Universidad
Católica del Norte

Fisher-Kolmogorov equations

applied to complex systems

S. Curilef, E. Larroza

80^{th} Birthday of Constantino

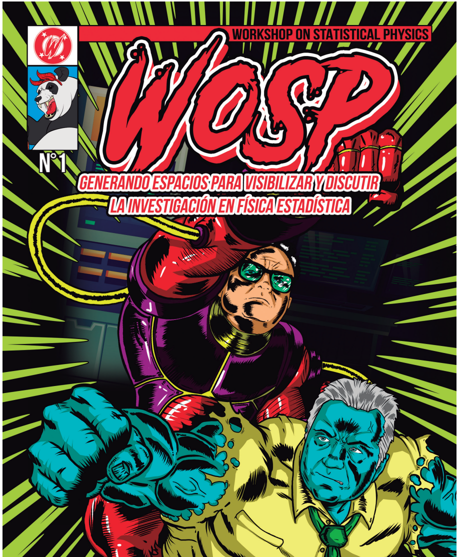
Contents

History

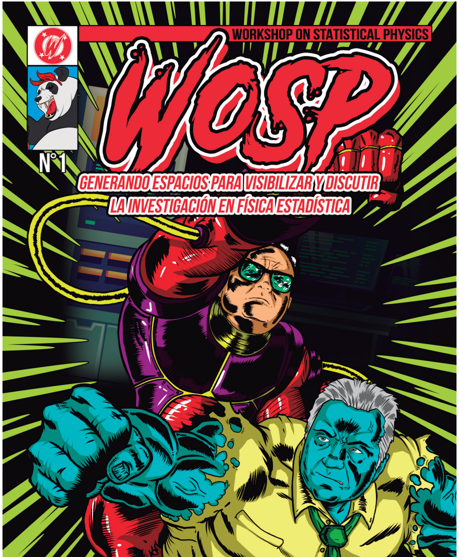
Summarizing my record with Constantino

- 1993 — I met him
- 1993-1997 — Ph.D. advisor
- 2002 — The Bachelor in Physics was created at UCN, he visited students.
- 2004 — He visited SOCHIFI (Chilean Physics society)
- 2019 — In Erice, we had an interesting conversation about social topics.
- 2021 — He has supported some meeting https://www.iwosp.cl/
- 2022 — He supported a Focus Issue in CHAOS.

https//www.iwosp.cl/



https//www.iwosp.cl/



IWoSP (hybrid)

International Workshop on Statistical Physics

Producto actual:

- AIP Conference Proceedings
- Journal of Physics: Conference Series
- Focus Issue in CHAOS
- Special Issue in ENTROPY

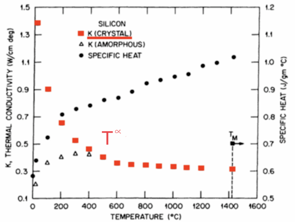
Birthday celebrations in 1993 - 2003 - 2013 - 2023 · · ·

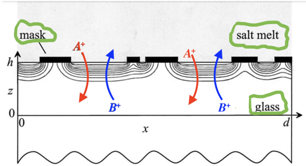
Motivation

Anomalous Diffusion

Case 1: Conductivity of Si

$$\kappa(T) = \mu T^{\alpha} \Rightarrow \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(\kappa(T)\frac{\partial T(x,t)}{\partial x}\right)$$





Case 2: Ion interchange in glass

$$\frac{\partial W}{\partial t} = D_A \nabla^2 \log W, \quad \text{where } W = (D_A/D_B - 1)C + 1$$

Case 3: Overdamped regime, $\mathbf{F} = 0$, short-range interactions $\phi = \delta(\mathbf{r} - \mathbf{r}')$

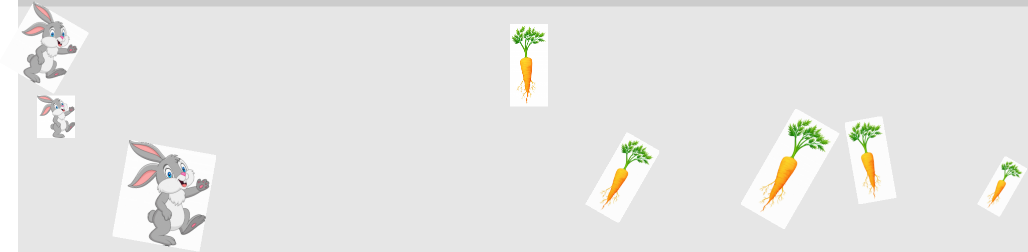
$$\mathbf{F} = -\gamma \mathbf{c} + \mathbf{F}_{\text{int}} + \mathbf{F}_{\text{ext}} \quad \Longrightarrow \quad \frac{\partial \rho}{\partial t} = \kappa \nabla^2 \rho^2 + \nabla \cdot (\rho \mathbf{G}) \, .$$

