Stochastic Bragg scattering in PT-symmetric photonic lattices

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Statistical Mechanics for complexity
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~ 1979 Kings College, London

- 1986 UFAL

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The bond-diluted interface between semi-infinite Potts bulks: criticality

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Abstract. Within a real-space renormalisation group framework, we discuss the criticality of a system constituted by two (not necessarily equal) semi-infinite ferromagnetic q-state Potts bulks separated by an interface. This interface is a bond-diluted Potts ferromagnet with a coupling constant which is in general different from those of both bulks. The phase diagram presents four physically different phases, namely the paramagnetic one, and the surface, single-bulk and double-bulk ferromagnetic ones. These various phases determine a multicritical surface which contains a higher-order multicritical line. Particular attention is devoted to the discussion of the critical concentration p_c. Here, p_c is the concentration of the interface bonds above which surface magnetic ordering is possible even if the bulks are disordered. An interesting feature comes out which is that p_c varies continuously with J_1/J_s and J_2/J_s. The standard two-dimensional percolation concentration is recovered for $J_1 = J_2 = 0$. From the analysis of the various fixed points obtained within the present formalism, a very rich set of critical universality classes emerges.

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Modulation instability in the region of minimum group-velocity dispersion of single-mode optical fibers via an extended nonlinear Schrödinger equation

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Modulation instability in the region of the minimum group-velocity dispersion is analyzed by means of an extended nonlinear Schrödinger equation. It is shown that the critical modulation frequency saturates at a value determined by the fourth-order dispersion. Experimental results demonstrate the viability of generating a train of femtosecond pulses with repetition rates of a few terahertz in reasonable agreement with the theory.

PHYSICAL REVIEW A VOLUME 51, NUMBER 5 MAY 1995

Noise amplification in dispersive nonlinear media

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(Received 4 June 1993; revised manuscript received 21 November 1994)

The propagation of a partially coherent optical beam through dispersive nonlinear media is investigated theoretically by using a phase-diffusion model for the laser beam. Changes in the second-order statistical properties during beam propagation depend on whether the nonlinear medium exhibits normal or anomalous group-velocity dispersion. In the case of normal dispersion, the coherence function and the corresponding optical spectrum remain unaffected. By contrast, modulation instability is found to be responsible for noise amplification in the anomalous dispersion regime, enhancing phase fluctuations and causing spectral distortion as well as coherence degradation. Under certain conditions, phase fluctuations exhibit temporal oscillations that lead to the characteristic spectral sidebands associated with modulation instability. The nonlinear Schrödinger equation is solved numerically to study the propagation regime in which the analytic theory becomes invalid.

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Valuable Team

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Outline

- Bragg scattering: power oscillations (coupled waves+ Gaussian num.)
  Hermitian x PT-symmetric
- Non-Hermitian photonics (PT-symmetry)
  in QFT, in Optics
- Results:
  non-reciprocity, non-trivial behavior at the critical point,
- Cross - spectral density approach
  Gaussian-Schell source, spectral degree of coherence
- Scattering of partially coherent light by a PT-symmetric medium

Optical Rabi Oscillations

1920 OPTICS LETTERS / Vol. 32, No. 13 / July 1, 2007

Bragg-resonance-induced Rabi oscillations in photonic lattices

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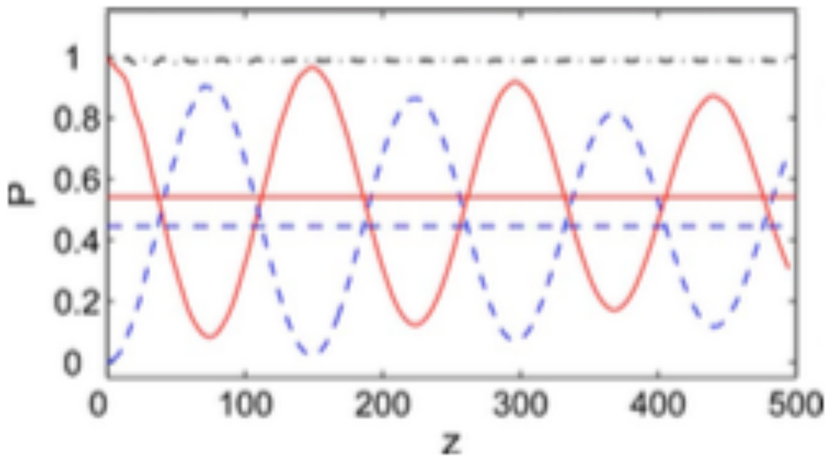


Fig. 2. (Color online) Fourier peak powers P_1 at $\boldsymbol{k}=(k_B,0)$ (solid line) and P_2 at $\boldsymbol{k}=(-k_B,0)$ (dashed line), their average values (straight lines), and the sum P_1+P_2 (dashed–dotted line). Here $V_0=0.05$, $\varepsilon=1$ and $E(\mathbf{r},0)=1/\sqrt{\pi}\sigma\exp\{ik_Bx-\mathbf{r}^2/(2\sigma^2)\}$ with $\sigma=50$.

