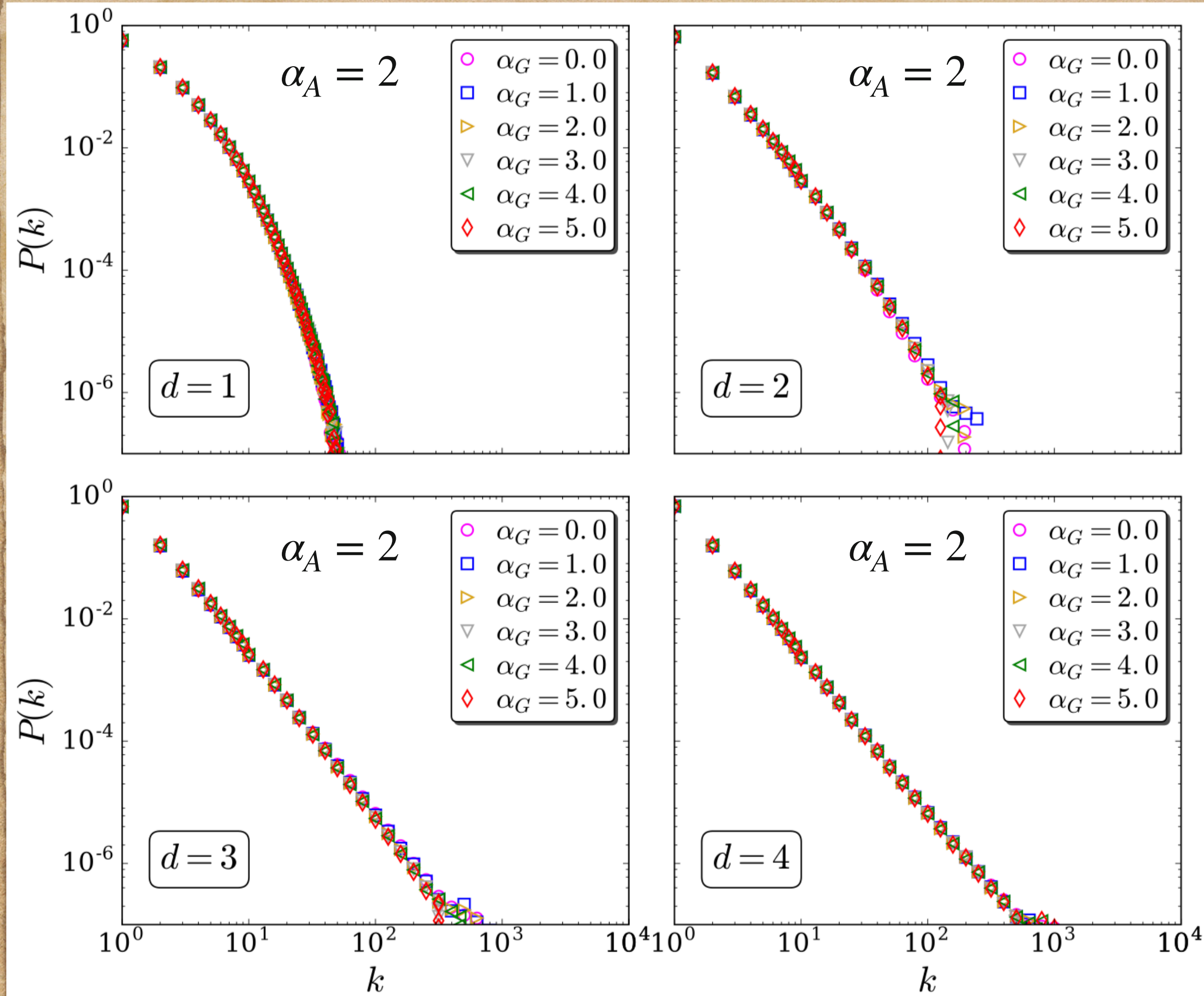
in 1d  $\rightarrow r = x$

in 2d  $\rightarrow r = \sqrt{x^2 + y^2}$

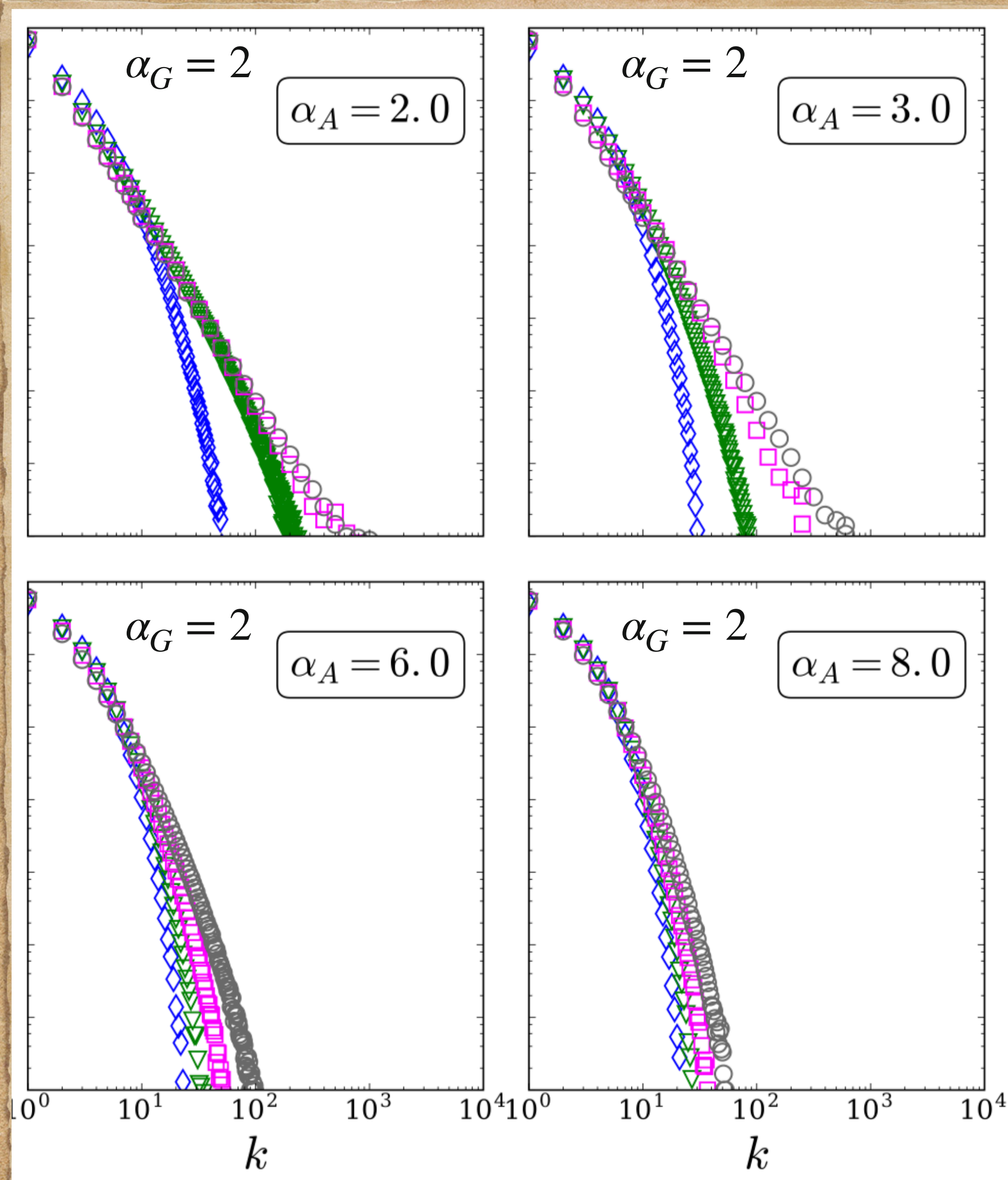
in 3d  $\rightarrow r = \sqrt{x^2 + y^2 + z^2}$

$\rightarrow$  Preferential attachment probability

$$p_A = \frac{k_i r_{ij}^{-\alpha_A}}{\sum_{j=1}^{N-1} k_j r_{ij}^{-\alpha_A}} \in [0,1]$$



$P(k)$  distribution is **independent** of  $\alpha_G$  for all dimensions.



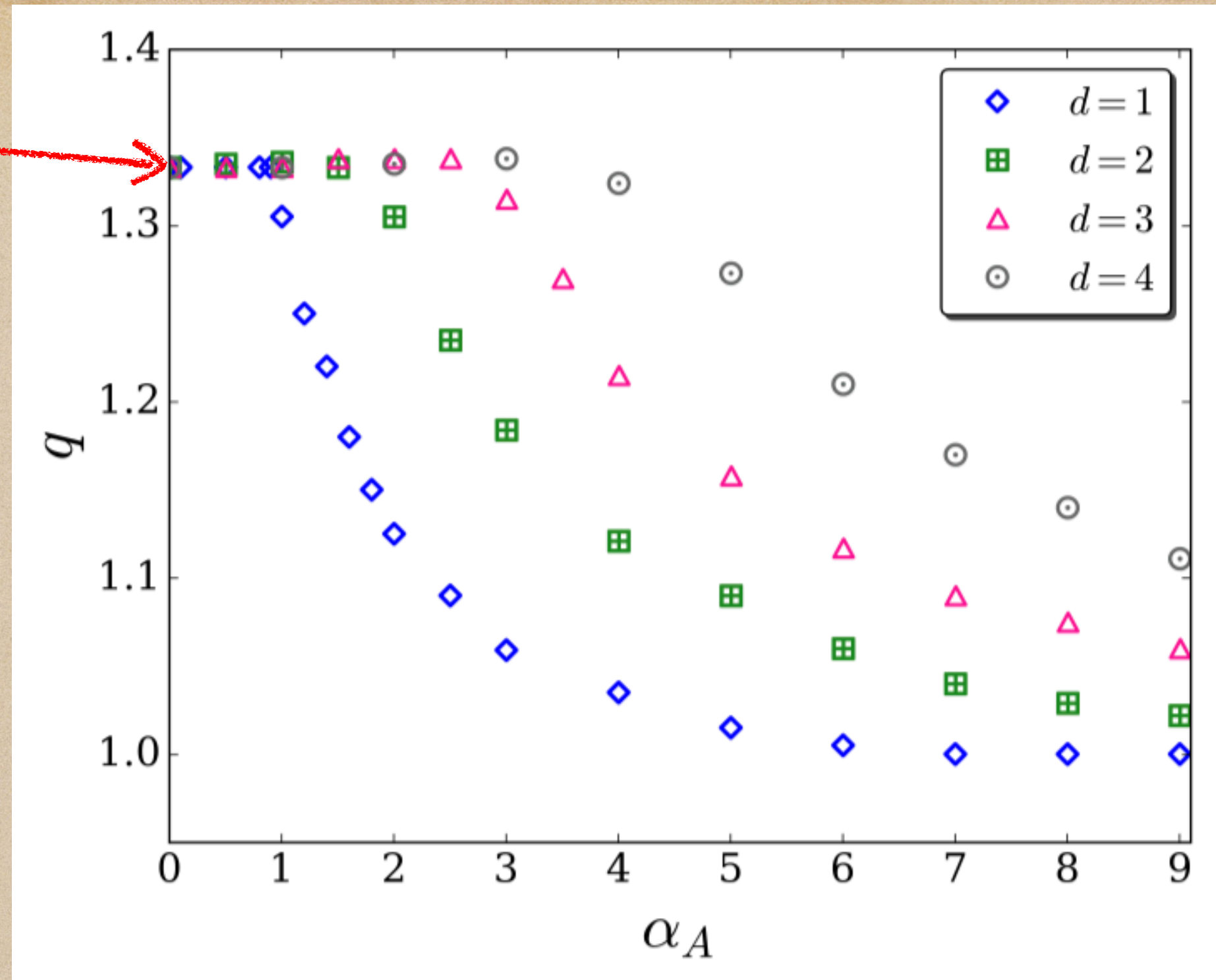
On the other hand,  $P(k)$  distribution is **dependent** of  $\alpha_A$  for all dimensions.

Each color indicates different dimensions.

Brito, da Silva and Tsallis, Sci. Rep. 6 (2016) 27992.