Population dynamics

Case 3: A diffusion-reaction problem

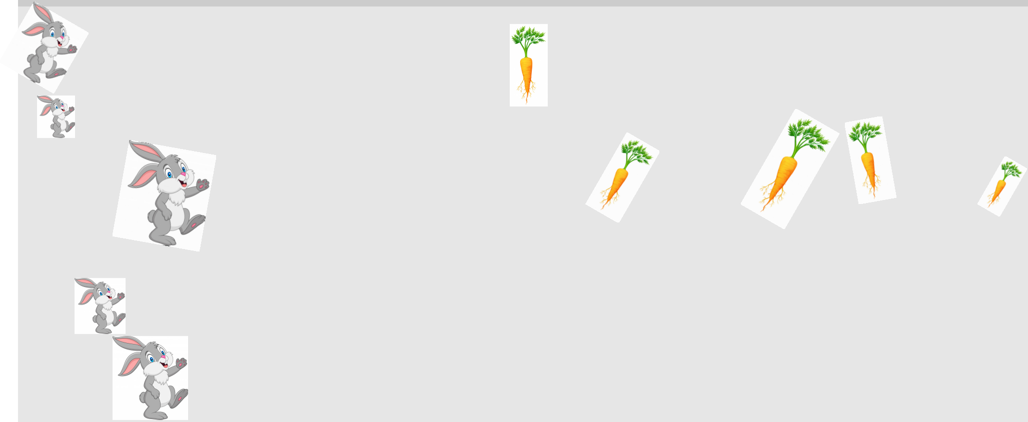
Population dynamics

Case 3: A diffusion-reaction problem



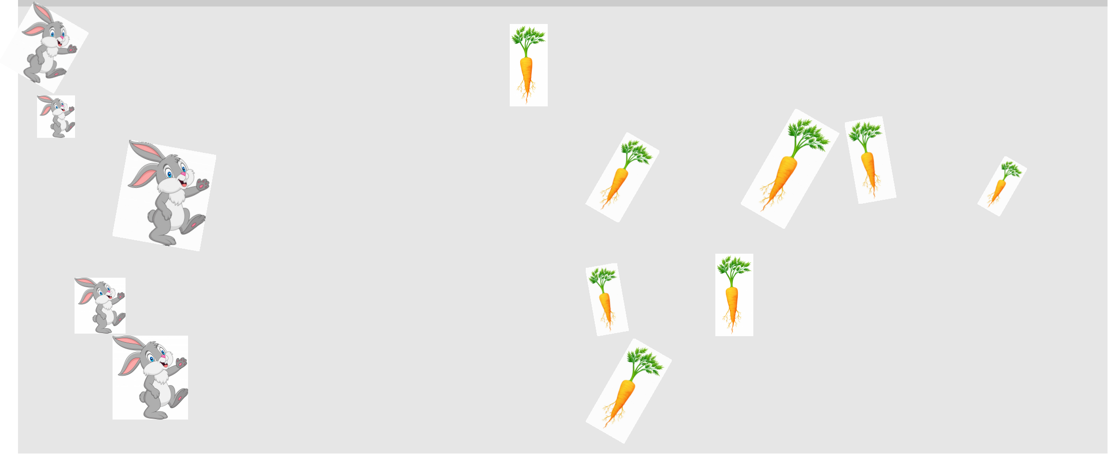
Population dynamics

Case 3: A diffusion-reaction problem



Population dynamics

Case 3: A diffusion-reaction problem



Equations that solve this problem

Extended Fisher-Kolmogorov equation (EFKE)

$$\frac{\partial}{\partial t}W(x,t) = \frac{\kappa}{2-q}\frac{\partial^2}{\partial x^2}W(x,t)^{2-q} + r^*(t)W(x,t) - \mu^*(t)c(x)W(x,t)^q.$$

See Fokker-Planck Video

Other equations

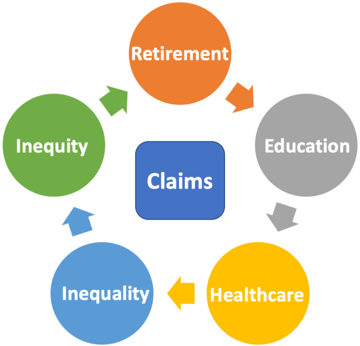
- Lotka-Volterra equations
- Verhulst Equations