Propagation Equation

described by the dimensionless paraxial wave equation

$$i\frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial x^2} - V(x)\psi = 0, \quad (1)$$

in arbitrary units. Let us define a potential function of the form $V(x) = \alpha[\cos^2 x + i\beta \sin(2x)]$, with α, β representing real parameters (with β positive). It is easy to verify that the so-defined potential satisfies $V(-x)^* = V(x)$, and there is a gain-loss parameter β that controls the degree of Hermiticity. This parameter defines a spontaneous symmetry-breaking point, $\beta_c = 1/2$, above which the spectrum undergoes a phase

Quantum field theory

Within the context of field theory

<u>Axiom of QM</u>

$H = H^+$ Hermitian $\longrightarrow$ $\varepsilon = \varepsilon^*$

✓ energy spectrum is real & time evolution is unitary (probability-preserving).

however...

Non-Hermitian Hamiltonians

Making sense of non-Hermitian Hamiltonians
Carl M Bender
Published 30 May 2007 • 2007 IOP Publishing Ltd
[Reports on Progress in Physics, Volume 70, Number 6]

(space-time reflection)

APS/Alan Stonebraker

Unbroken PT symmetry

REAL SPECTRUM !!!!

PT symmetric Hamiltonians

PT = Parity time symmetry

P spatial inversion

X → -X P → -P

T time reversal

X → X P → -P i → -i

PT

X → -X P → P i → -i

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial^2 x}\right) + V(\mathbf{x})\psi$$

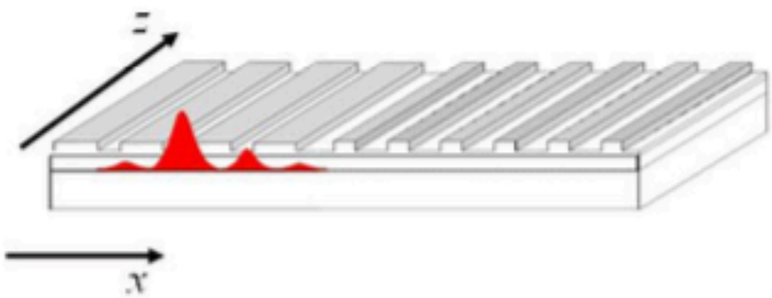
Schrödinger's equation

symmetric Hamiltonian
$(PT)^{-1}$ H (PT)= H → $V(x) = V^*(-x)$

PT symmetry condition

Optical analogy

Paraxial Optics

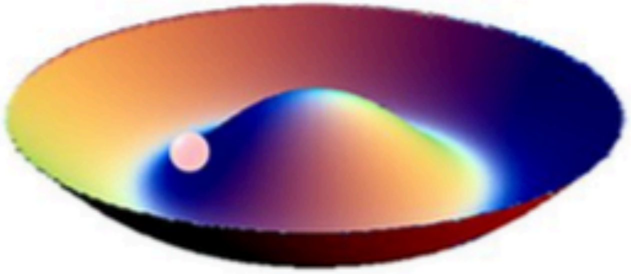


Paraxial equation of diffraction

$$i\frac{\partial E}{\partial z}+\frac{1}{2k}\frac{\partial^2 E}{\partial x^2}+k_0 n(x)E=0$$

Propagation constants

Quantum Mechanics



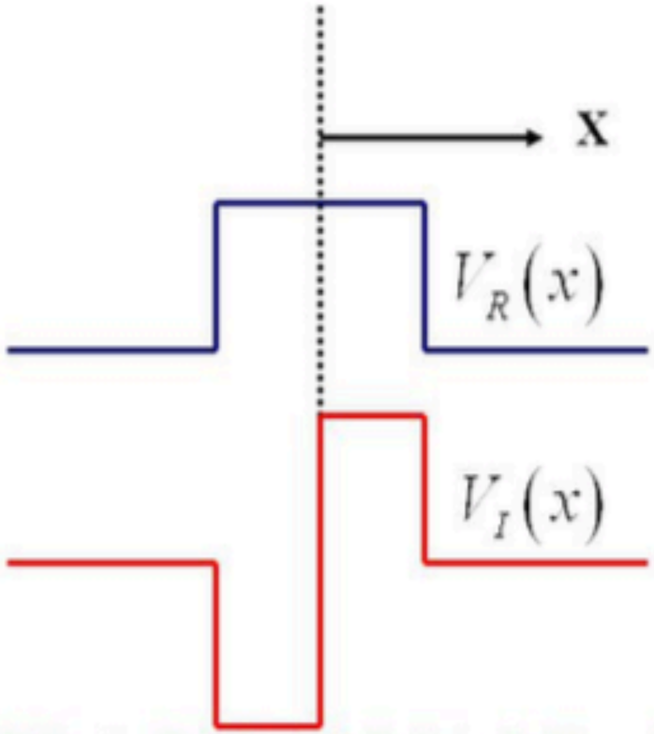
Schrödinger equation

$$i\hbar\frac{\partial \psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}+V(x)\psi$$