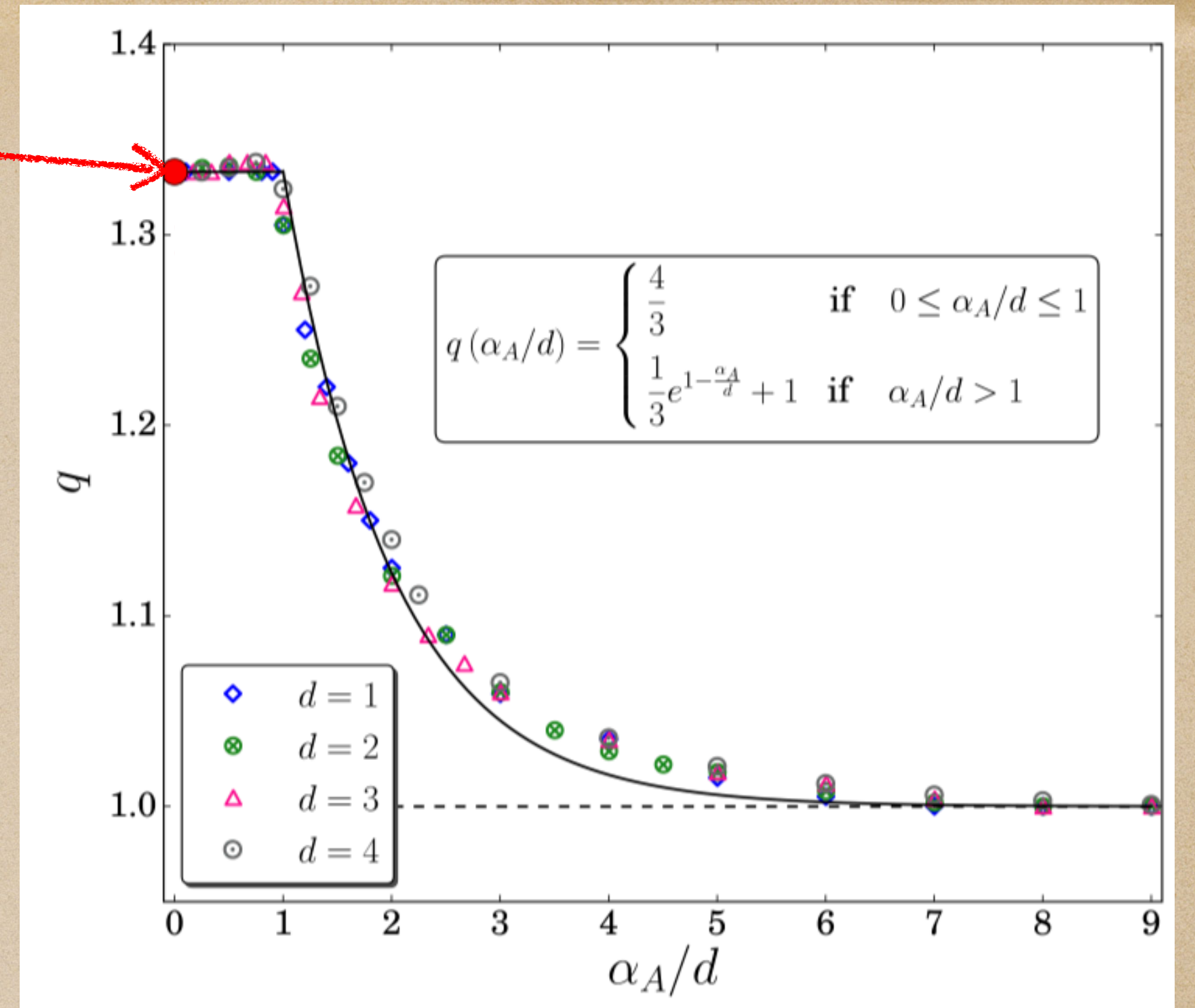


BA



$q$  versus  $\alpha_A$

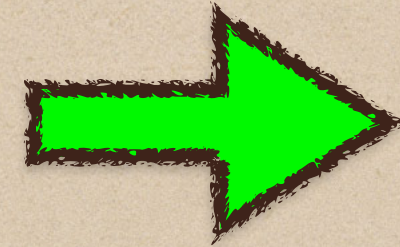
BA



$q$  versus  $\alpha_A/d$

# EPISODE 3

Exploring 'energy' distribution



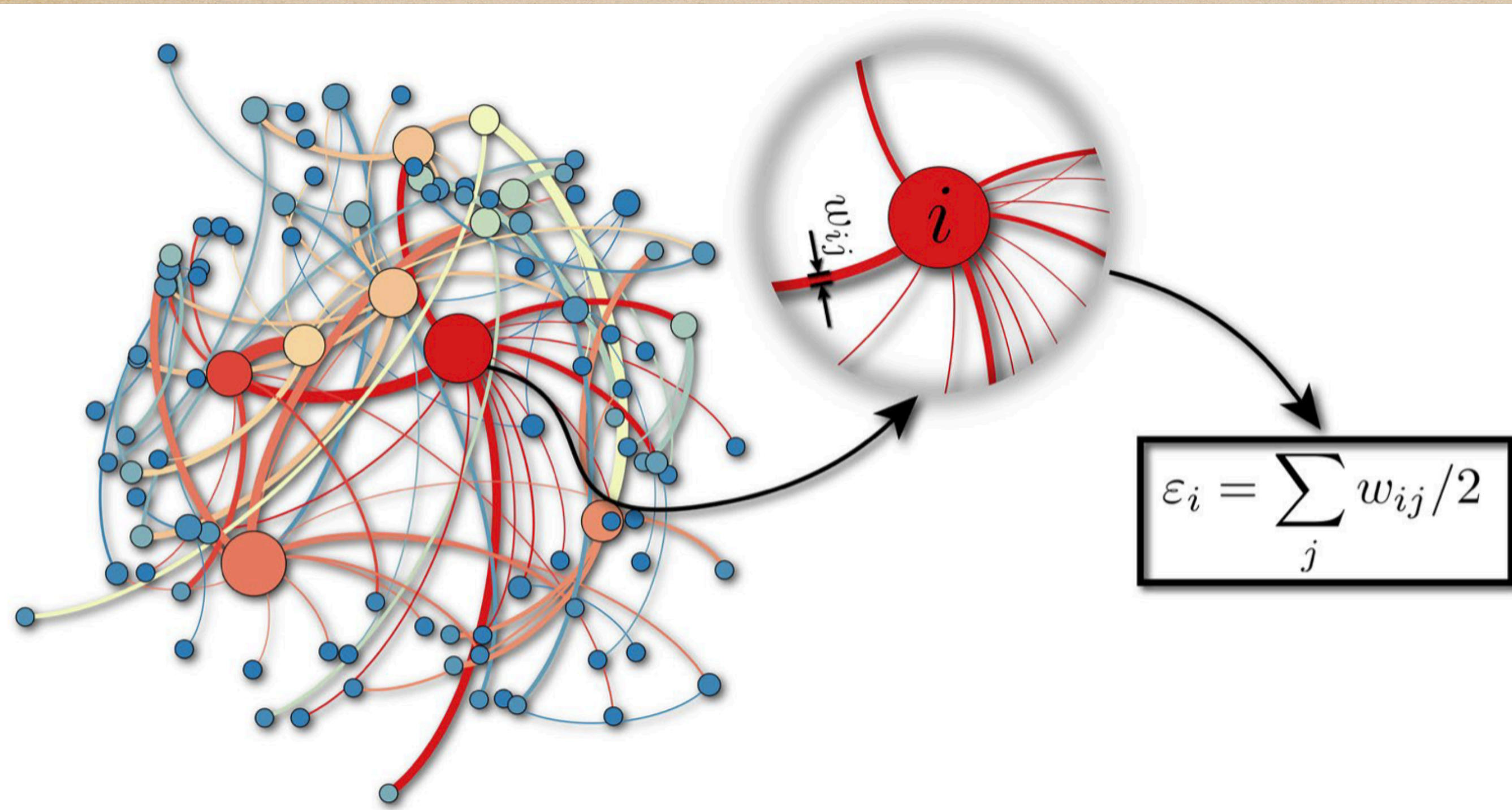
scientific reports

 Check for updates

OPEN

## Connecting complex networks to nonadditive entropies

R. M. de Oliveira<sup>1</sup>, Samurá Brito<sup>2✉</sup>, L. R. da Silva<sup>1,3</sup> & Constantino Tsallis<sup>3,4,5,6</sup>



Hubs emerge in the network

Each link has a specific width  $w_{ij}$

'Energy'  $\epsilon_i$  of a site  $i$  is given by the half of the sum over all link widths connected that site.

# GROWTH MODEL

→ Locate each site using probability  $P_G(r) \propto \frac{1}{r^{d+\alpha_G}}$

→ Choose a random number from a distribution  $P(w)$  which gives the corresponding link weight.

Energy of a site depends on how many links it has ( $k_i$ ) and the widths of those links ( $\{w_{ij}\}$ ).

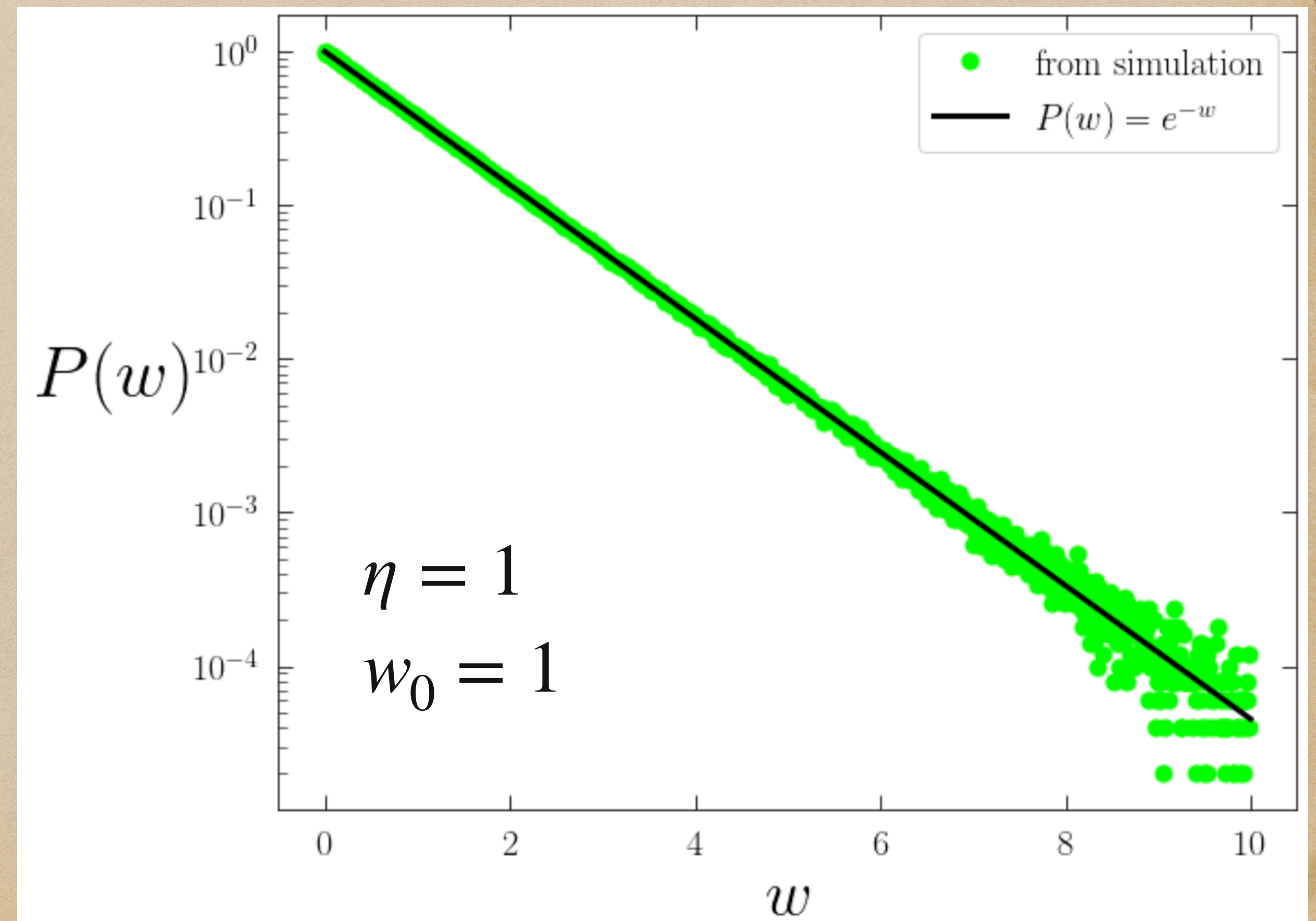
→ Sites  $i = 3, 4, \dots$  will be linked to the previous ones with probability