Context

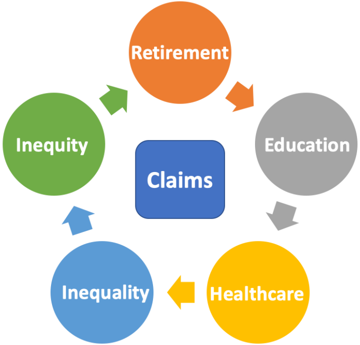
Social motivation

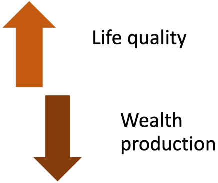
Social motivation

Social context

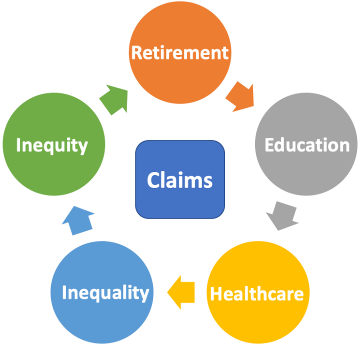


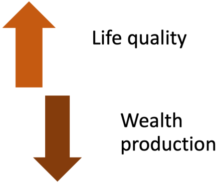
Social context





Social context





Relative Deprivation Theory

Income inequalities $\Rightarrow$ $\Leftarrow$ Well-being perception

PLOS ONE

RESEARCH ARTICLE

Analyzing the 2019 Chilean social outbreak: Modelling Latin American economies

Sergio Curilef[1☯*], **Diego González**[1,2☯], **Carlos Calderón**[3☯]

1 Departamento de Física, Universidad Católica del Norte, Antofagasta, Chile, 2 Banco Itaú-Corpbanca, Santiago, Chile, 3 Escuela de Psicología, Universidad Católica del Norte, Antofagasta, Chile

☯ These authors contributed equally to this work.
* scurilef@ucn.cl

Check for updates

Abstract

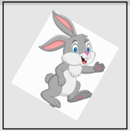
In this work, we propose a quantitative model for the 2019 Chilean protests. We utilize public data for the consumer price index, the gross domestic product, and the employee and per capita income distributions as inputs for a nonlinear diffusion-reaction equation, the solutions to which provide an in-depth analysis of the population dynamics. Specifically, the per capita income distribution stands out as a solution to the extended Fisher-Kolmogorov equation. According to our results, the concavity of employee income distribution is a decisive input parameter and, in contrast to the distributions typically observed for Chile and other countries in Latin America, should ideally be non-negative. Based on the results of our model, we advocate for the implementation of social policies designed to stimulate social mobility by broadening the distribution of higher salaries.

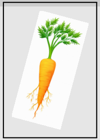
OPEN ACCESS

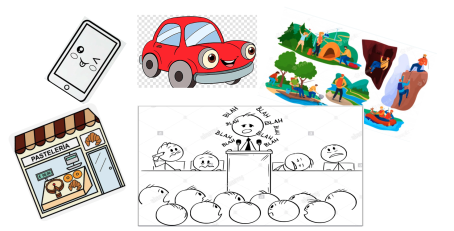
Citation: Curilef S, González D, Calderón C (2021) Analyzing the 2019 Chilean social outbreak: Modelling Latin American economies. PLoS ONE 16(8): e0256037. https://doi.org/10.1371/journal.pone.0256037

Editor: Petre Caraiani, Institute for Economic

Mapping the social outbreak into the population dynamics







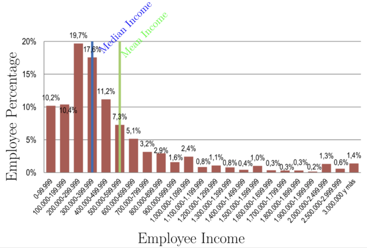
Fisher-Kolmogorov equation

$$\frac{\partial}{\partial t} W(x,t) = \kappa \frac{\partial^2}{\partial x^2} \log W(x,t) + r(t) W(x,t) - \mu(t) c(x) W(x,t)^2,$$

Useful Distributions: $c(x)$ y $W(x, 0)$

Income

$$c(x) = 0.197\, \mathbf{e}_2^{-\frac{(x-2.5)^2}{4.51}}$$



Per-capita Income

$$W(x, 0) = 0.359\, \mathbf{e}_2^{-\frac{(x-1)^2}{1.57}}, \quad x = 10^5 \textbf{ CLP}$$