Energy eigenvalues

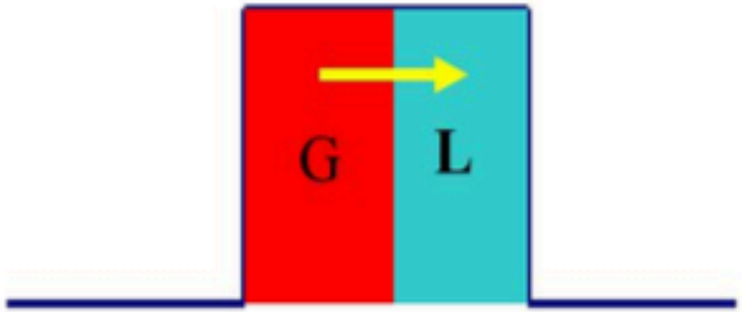
PT symmetry in optics

$$V(x) = V^*(-x)$$ *PT*-symmetric potential



Typical parameters

$0.5\mu m < \lambda_0 < 1.6\mu m$	$\Delta n_R^{max} \approx 10^{-3}$
$g = -a \approx 30cm^{-1}$	$\Delta n_I^{max} \approx 5 \times 10^{-4}$



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PT- symmetric experimental optics

PRL **103,** 093902 (2009) PHYSICAL REVIEW LETTERS week ending 28 AUGUST 2009

Observation of $\mathcal{PT}$-Symmetry Breaking in Complex Optical Potentials

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LETTERS
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nature physics

Observation of parity–time symmetry in optics

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Demonstration of a parity–time symmetry breaking phase transition using superconducting and trapped-ion qutrits

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(Dated: November 1, 2023)

Scalable quantum computers hold the promise to solve hard computational problems, such as prime factorization, combinatorial optimization, simulation of many-body physics, and quantum chemistry. While being key to understanding many real-world phenomena, simulation of non-conservative quantum dynamics presents a challenge for unitary quantum computation. In this work, we focus on simulating non-unitary parity-time symmetric systems, which exhibit a distinctive symmetry-breaking phase transition as well as other unique features that have no counterpart in closed systems. We show that a qutrit, a three-level quantum system, is capable of realizing this non-equilibrium phase transition. By using two physical platforms – an array of trapped ions and a superconducting transmon – and by controlling their three energy levels in a digital manner, we experimentally simulate the parity–time symmetry-breaking phase transition. Our results indicate the potential advantage of multi-level (qudit) processors in simulating physical effects, where additional accessible levels can play the role of a controlled environment.

PHYSICAL REVIEW A **96**, 053841 (2017)

Bragg-induced power oscillations in $\mathcal{PT}$-symmetric periodic photonic structures

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We study Bragg-induced power oscillations in Fourier space between a pair of optical resonant transverse modes propagating through a periodic $\mathcal{PT}$-symmetric lattice, represented by a refractive index that includes gain and loss in a balanced way. Our results imply that the $\mathcal{PT}$-symmetric system shows exceptionally rich phenomena absent in its Hermitian counterpart. It is demonstrated that the resonant modes exhibit unique characteristics, such as Bragg power oscillations controlled via the $\mathcal{PT}$ symmetry, severe asymmetry in mode dynamics, and trapped enhanced transmission. We have also performed numerical simulations in (1+1) and (2+1) dimensions of propagating Gaussian beams to compare with analytical calculations developed under a two-waves model.

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Two-waves model

equation by writing the field as

$$\psi(x,z) = \sum_{n=\pm 1,\pm 2...} \psi_n(z)\exp(ink_b x), \quad (2)$$

where $k_b = 1$ is the resonant transverse wave vector and the index n indicates the nth mode that is Bragg resonant with the lattice. Substituting (2) into (1), one arrives at the following set of coupled differential equations:

$$i\frac{d\psi_n}{dz} = a_n\psi_n + b\psi_{n-2} + c\psi_{n+2} \quad (3)$$

$$a_n = \left(n^2 + \frac{\alpha}{2}\right) \qquad b = \frac{\alpha}{2}(\beta_c + \beta), \qquad c = \frac{\alpha}{2}(\beta_c - \beta).$$