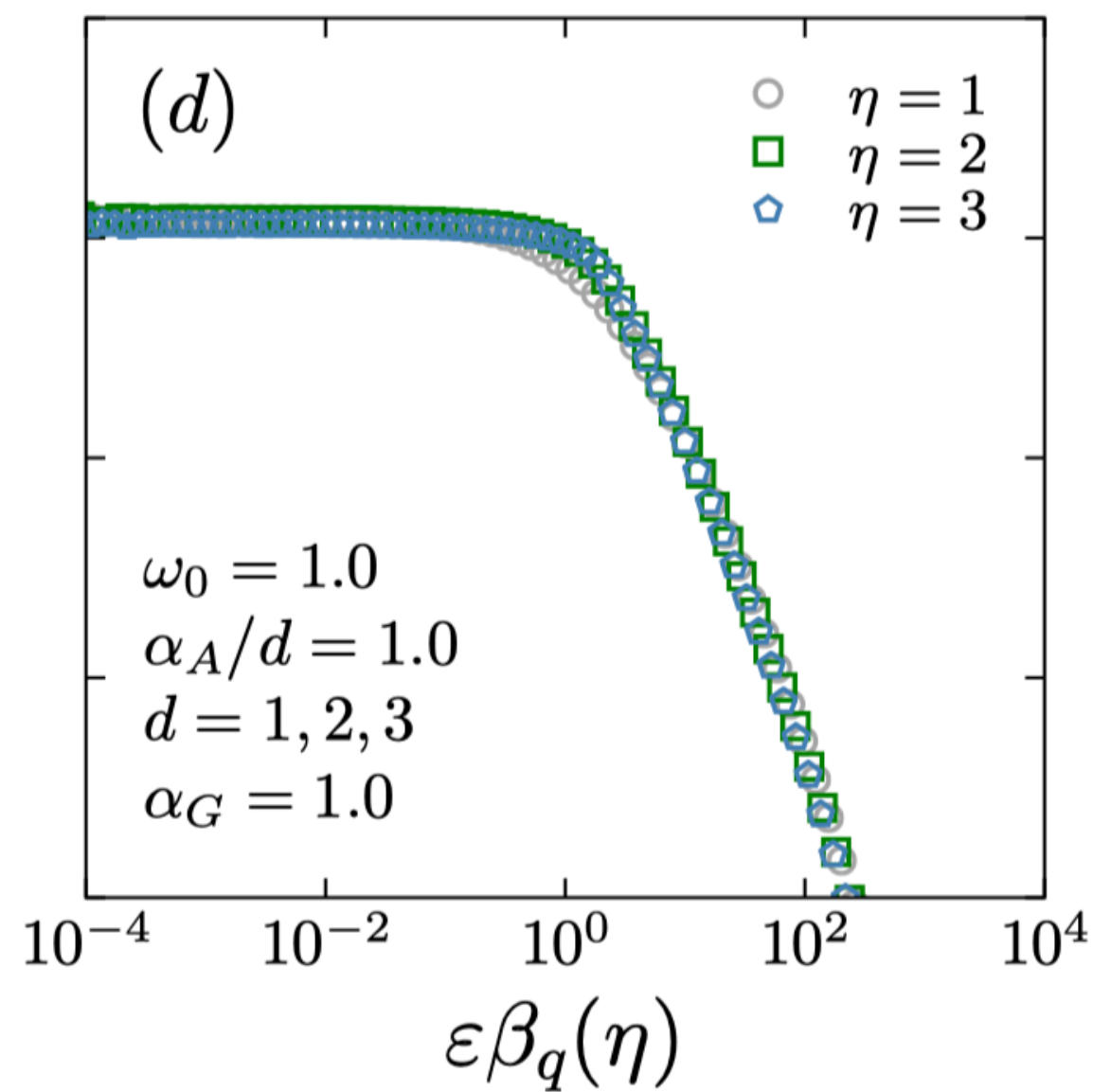
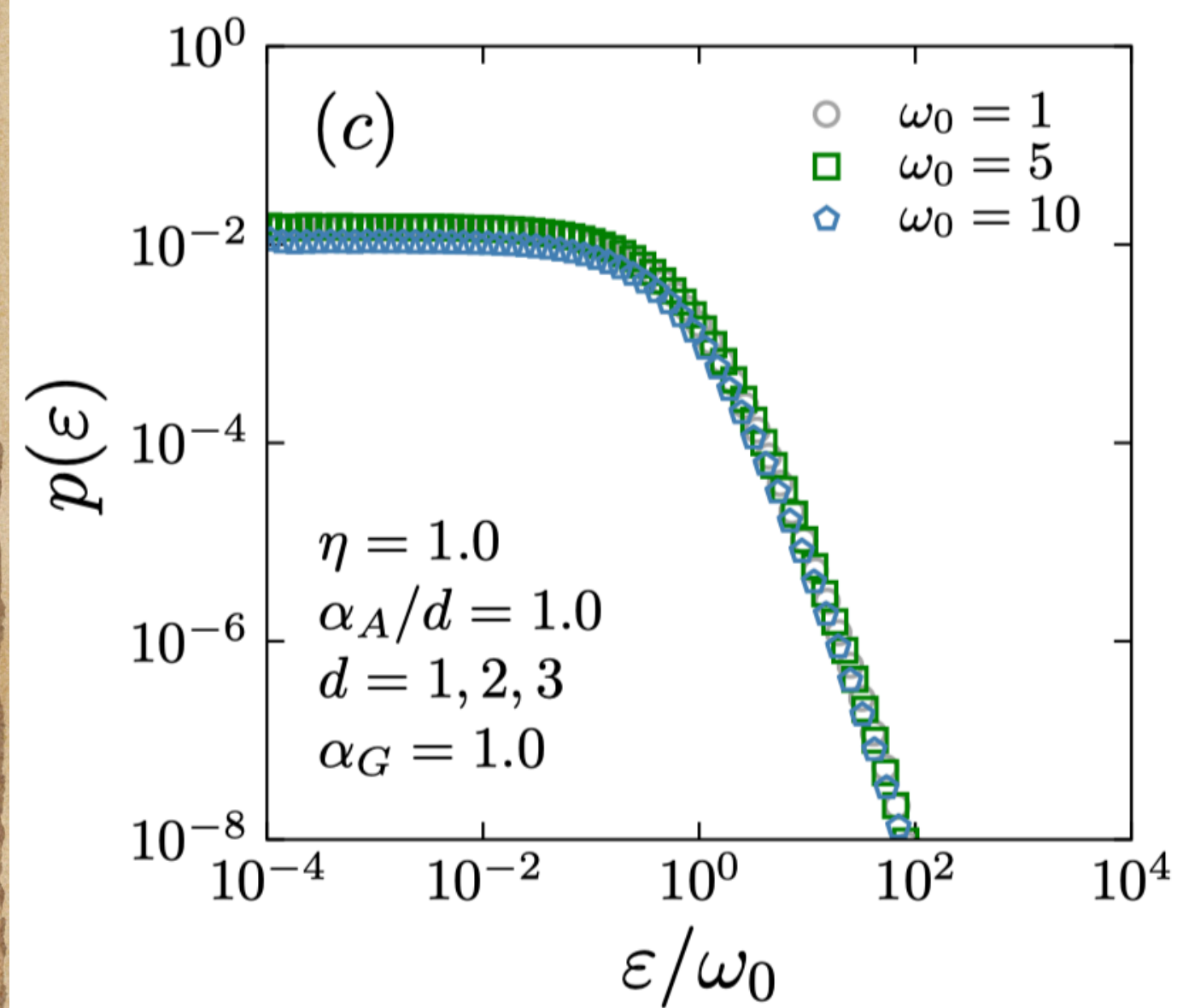
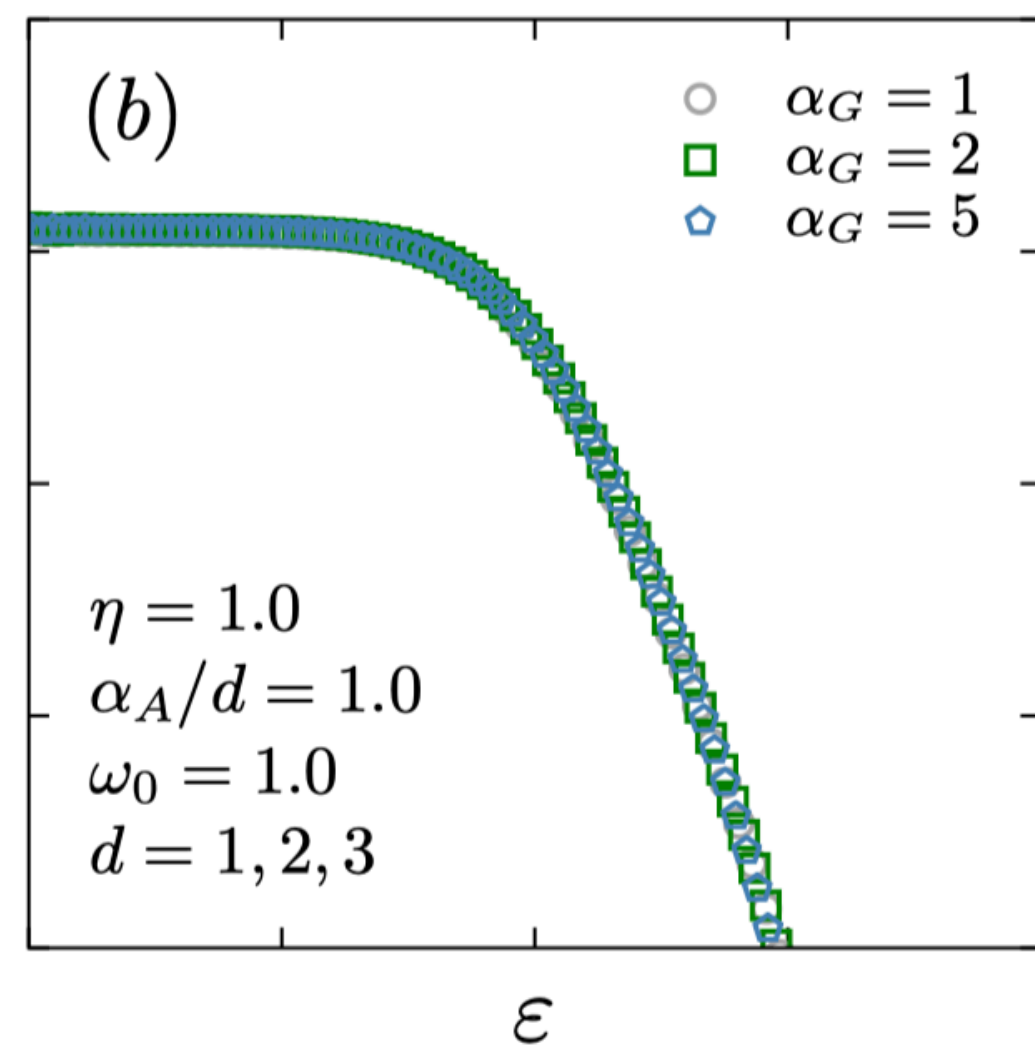
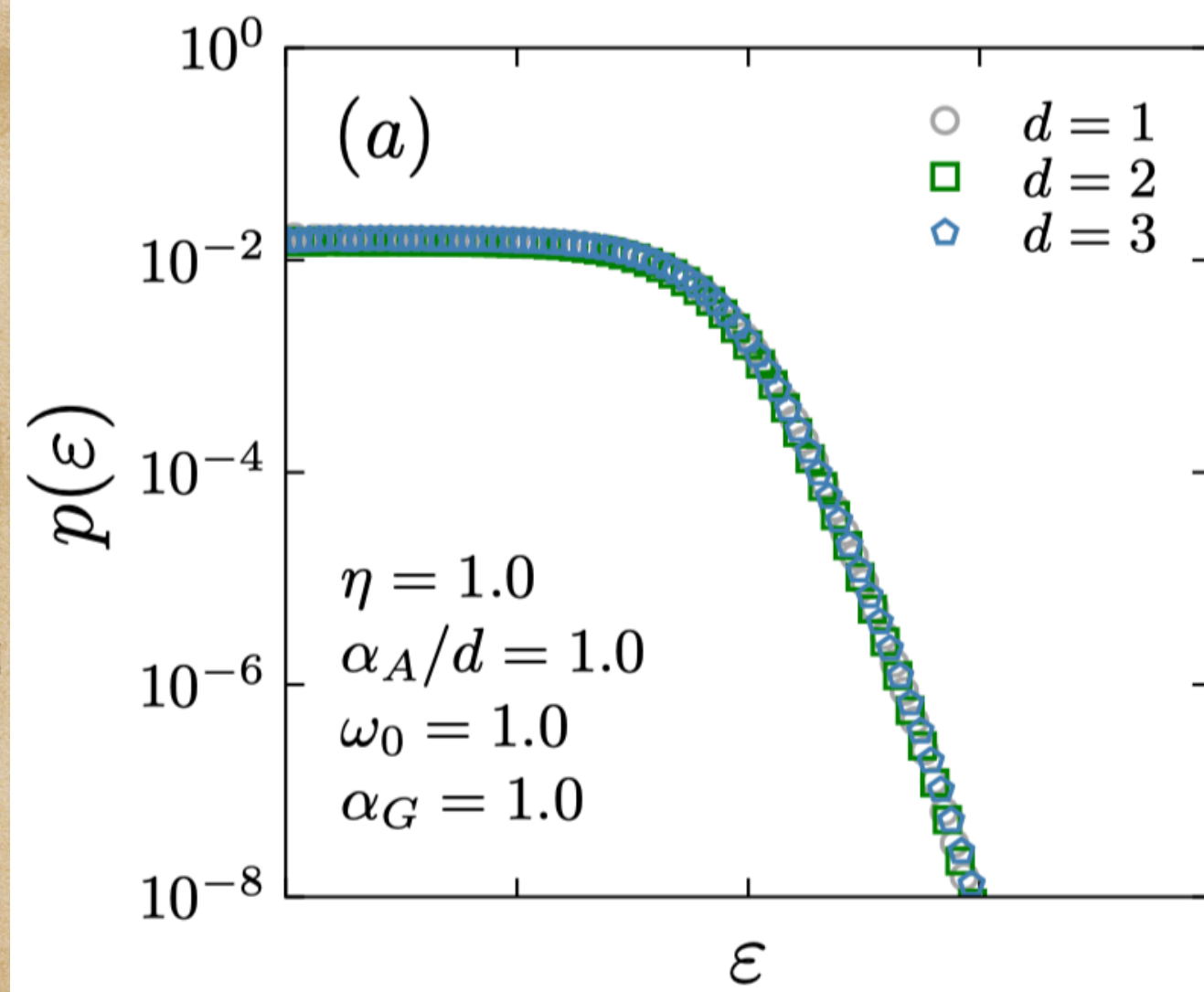
$$\Pi_{ij} \propto \frac{\epsilon_i}{r_{ij}^{\alpha_A}}$$

# GROWTH MODEL

➔ Let's take the case where  $w$  is given by the stretched-exponential distribution:

$$P(w) = \frac{\eta}{w_0 \Gamma(1/\eta)} e^{-(w/w_0)^\eta}$$

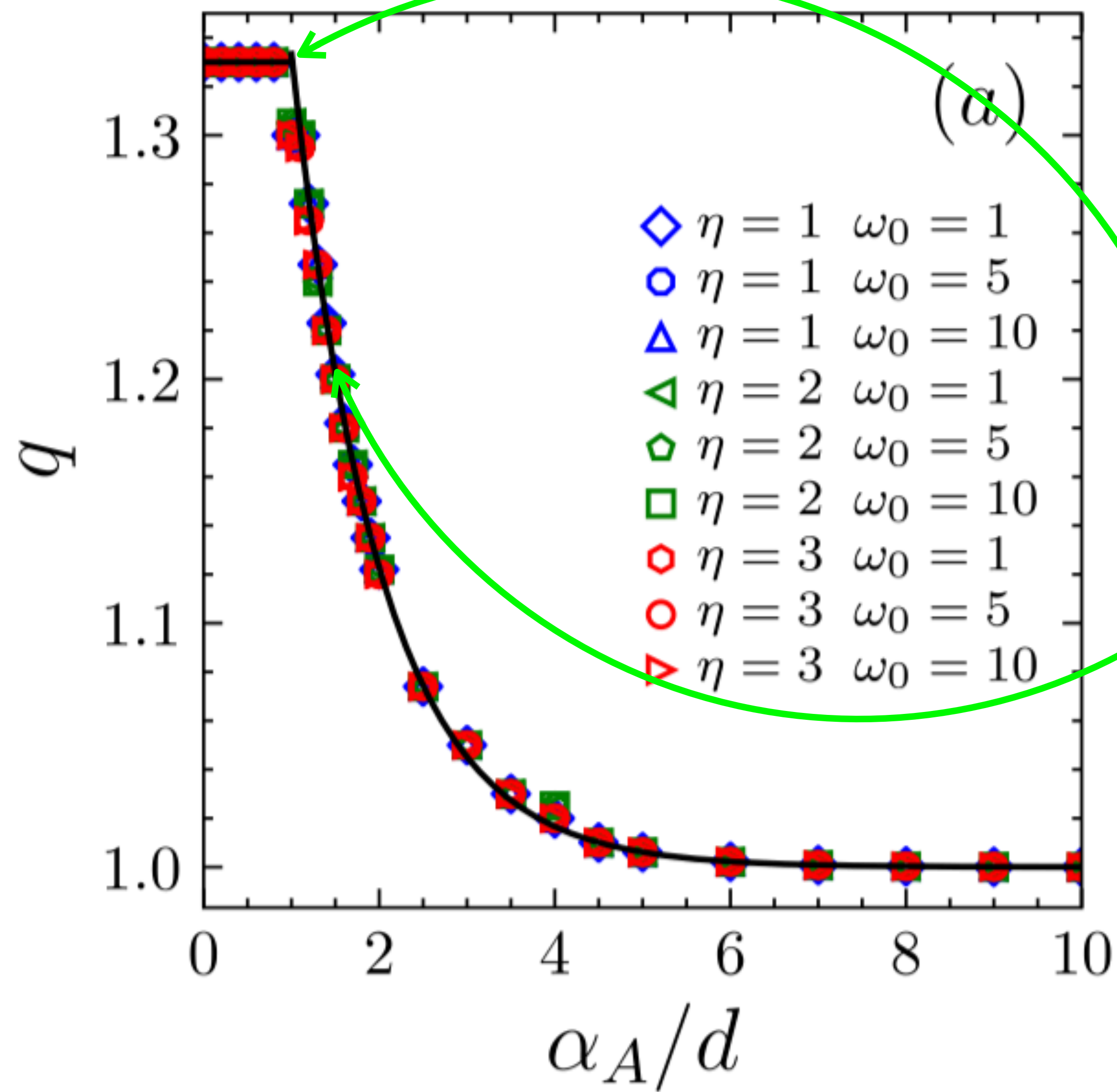




**q-exponentials** are good also to fit these results!!

$$P(k) = P(0) e_q^{-\beta_q \varepsilon}$$

$$e_q^x = [1 + (1 - q)x]^{1/(1-q)} \quad (e_1^x = e^x)$$



$$q = \begin{cases} \frac{4}{3} & \text{if } 0 \leq \frac{\alpha_A}{d} \leq 1 \\ \frac{1}{3} e^{1-\alpha_A/d} + 1 & \text{if } \frac{\alpha_A}{d} > 1 \end{cases}$$

For example :

$$q = 1.333 \text{ if } \alpha_A = 2 \text{ for } d = 2$$

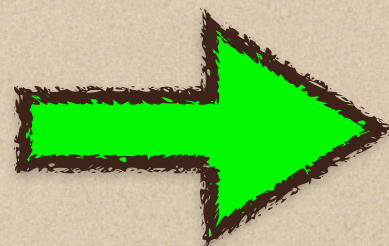
$$q = 1.202 \text{ if } \alpha_A = 3 \text{ for } d = 2$$

$q$  versus  $\alpha_A/d$

# EPISODE 4

Exploring finite-size effects :

starts with this



Chaos

ARTICLE

[scitation.org/journal/cha](https://scitation.org/journal/cha)

**Complex network growth model: Possible isomorphism between nonextensive statistical mechanics and random geometry**

Cite as: Chaos 32, 053126 (2022); doi: [10.1063/5.0090864](https://doi.org/10.1063/5.0090864)

Submitted: 9 March 2022 · Accepted: 2 May 2022 ·

Published Online: 19 May 2022



View Online



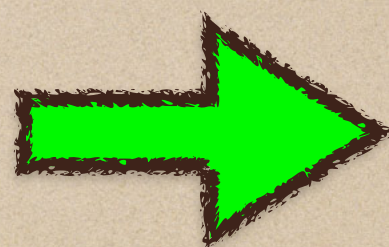
Export Citation



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Constantino Tsallis<sup>1,2,3,a)</sup> and Rute Oliveira<sup>4</sup>

and more to come



Tirnakli & Tsallis, preprint (2023), in preparation.

# FINITE-SIZE EFFECTS

Conjecture:

**finite-size effects** can satisfactorily be described by the equation:

$$\frac{d\xi}{d\varepsilon} = -\mu_r \xi^r - (\beta_q - \mu_r) \xi^q \quad (r \leq q; \varepsilon \geq 0)$$