Non-reciprocity

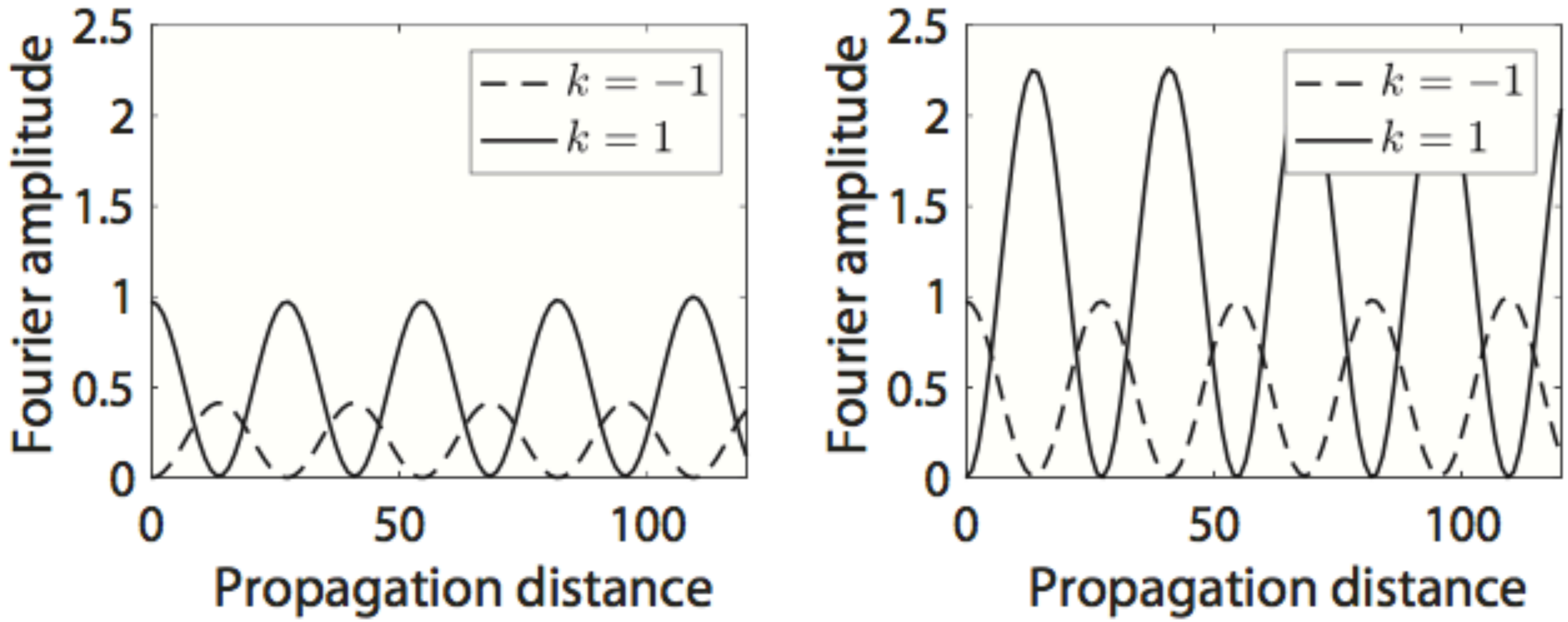


FIG. 2. Gaussian beam propagation below the phase transition point ($\beta < \beta_c$) in both real (upper panels) and Fourier spaces (lower panels), illustrating power oscillations with $\beta = 0.2$ and $\alpha = 0.5$. Left column: $\{\psi_{-1}(0), \psi_1(0)\} = \{0,1\}$. Right column: $\{\psi_{-1}(0), \psi_1(0)\} = \{1,0\}$. Excited modes outside the first Brillouin zone are indistinguishable from the horizontal zero axis line.

Wide Gaussian beam propagation at the critical phase point ($\beta = \beta_c$)