where

$$\xi(\varepsilon) = \frac{p(\varepsilon)}{(2 - q) \beta_q}$$

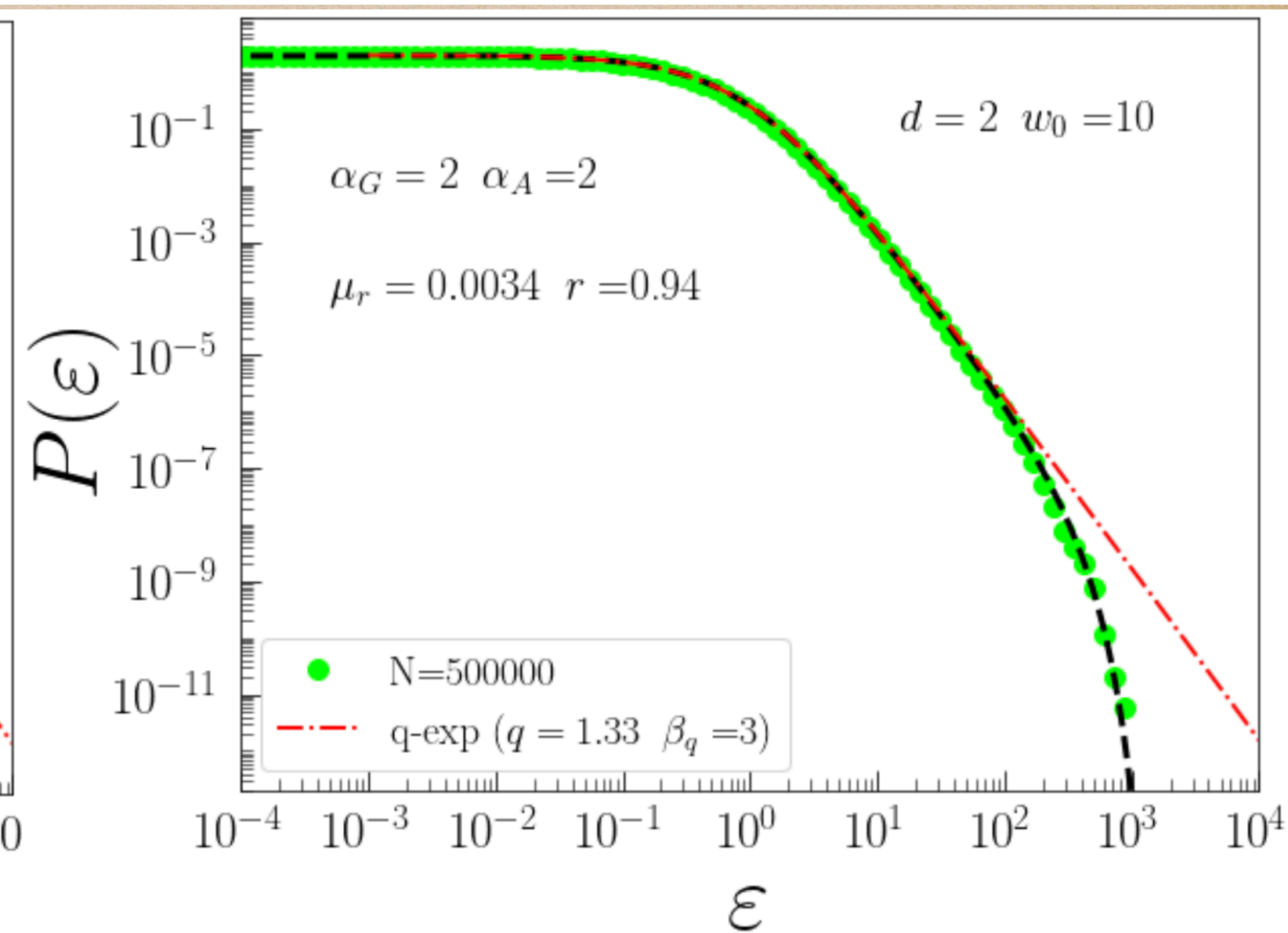
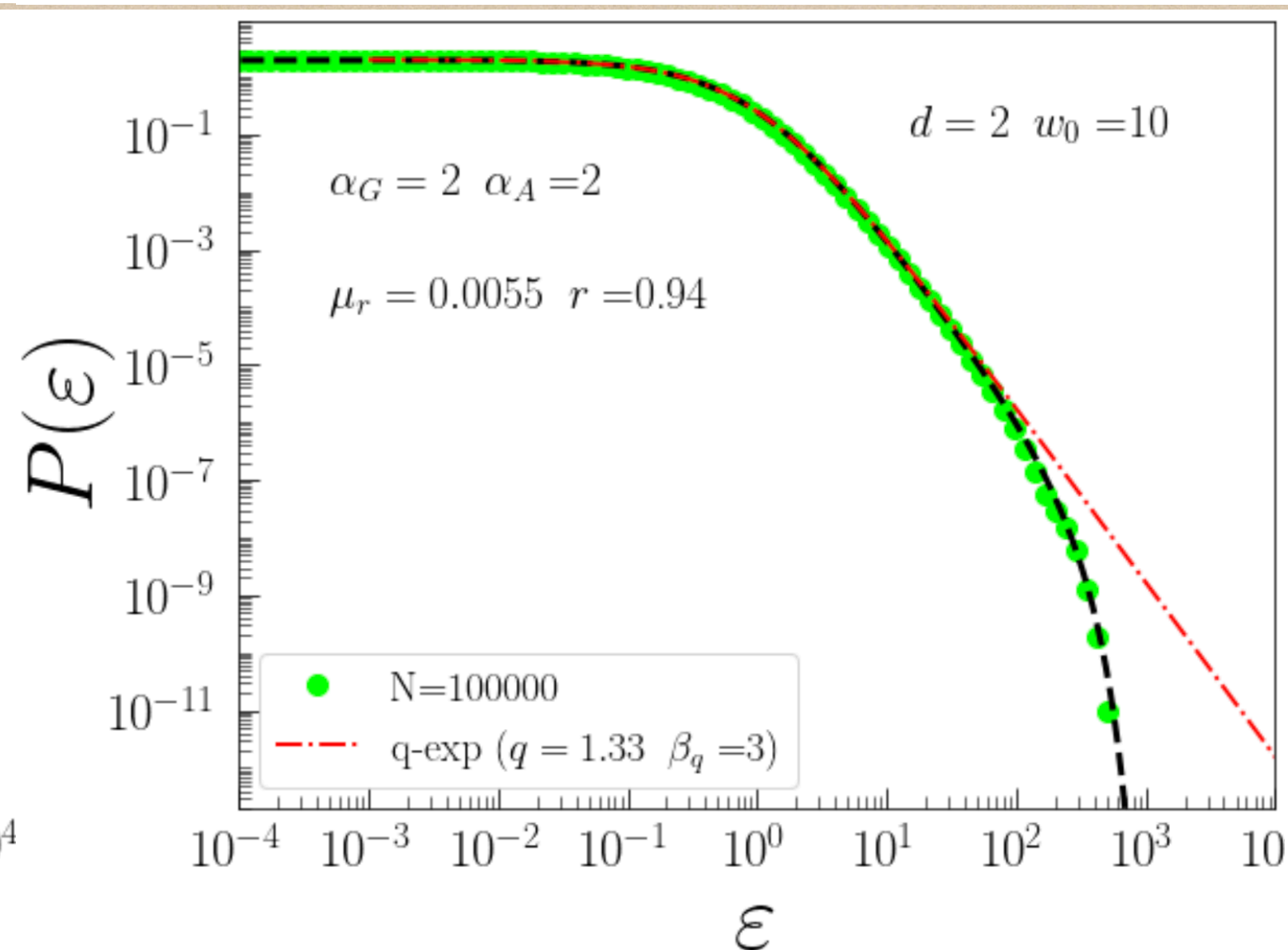
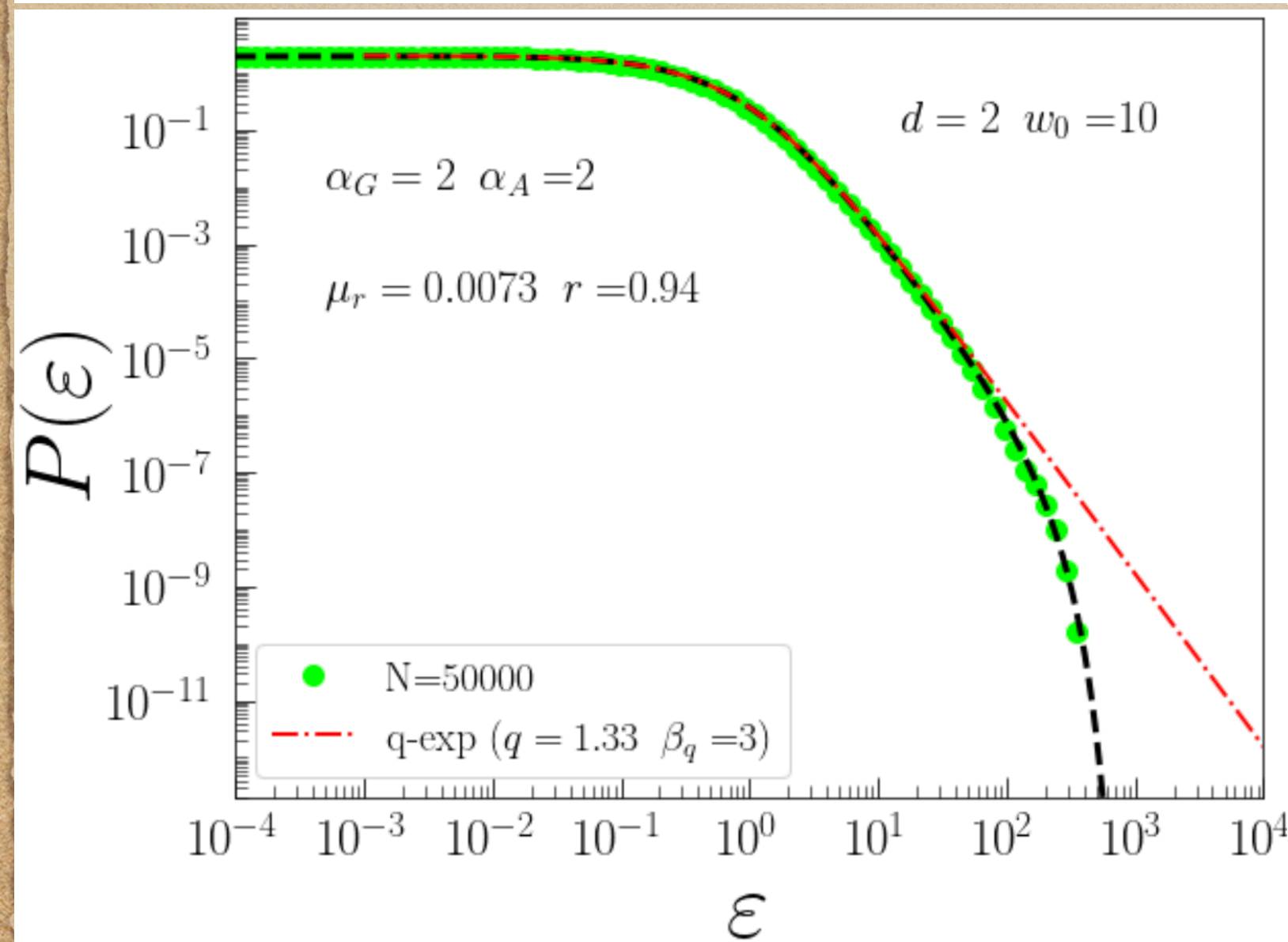
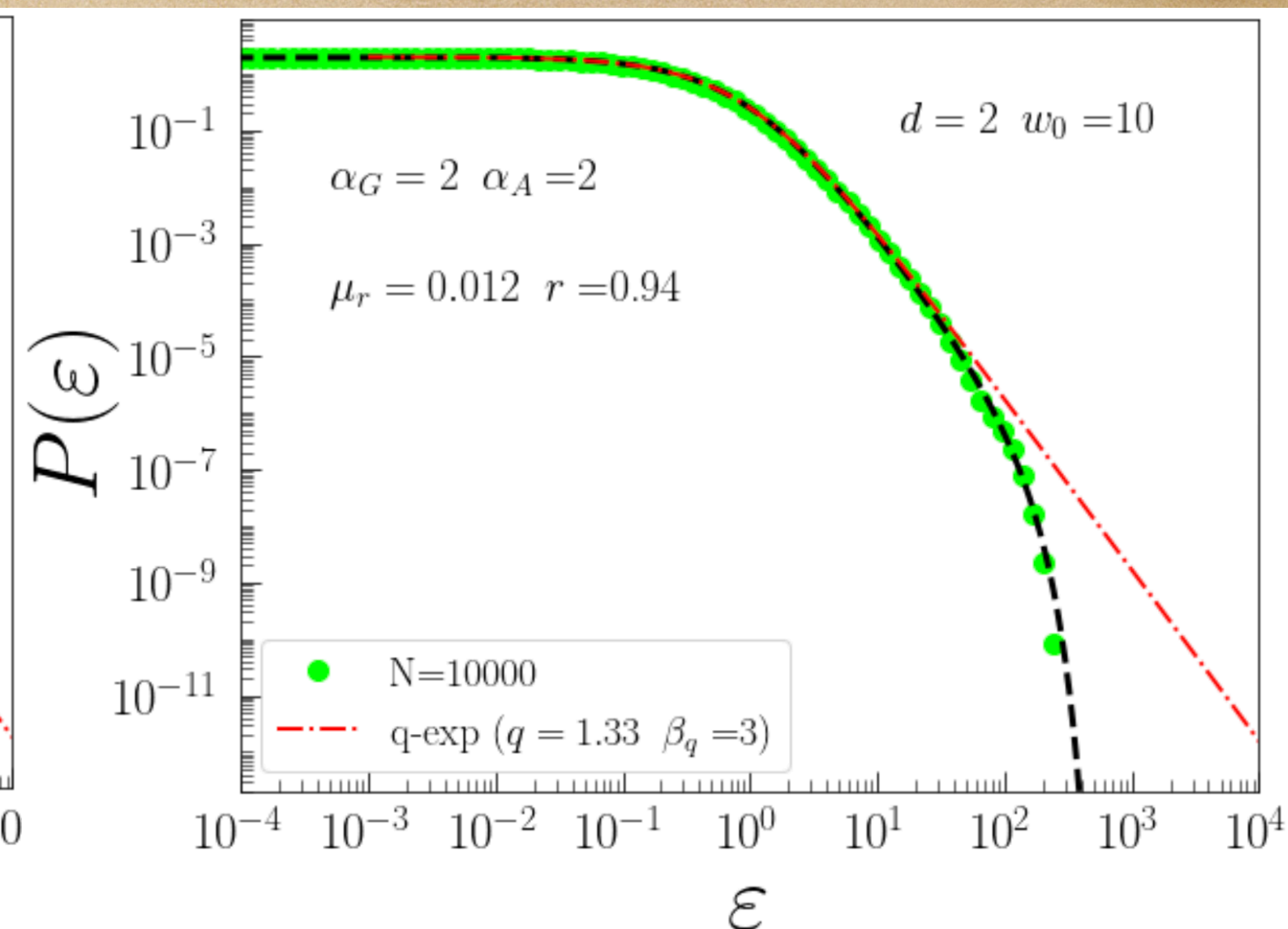
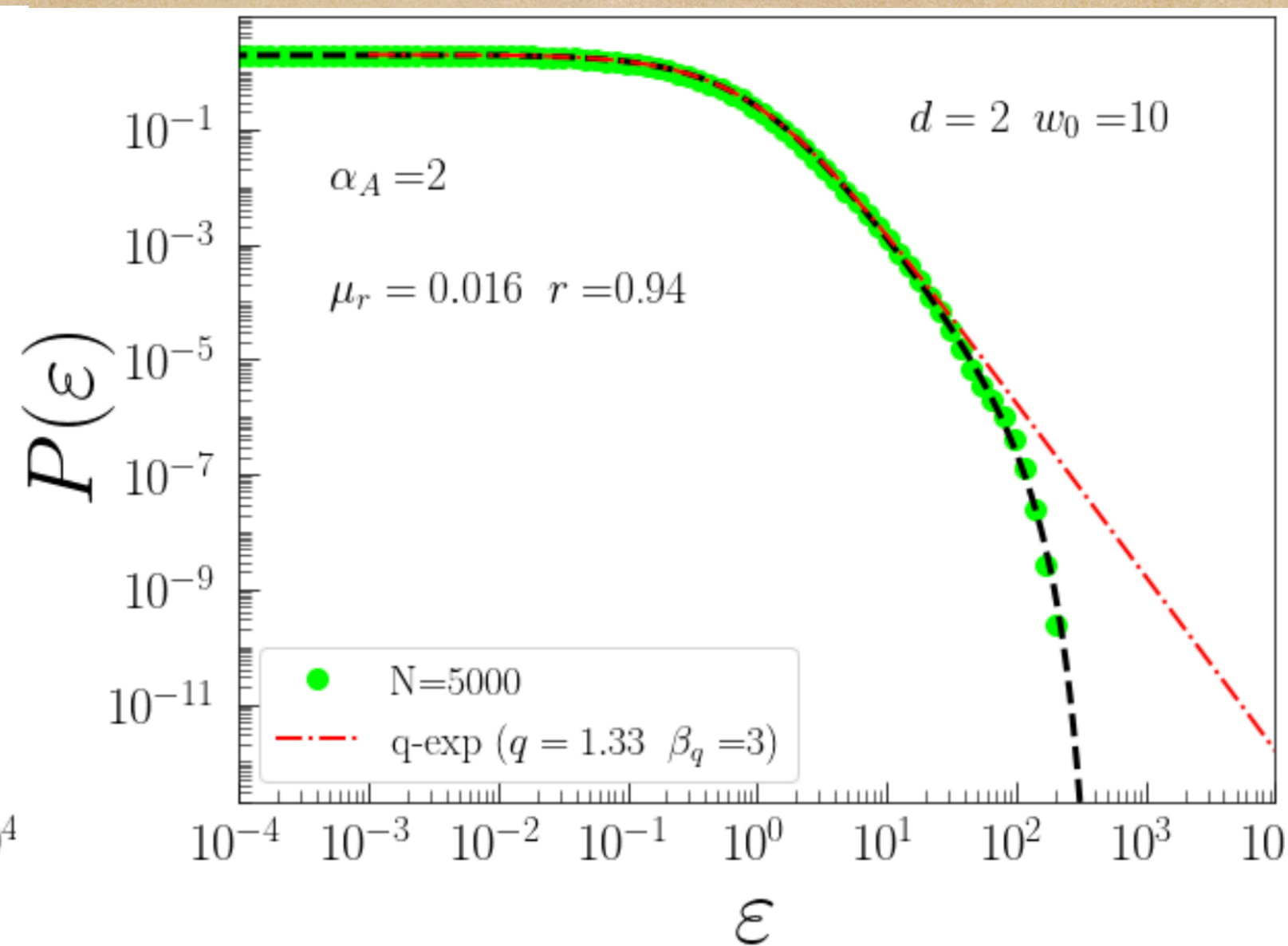
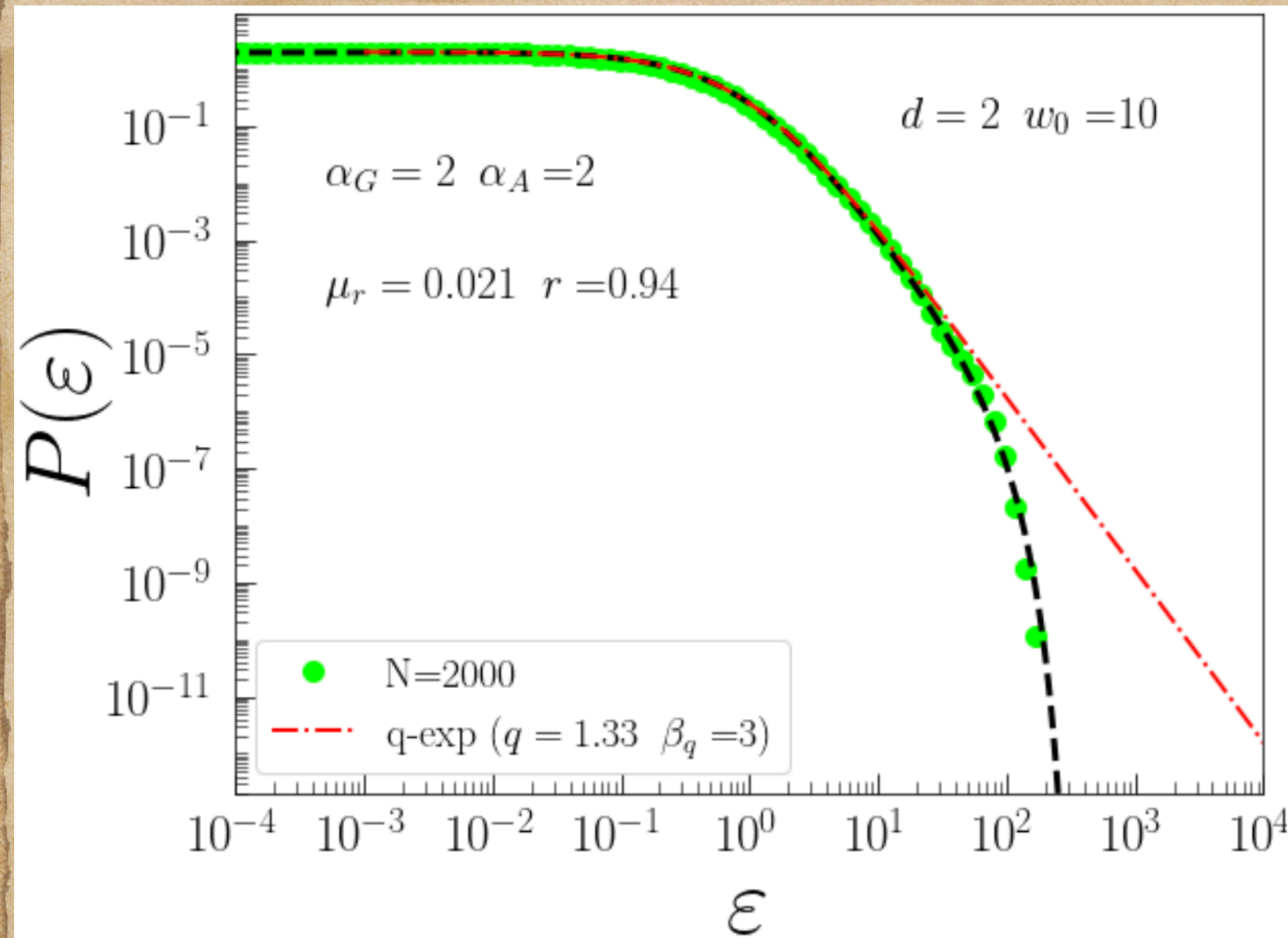
Consequently we have the solution as

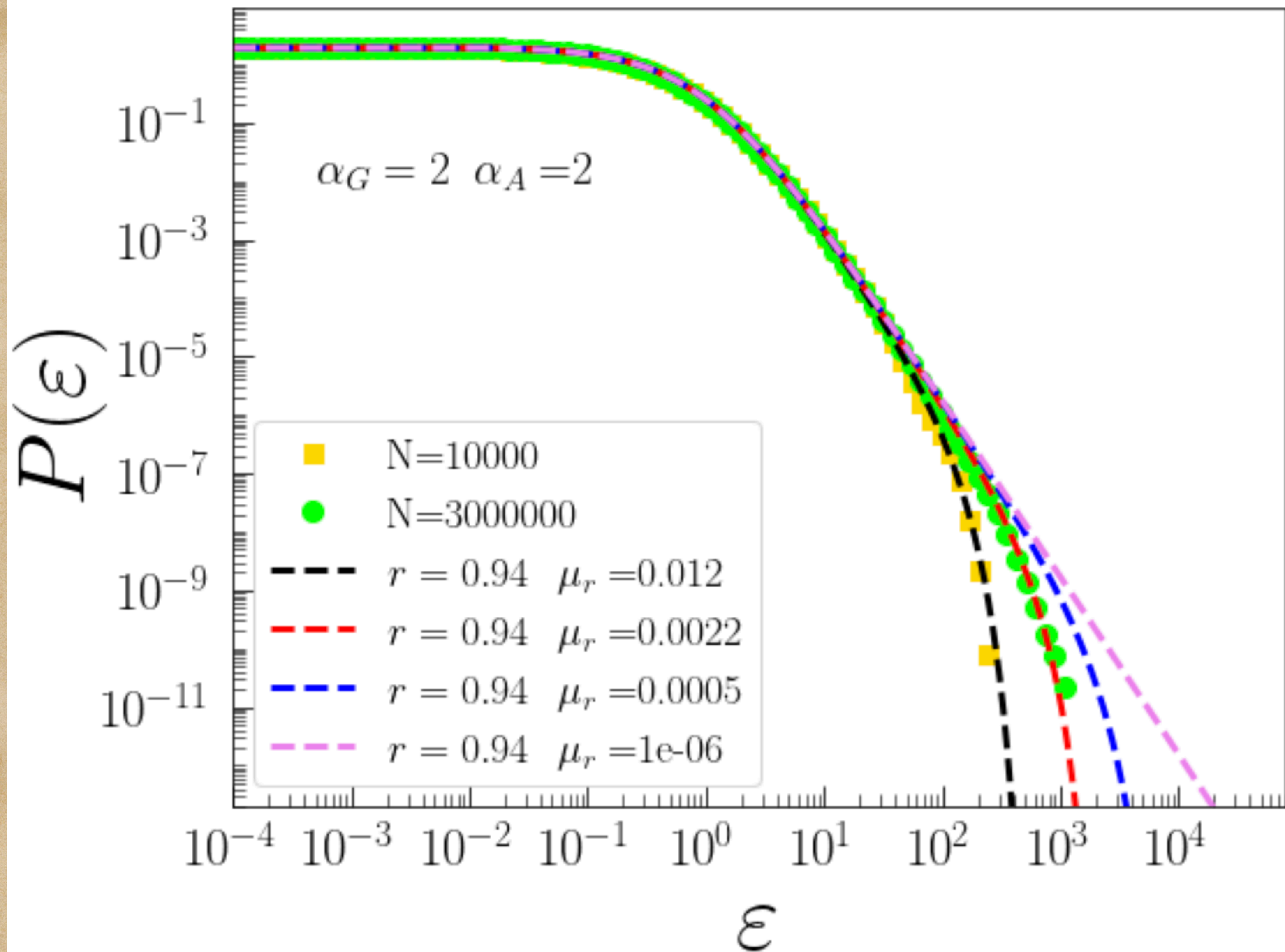
$$\varepsilon = \int_{\xi}^1 \frac{dx}{\mu_r x^r + (\beta_q - \mu_r) x^q}$$



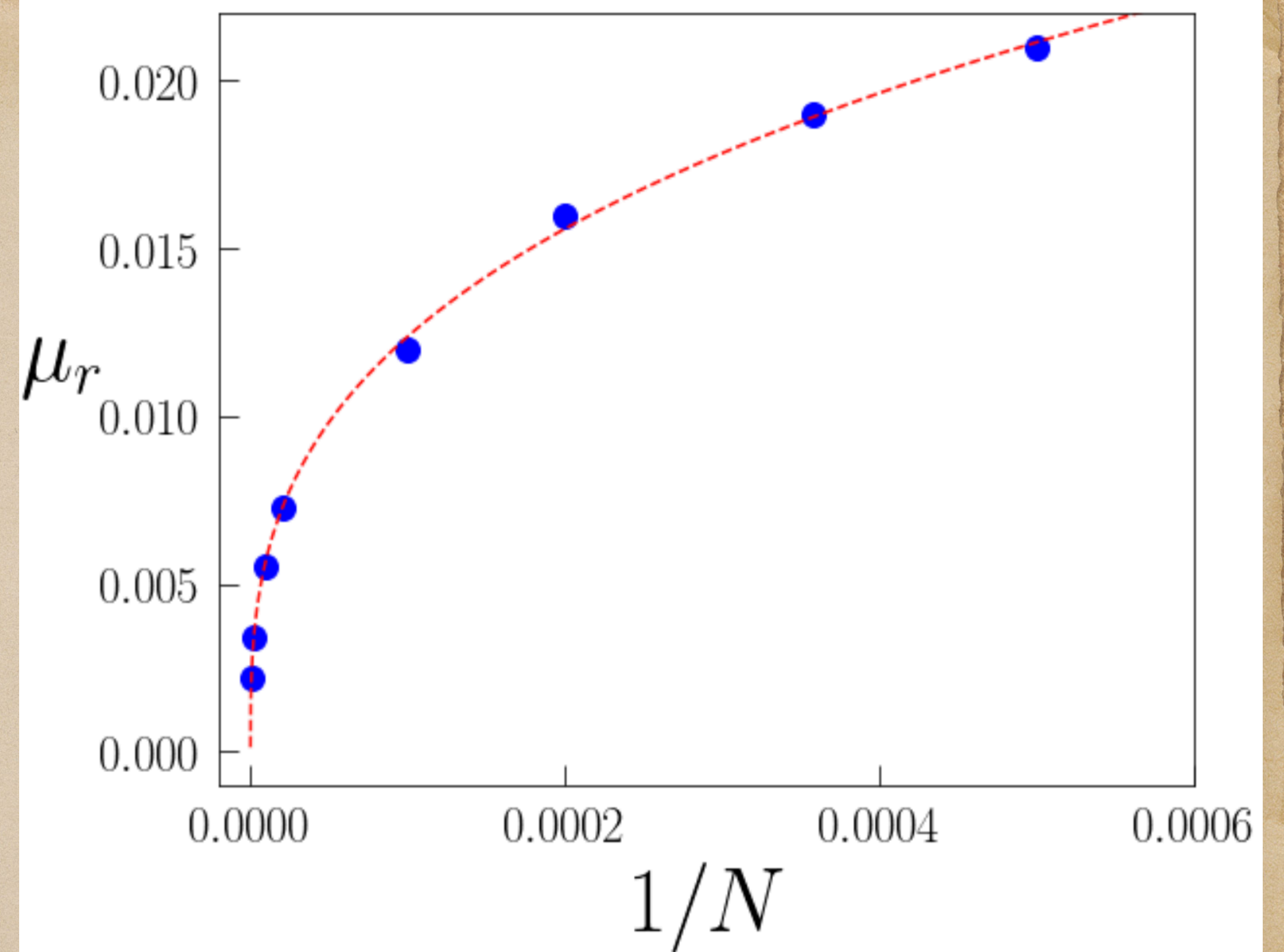
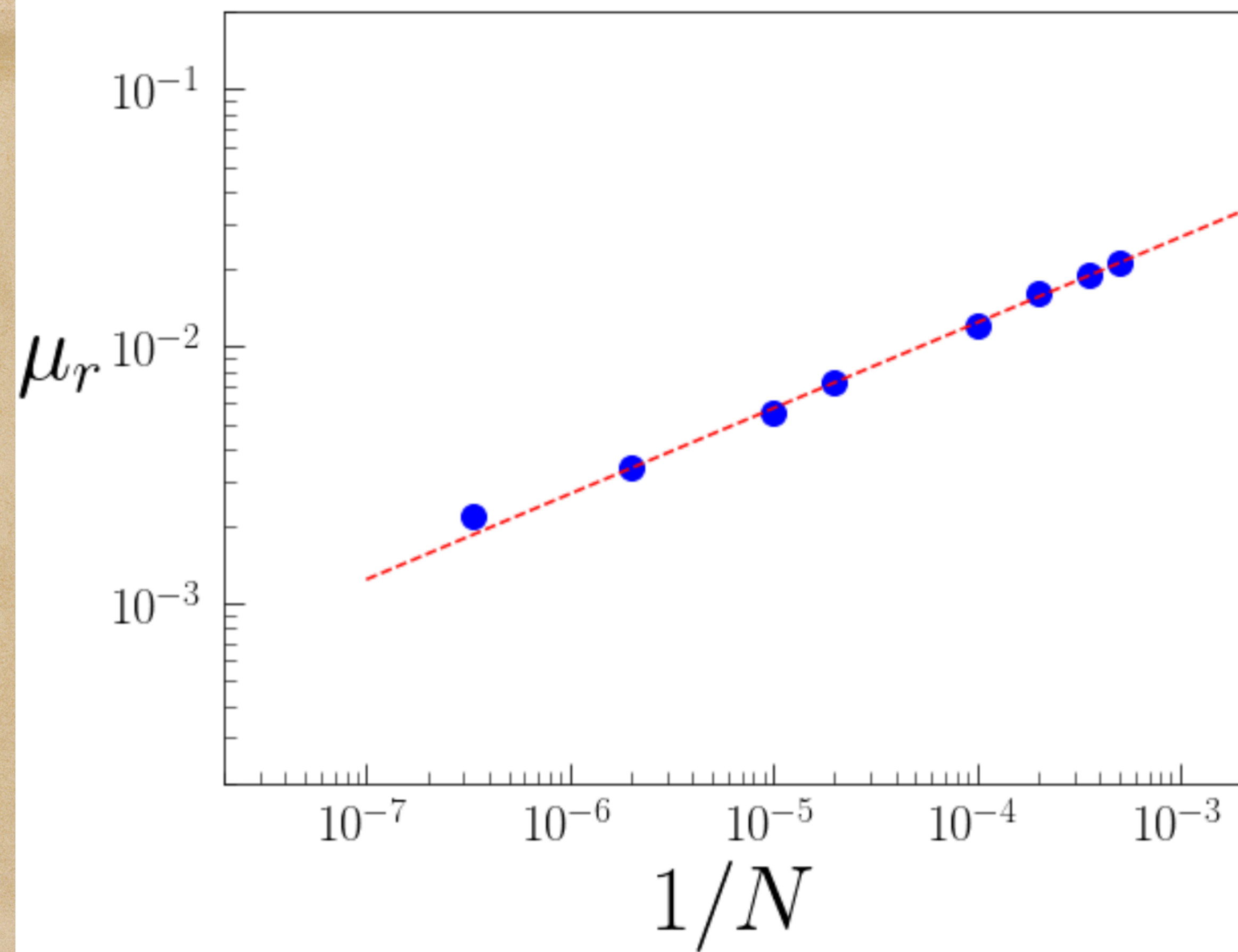
Numerically solve  
this integral







As  $N$  grows, the tendency to the exact  $q$ -exponential is evident.

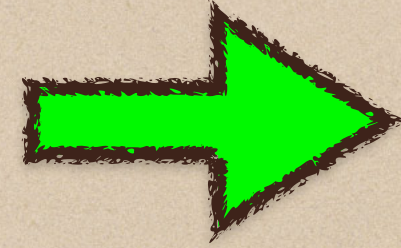


As  $N$  grows, the tendency to the exact  $q$ -exponential is more evident here.

Tirnakli & Tsallis, preprint (2023), in preparation.

# EPISODE 5

## Exploring crossover phenomenon



Tirnakli & Tsallis, preprint (2023), in preparation.

# CROSSOVER PHENOMENON

REMEMBER → Conjecture:

finite-size effects can satisfactorily be described by the Eq.:

$$\frac{d\xi}{d\varepsilon} = -\mu_r \xi^r - (\beta_q - \mu_r) \xi^q \quad (r \leq q; \varepsilon \geq 0)$$

where

$$\xi(\varepsilon) = \frac{p(\varepsilon)}{(2 - q) \beta_q}$$

Consequently we have the solution as

$$\varepsilon = \int_{\xi}^1 \frac{dx}{\mu_r x^r + (\beta_q - \mu_r) x^q}$$

Numerically solve  
this integral

# GROWTH MODEL

→ Locate each site using probability  $P_G(r) \propto \frac{1}{r^{d+\alpha_G}}$

→ Choose a random number from a distribution  $P(w)$  which gives the corresponding link weight.

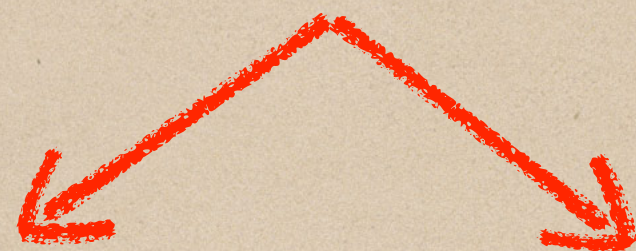
Energy of a site depends on how many links it has ( $k_i$ ) and the widths of those links ( $\{w_{ij}\}$ ).

→ Sites  $i = 3, 4, \dots$  will be linked to the previous ones with probability

$$\cancel{\Pi_{ij} \propto \frac{\varepsilon_i}{r_{ij}^{\alpha_A}}} \rightarrow \Pi_{ij} \propto \frac{\varepsilon_i}{r_{ij}^{\alpha_A} + c r_{ij}^{\gamma_A}}$$

# GROWTH MODEL

$$\Pi_{ij} \propto \frac{\varepsilon_i}{r_{ij}^{\alpha_A} + c r_{ij}^{\gamma_A}}$$



If  $c = 0$ , previous case is obtained:

$$\Pi_{ij} \propto \frac{\varepsilon_i}{r_{ij}^{\alpha_A}}$$



This corresponds one  $q$ -exponential depending on  $\alpha_A$ .

If  $c$  is too large, then

$$\Pi_{ij} \propto \frac{\varepsilon_i}{c r_{ij}^{\gamma_A}}$$



This corresponds another  $q$ -exponential depending on  $\gamma_A$ .