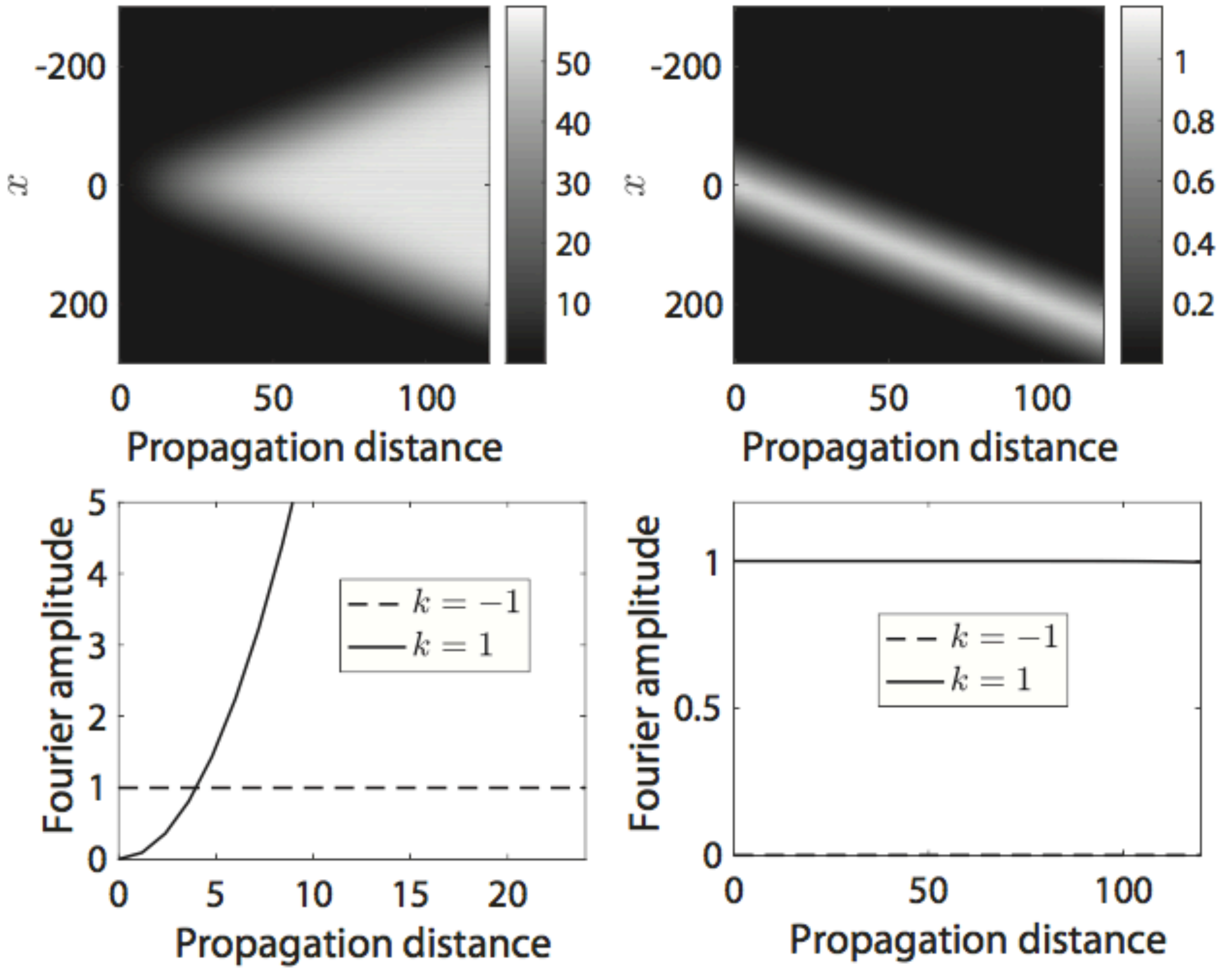


FIG. 3. Wide Gaussian beam propagation at the critical phase point ($\beta = \beta_c$) in both real (upper panels) and Fourier (lower panels) spaces, with initial width $X_0 = 15p$ (p is the lattice period) incident on a $\mathcal{PT}$-symmetric lattice at the critical point. Left column: $\{\psi_{-1}(0), \psi_1(0)\} = \{1,0\}$. Right column: $\{\psi_{-1}(0), \psi_1(0)\} = \{0,1\}$. Excited modes outside the first Brillouin zone are indistinguishable from the zero horizontal axis line.

Stochastic Theory

function. Under general conditions, likely to be valid in many systems of interest, the cross-spectral density of a statistical stationary source is defined as

$$W(x_1, x_2, z) = \langle \psi^*(x_1, z)\psi(x_2, z)\rangle_\omega, \qquad (10)$$

where $\langle\cdot\rangle_\omega$ implies an ensemble average of monochromatic realizations of the incident optical field.

Wave equation for the cross- spectral density

Let us begin by considering a statistically stationary optical field $\psi(x,z,t)$ characterized by its cross-spectral density $W_{12}(z,\omega) = W(x_1,x_2,z,\omega)$, defined as the Fourier transform of the mutual coherence function $\Gamma(x_1,x_2,\tau) = \langle\psi^*(x_1,t)\psi(x_2,t+\tau)\rangle$. The field varies in the (x,z) plane, with z being the main propagation direction, and ω the angular frequency, which we omit from now on. The cross-spectral density characterizes field correlations between the transverse positions x_1 and x_2 at frequency ω. Assuming that each element of the ensemble representing the optical field satisfies the paraxial wave equation

Effective potential

$i\psi_z(x,z) + \psi_{xx}(x,z) + V(x)\psi(x,z) = 0$, it is easy to demonstrate that the cross-spectral density evolves according to [11]

$$i\frac{\partial W_{12}(z)}{\partial z} + \left(\frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_1^2}\right) W_{12}(z) + \mathcal{V}_{12} W_{12}(z) = 0, \qquad \textbf{(1)}$$

where $\mathcal{V}_{12} = V(x_2) - V^*(x_1)$ is the effective potential representing the transverse variations of the refractive index profile $V(x)$ relative to a substrate where the heterogeneous material is deposited.

Gaussian-Schell Beam

The simplest nontrivial model of a random optical beam is the Gaussian-Schell beam, and therefore we choose one for which the initial correlation profile is given by,

$$W_{12}(0) = e^{-(x_1^2+x_2^2)/4\sigma^2} e^{-(x_1-x_2)^2/2\delta^2} e^{-iq(x_1-x_2)}, \qquad \textbf{(2)}$$

where σ is related to the initial beam width, δ is the coherence parameter and q the incident transverse wavevector. The beam is fully coherent in the limit $\delta \to \infty$. The initial spectral density is described by a Gaussian function $S(x,0) = W(x,x,0) = e^{-x^2/2\sigma^2}$. To numerically solve Eq.

"…sources of G-S beam are frequently encountered in nature and can readily be produced in the lab." (excerpt from *Introduction to the theory of coherence and polarization of light, by Emil Wolf, Cambridge* .)

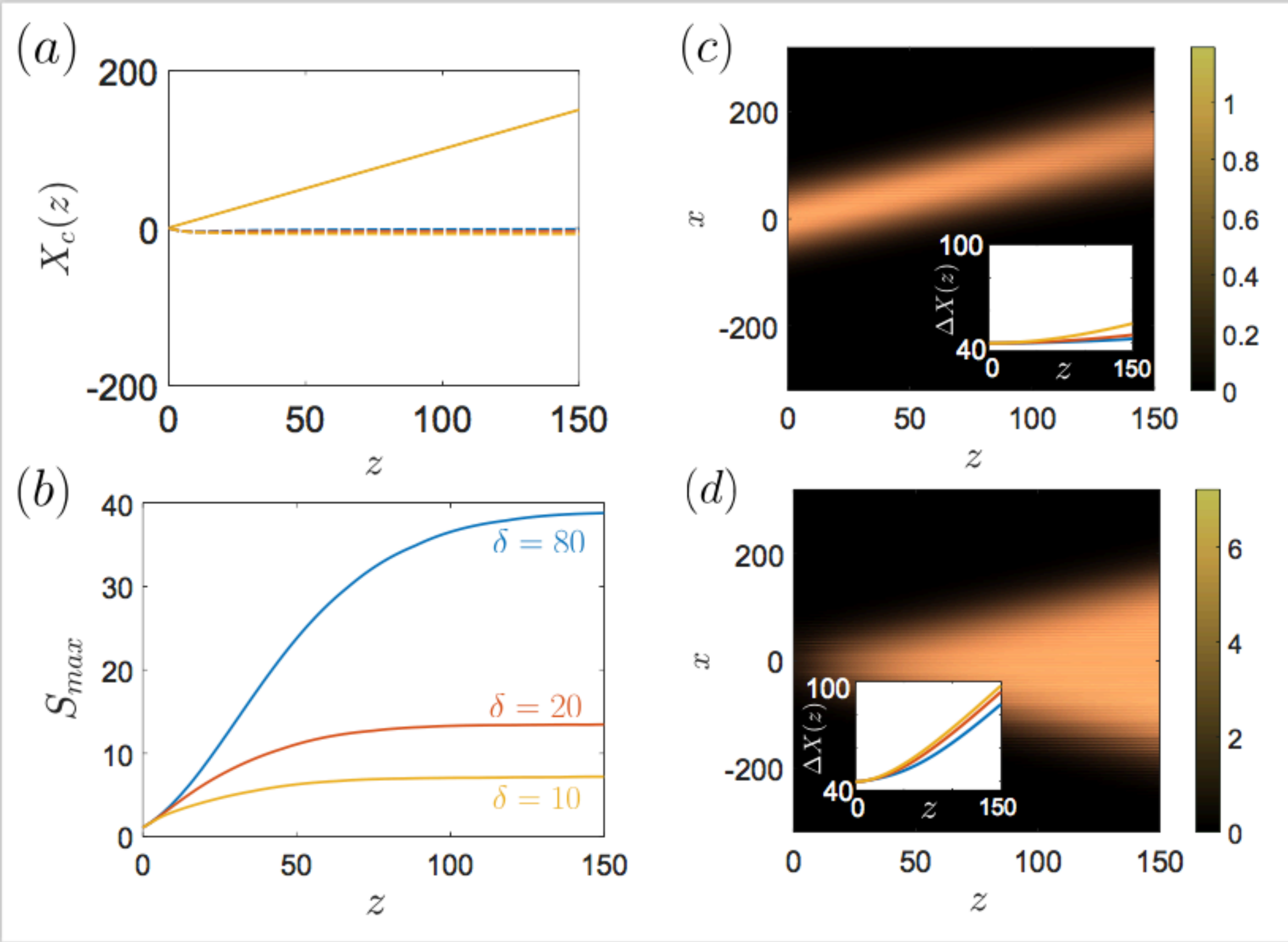
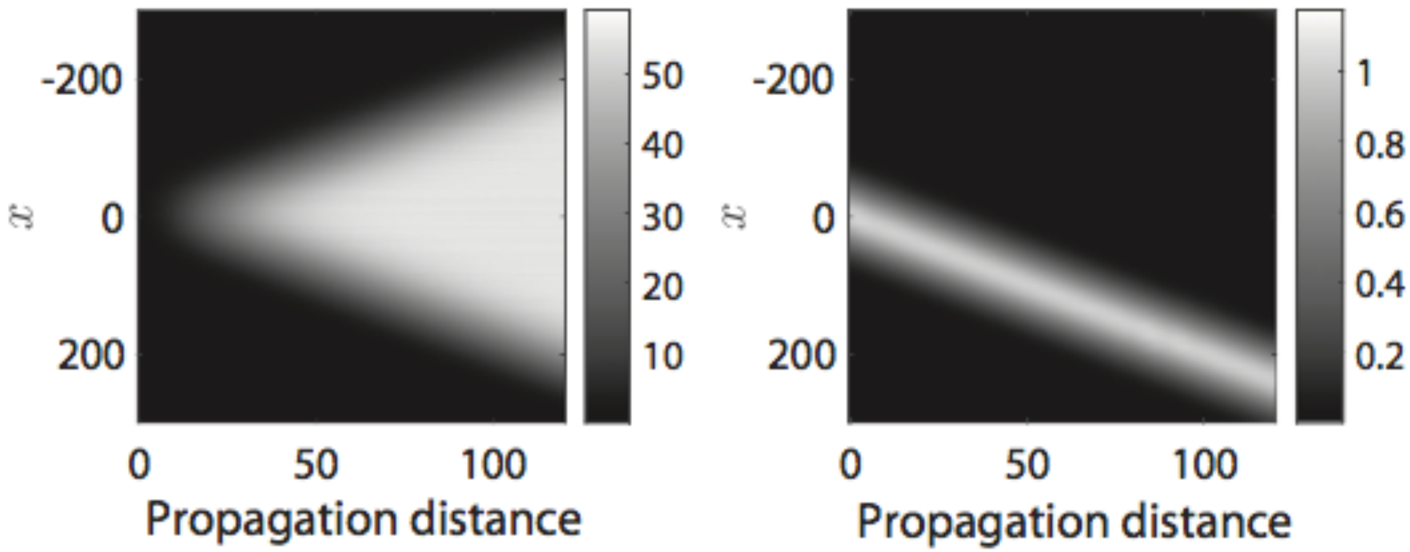


Fig. 4. Beam evolution of partially coherent light in a periodic potential at the symmetry breaking point $\beta = 1$. (a) Beam center. Continuous (dashed) lines represent $q = 0.5$ ($q = -0.5$). (b) Evolution of the spectral density for various values of the coherence parameter. (c) and (d) show the spectral density in the plane (x, z) at $q = 0.5$ and $q = -0.5$, respectively. The coherence parameters are $\delta = 10$ (yellow), $\delta = 20$ (orange) and $\delta = 80$ (blue).