# CROSSOVER PHENOMENON

Conjecture:

crossover phenomenon together with finite-size effects can satisfactorily be described by the equation:

$$\frac{d\xi}{d\varepsilon} = -\mu_r \xi^r - (\beta_q - \mu_r) \xi^q - (\beta_{q'} - \beta_q + \mu_r) \xi^{q'}$$

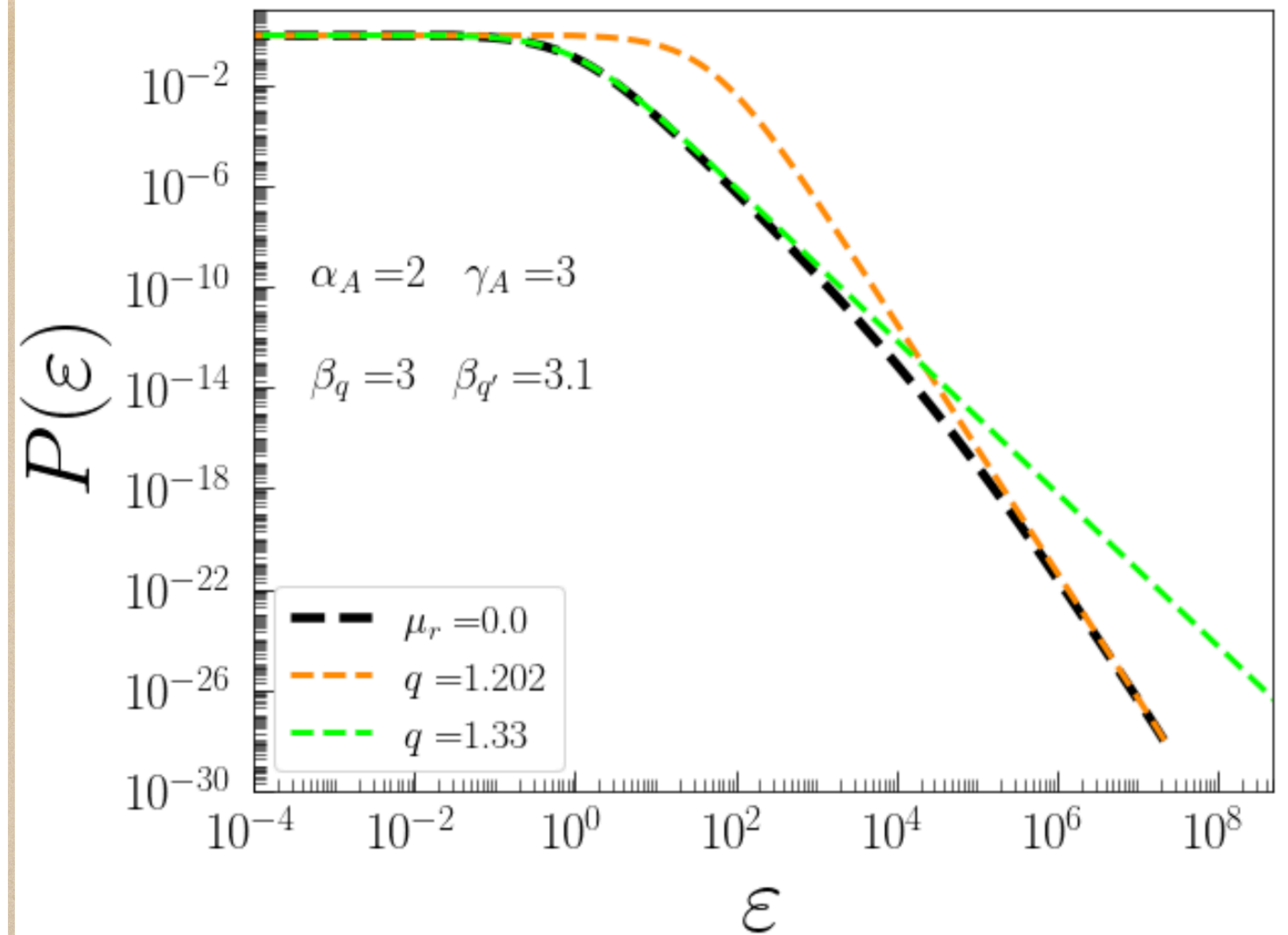
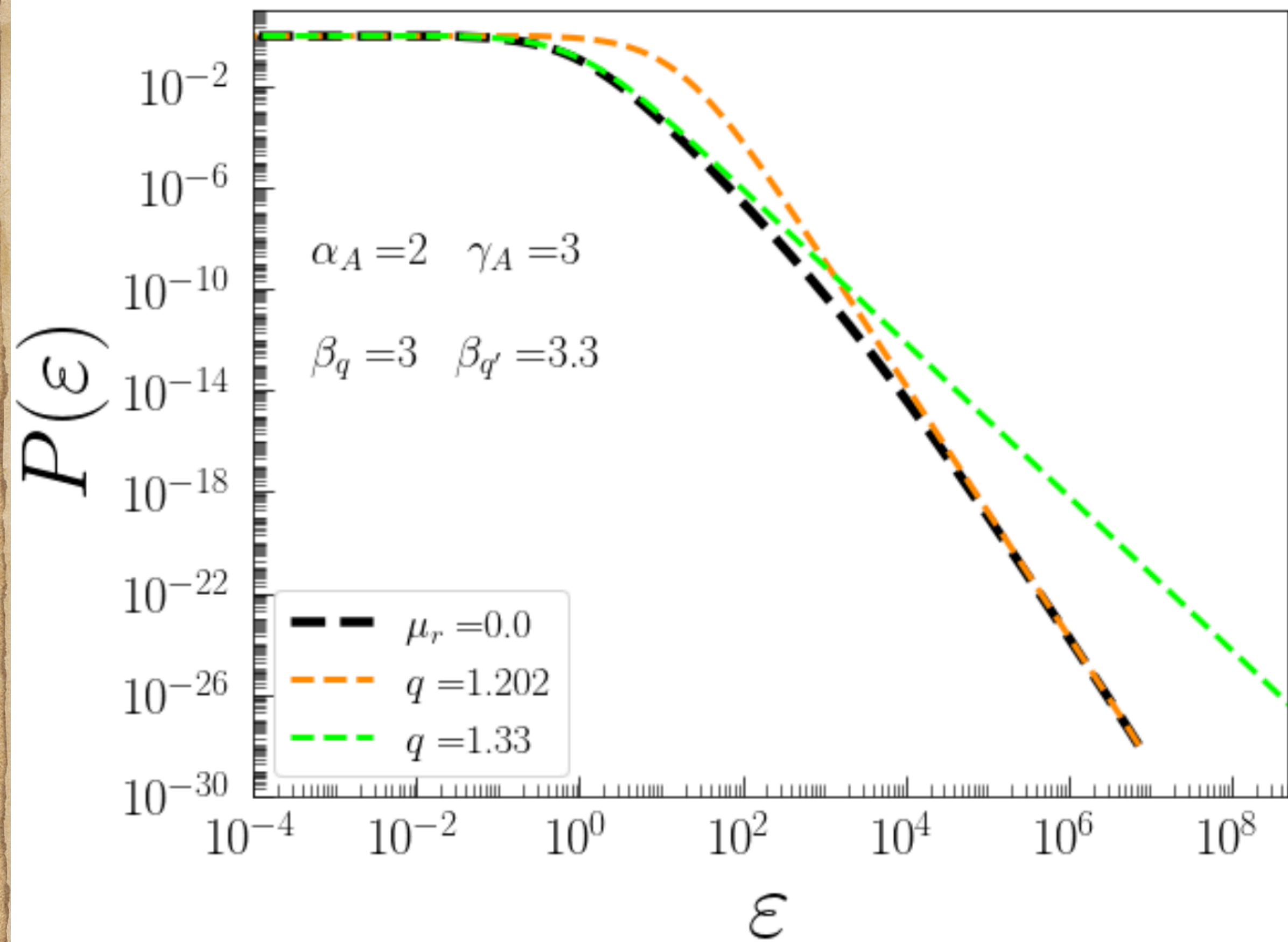
where  $\xi(\varepsilon) = \frac{p(\varepsilon)}{(2-q)\beta_q}$

Consequently we have the solution as

$$\varepsilon = \int_{\xi}^1 \frac{dx}{\mu_r x^r + (\beta_q - \mu_r) x^q + (\beta_{q'} - \beta_q + \mu_r) x^{q'}}$$

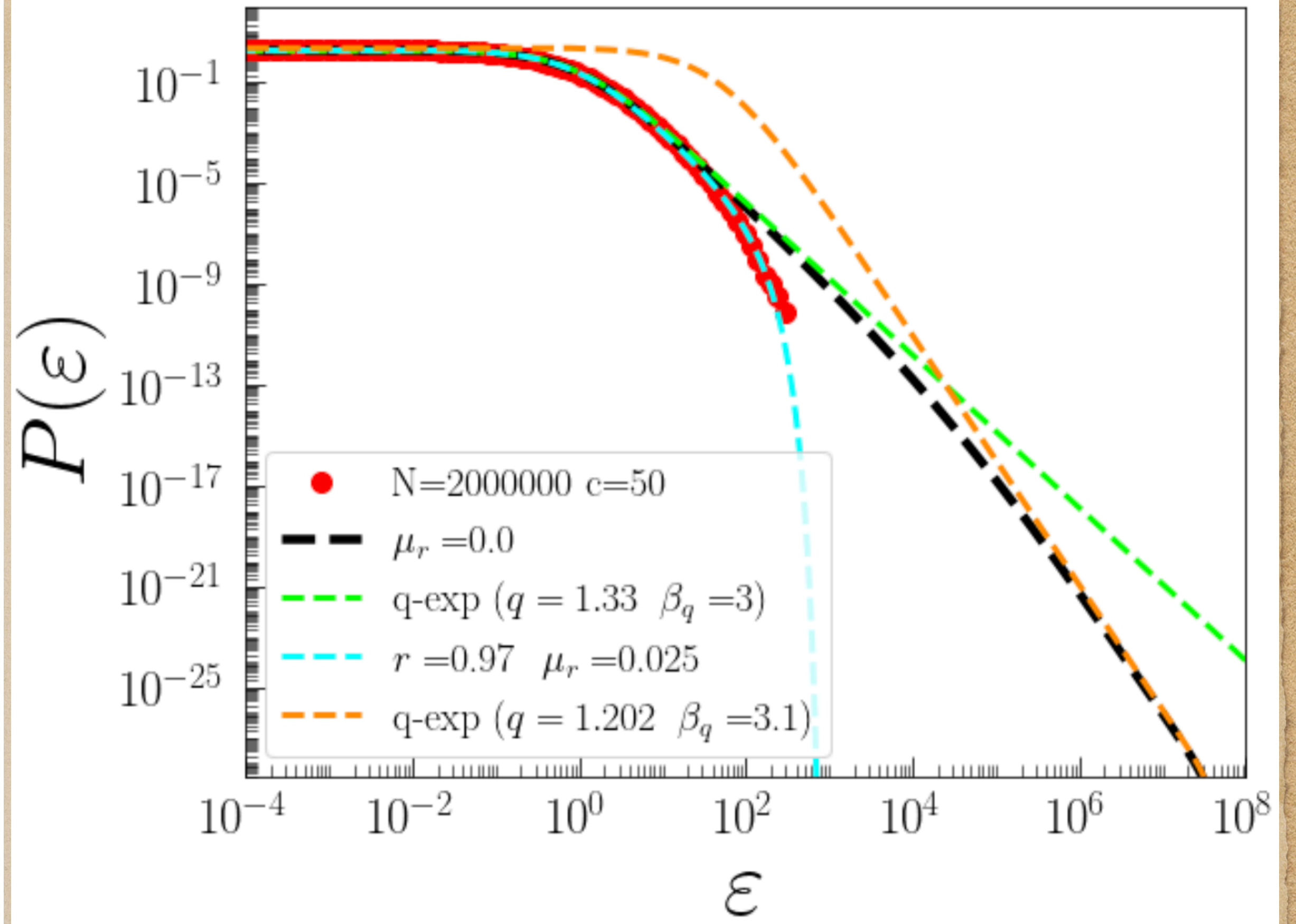
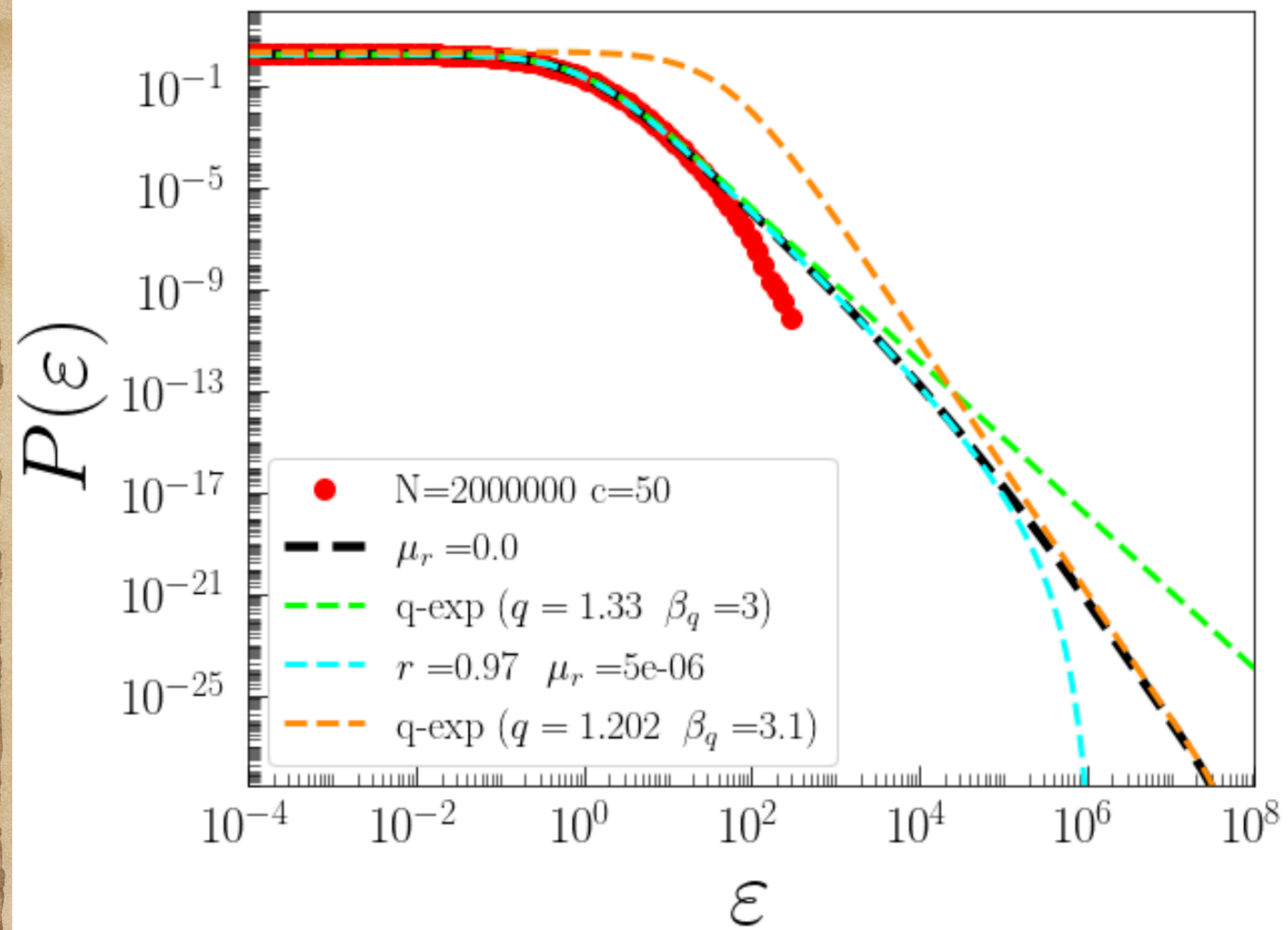
Numerically solve this integral





$q = 1.333$  if  $\alpha_A = 2$  for  $d = 2$

$q = 1.202$  if  $\gamma_A = 3$  for  $d = 2$



... more to come soon ...