Beam center and beam width are quite robust under the lack of correlations in contrast with S_{max}



Non-Hermitian, at the critical $\beta_c = 1$, fully coherent

Spectral degree of coherence

$$\mu^2(z) = \frac{\int |W(x_1, x_2, z)|^2 \, dx_1 dx_2}{\left[\int S(x, z) \, dx \right]^2}.$$

G-S sources: spectral degree of coherence $\mu(x_1, x_2)$ depends only on the distance between the two points

- partial correlation modifies the medium response
- correlations are induced into an uncorrelated beam by the non-Hermitian medium!

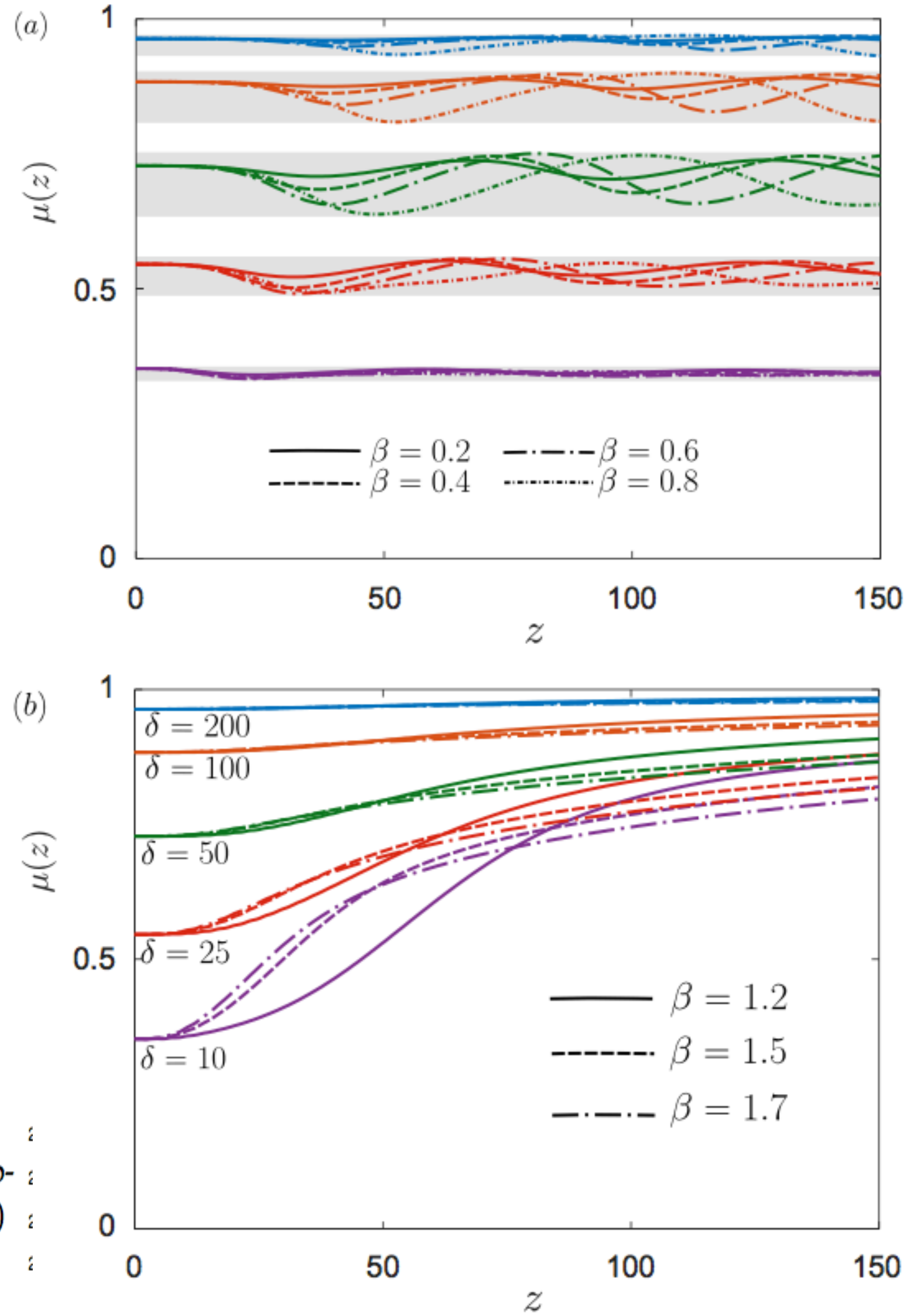


Fig. 5. Effective degree of coherence $\mu(z)$ as a function of propagated distance z for a PT-symmetric lattice (a) below and (b) above the symmetry breaking point.