*entropy*

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## Nonadditive Entropies and Nonextensive Statistical Mechanics—Dedicated to Professor Constantino Tsallis on the Occasion of his 80th Birthday

### **Guest Editors**

Ugur Tirnakli, Christian Beck, Hans J. Herrmann, Airton Deppman, Henrik Jeldtoft Jensen, Evaldo M. F. Curado, Fernando D. Nobre, Angelo Plastino, Astero Provata and Andrea Rapisarda

### **Deadline**

31 December 2023

[mdpi.com/si/146224](https://mdpi.com/si/146224)

# Special Issue

Invitation to submit

## Nonadditive Entropies and Nonextensive Statistical Mechanics—Dedicated to Professor Constantino Tsallis on the Occasion of his 80th Birthday

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**Fernando D. Nobre**

**Angelo Plastino**

**Astero Provata**

**Andrea Rapisarda**

Deadline for  
manuscript submissions:  
**31 December 2023**

### Message from the Guest Editors

The aim of this Special Issue is to collect original research articles on the most recent research in nonadditive entropies and nonextensive statistical mechanics with their applications in physics and elsewhere, as well as comprehensive review articles covering these topics from a theoretical, experimental, or computational viewpoint.

This generalization of the centennial Boltzmann-Gibbs statistical mechanics and of the entropy upon which it is based were proposed in 1988 and have received, since then, many applications in natural, artificial, and social sciences. The undeniable success of the Boltzmann-Gibbs theory is deeply related to strongly chaotic nonlinear dynamical systems. In particular, for classical systems, the standard requirement is that the maximal Lyapunov exponent is positive. At the edge of chaos, where the maximal Lyapunov exponent vanishes, the need emerges for nonadditive entropies and consistent generalizations of quantities such as the Maxwellian distributions of velocities, the celebrated Boltzmann-Gibbs weight for energies, and Pesin-like identities. This generalized theory has received uncountable validations in complex systems.

Professor Constantino Tsallis has had an outstanding global impact on physics, astrophysics, geophysics, economics, mathematics, and computational sciences, among others. In recognition of his extraordinarily creative and productive scientific life and innumerable contributions to the field of statistical physics of complex systems, this Special Issue is dedicated to him on the occasion of his 80th birthday (5 November 2023).



[mdpi.com/si/146224](https://mdpi.com/si/146224)

# Special Issue

### Editor-in-Chief

**Prof. Dr. Kevin H. Knuth**

Department of Physics,  
University at Albany, 1400  
Washington Avenue, Albany,  
USA

### Message from the Editor-in-Chief

The concept of entropy is traditionally a quantity in physics that has to do with temperature. However, it is now clear that entropy is deeply related to information theory and the process of inference. As such, entropic techniques have found broad application in the sciences.

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# MY TALK

## STATISTICAL MECHANICS FOR COMPLEXITY

A CELEBRATION OF THE 80TH BIRTHDAY OF CONSTANTINO TSALLIS

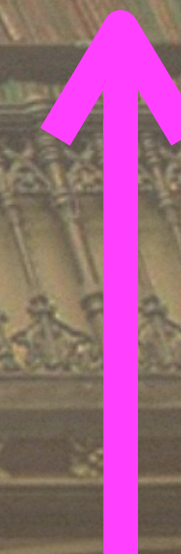
~~1<sup>st</sup> part~~

RIO DE JANEIRO, 6 TO 10 NOVEMBER 2023

2<sup>nd</sup> part

~~about PHYSICS~~

about HISTORY



# A SCIENTIFIC LIFE WITH CONSTANTINO

Story begins in **1997** when he visited us in Izmir during my PhD (no photo is available from that archaic time 😊).

Then, at the end of my PhD, I received a scholarship from TUBITAK for a short visit to CBPF in **1998**.

Eur. Phys. J. B 11, 309–315 (1999)

**1<sup>st</sup> joint paper**

THE EUROPEAN  
PHYSICAL JOURNAL B

EDP Sciences  
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Springer-Verlag 1999

**Circular-like maps: sensitivity to the initial conditions, multifractality and nonextensivity**

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<sup>2</sup> Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro - RJ, Brazil

<sup>3</sup> Departamento de Física, Universidade Federal de Alagoas, 57072-970 Maceio-AL, Brazil

**Rio 1998**



# A SCIENTIFIC LIFE WITH CONSTANTINO

During my 1-month stay there, we have discussed various subjects but one day he told me:

“Ugur, if you continue your career in statistical mechanics, surely you will not be a rich guy, but I assure you that you will enjoy it”.!!!



# A SCIENTIFIC LIFE WITH CONSTANTINO

Then he immediately offered to me a Post-doc position supported by CNPq (and also by TUBITAK).

Then, I end up in a one-year Postdoc position in CBPF in **1999-2000**.  
(starting to enjoy life 🥰👁️)





# A SCIENTIFIC LIFE WITH CONSTANTINO

After finishing my post-doc period there, I became an Ass. Prof. in Izmir at Ege Uni.

But, we keep on collaborating since then without any interruption.

During the Conference

“International School and Conference on  
Nonextensive Thermodynamics and Physical  
Applications”, Villasimius, Italy, **Summer 2001.**



# A SCIENTIFIC LIFE WITH CONSTANTINO

During the Workshop  
“Trends and Perspectives on Extensive and  
Non-extensive Statistical Mechanics”,  
Angra dos Reis, Brazil, 2003.



# A SCIENTIFIC LIFE WITH CONSTANTINO



During the Conference  
“Complexity and Nonextensivity: New  
Trends in Statistical Mechanics”,  
Kyoto, Japan, **March 2005.**

... several conferences ...

During the Conference  
“Complexity, Metastability and  
Nonextensivity”, Erice, Italy,  
**Summer 2004.**



# A SCIENTIFIC LIFE WITH CONSTANTINO



During the Conference  
“LAWN’09”, Buzios,  
2009.

... more  
conferences ...

During the Conference  
“STATPHYS”, Genova, Italy, 2007.



# A SCIENTIFIC LIFE WITH CONSTANTINO

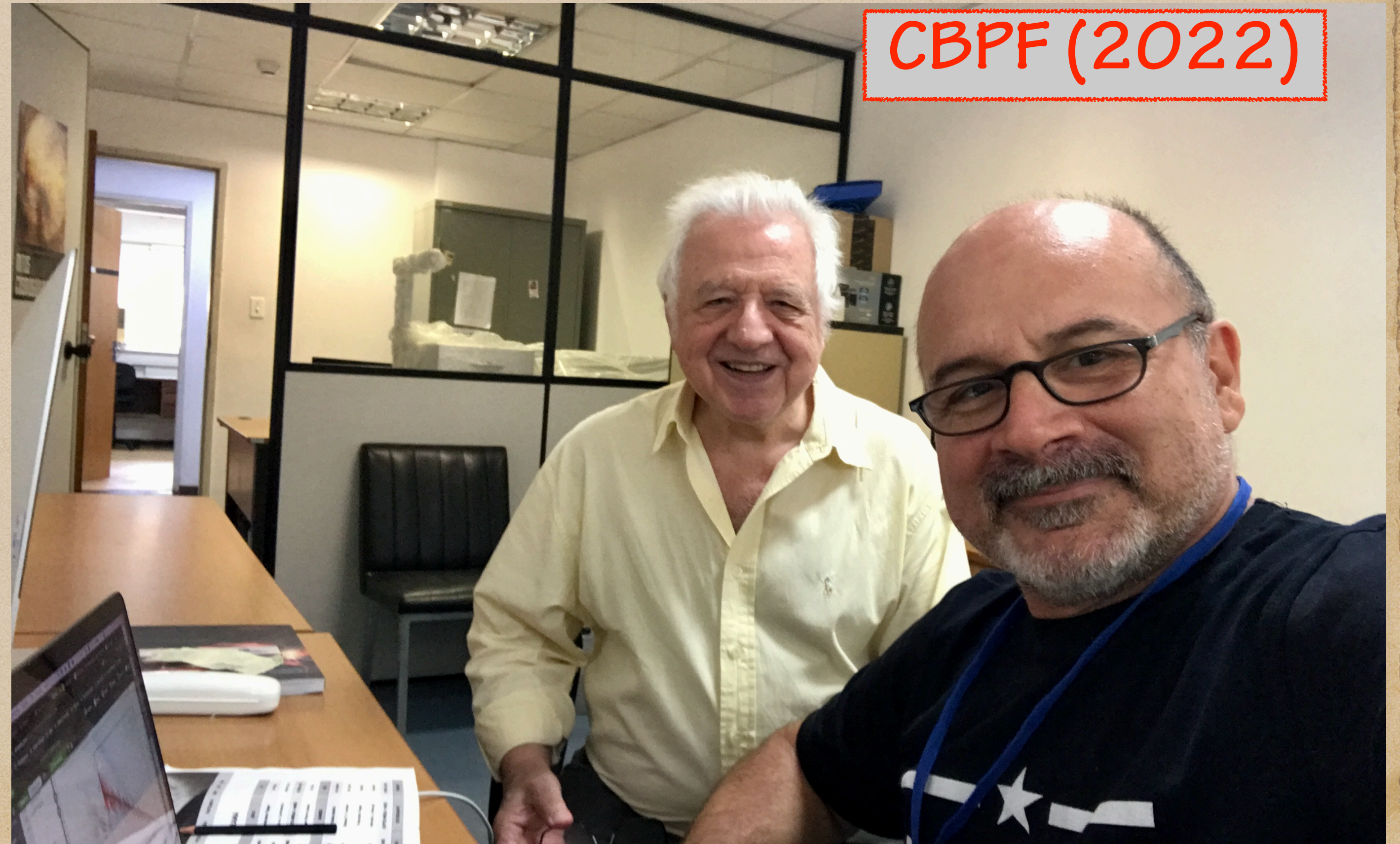
... as you see, sometimes we were working hard ...

During the Conference  
“School and Conference on Complex  
Systems and Nonextensive Statistical  
Mechanics”, Trieste, Italy, **Summer 2006.**



# A SCIENTIFIC LIFE WITH CONSTANTINO

... not only in those days we were young, bu also recently as we are getting older ...



# A SCIENTIFIC LIFE WITH CONSTANTINO

... but there are even other difficult times that we were even working harder ...

During the Conference

“3. International Conference NEXT-SigmaPhi:  
News, Expectations and Trends in Statistical  
Physics”, Crete, Greece, **Summer 2005**.



# A SCIENTIFIC LIFE WITH CONSTANTINO

... even harder ...

During the Conference  
“Greek-Turkish Meeting on  
Statistical Mechanics and Dynamical  
Systems”, Turunç, Turkey, **Summer 2008.**





# A SCIENTIFIC LIFE WITH CONSTANTINO

... even exceptionally harder ...



During the Conference  
“International Workshop on Foundations of Complexity”, Rio, Brazil, **October 2015**.



During the Conference  
“International School on Complexity”, Erice, Italy, **2015**.

# A SCIENTIFIC LIFE WITH CONSTANTINO

... as time goes by, we arrived 2013, remember?



# A SCIENTIFIC LIFE WITH CONSTANTINO

... then we realized that we are getting older, and decided to work harder ...

Rio 2016



Bodrum 2019



Istanbul 2019



# A SCIENTIFIC LIFE WITH CONSTANTINO

... believe it or not, even harder ...



Chios 2019



Rio 2019



Crete 2023

Istanbul 2019



# A SCIENTIFIC LIFE WITH CONSTANTINO

... but don't think that we are just workaholic guys, we are also very good in sports ...



# A SCIENTIFIC LIFE WITH CONSTANTINO

... and finally ...

Tsallis 80



Lagoa, Rio de Janeiro,  
2023 (yesterday) 🎉

# A SCIENTIFIC LIFE WITH CONSTANTINO

... and if I come back to **my first slide** ...

During my 1-month stay there, we have discussed various subjects but one day he told me:

“Ugur, if you continue your career with statistical mechanics, surely you will not be a rich guy, but I assure you that you will enjoy it” .!!!

**YES CONSTANTINO,  
ONCE AGAIN, YOU WERE RIGHT !! WE REALLY ENJOYED OUR LIFES,  
AS YOU PROMISED, AND IT WAS REALLY A PLEASURE AND PRIVILEGE  
FOR ME TO WALK TOGETHER DURING ALL THESE 26 YEARS.**

Rio 1997



!! 26 years !!  
(20 papers)

Rio 2023



obrigadao