Spatial coherence from ducks

It is not generally appreciated that radiation from uncorrelated random sources—for example, radiation generated by spontaneous emission of light by atoms—can produce a well-behaved, spatially coherent field over large regions. An illustration of this fact is the diffraction image of a star in the focal plane of a telescope. On a good observing night, the image will consist of a bright central spot surrounded by dark rings that represent regions in the focal plane where destructive interference cancels the light. This is a manifestation of strong correlation—a high degree of spatial coherence—between light fluctuations in the aperture of the telescope. The phenomenon illustrates the so-called van Cittert–Zernike theorem of optical coherence theory.[1,2]

In this letter we provide an example of the generation of spatial coherence. Thirteen Rouen ducks jump into a still one-acre pond, disturbing the surface at randomly distributed positions and times. The water surface exhibits an irregular, rather incoherent spatial pattern, as seen in panel a of the figure.[3] With increasing distance and time, the pattern evolves into a more regular one, as captured in panels b, c, and d, which clearly indicate the generation of spatial coherence in the far field from randomly distributed sources.

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2. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light*, Cambridge U. Press, Cambridge, UK (2007), sec. 3.2.
3. The pictures are from a 28-second video clip, available at http://www.youtube.com/watch?v=4o48J4streE.

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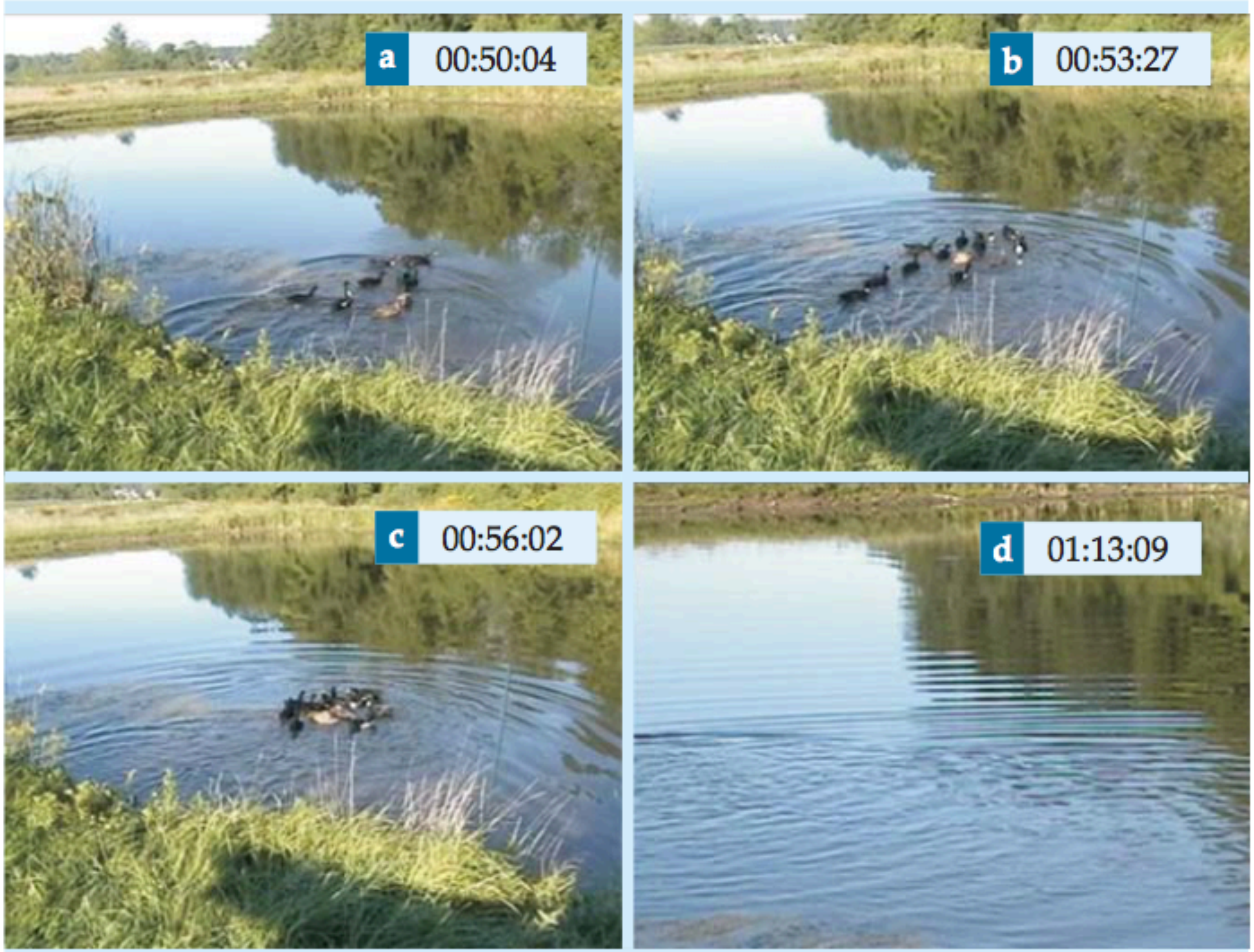

Generation of spatially coherent water waves from randomly distributed wave disturbances produced by 13 ducks jumping into a pool at time 00:47:12. The frame times are indicated.

Perspectives

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Tsallis statistics in optics?