Decile	*Quintile*	Average Income	
		Since	To
1	1	0	48.750
2		48.751	74.969
3	2	74.970	100.709
4		100.710	125.558
5	3	125.559	154.166
6		154.167	193.104
7	4	193.105	250.663
8		250.664	352.743
9	5	352.744	611.728
10		611.729	—

Relevant parameters

GDP (Gross domestic product)

$$\text{GDP} = C + I + G + (X - M)$$

C	Comsumption
I	Investiment
G	Goverment spending
X	Exports
I	Imports

CPI (Consumer price index)

$$\text{CPI} = \frac{\text{Updated cost}}{\text{Base period cost}} \times 100$$

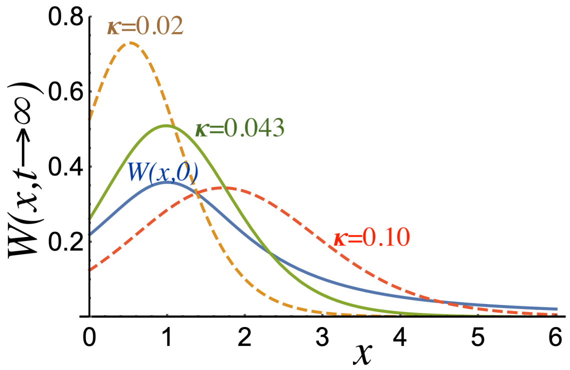
CPI measures change over time in the prices paid by consumers for a representative basket of goods and services.

$$\frac{\partial}{\partial t}\overbrace{W(x,t)}^{\text{Per capita income}} = \kappa \frac{\partial^2}{\partial x^2} \log W(x,t) + \underbrace{r(t)}_{\text{GDP variation}} W(x,t) - \underbrace{\mu(t)}_{\text{CPI}} \overbrace{c(x)}^{\text{Income}} W(x,t)^2,$$

Results

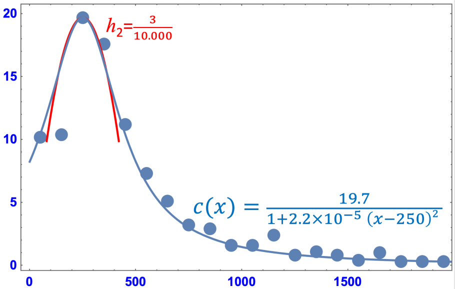
Stationary numerical solutions

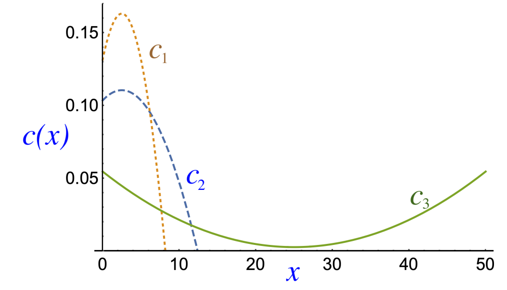
$$0 = \kappa \frac{\partial^2}{\partial x^2} \log W + rW - \mu c(x) W^2,$$



Analytical proposal

Income distribution as a quadratic potential





$$c(x) = h_1 + h_2\left(x - x_p\right)^2,$$

Analytical solutions

Considering the income distribution proposal $c(x) = h_1 + h_2\left(x - x_p\right)^2$,

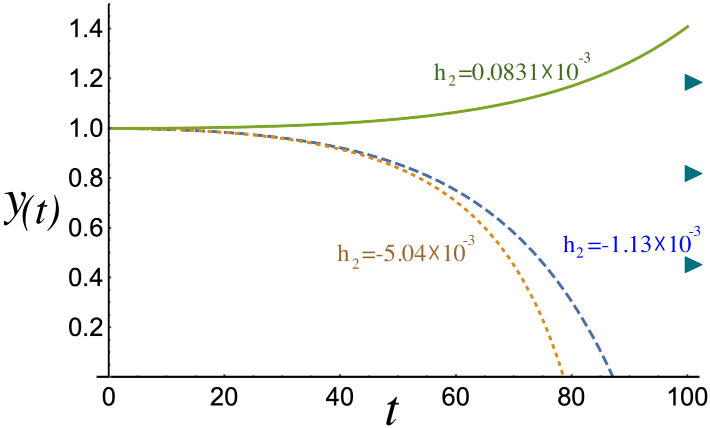
Ansatz

$$W(x,t) = A(t)^{-1}\exp_2\left(-\frac{(x-y(t))^2}{4A(t)S(t)}\right)$$

Motion equations

$$\begin{aligned} S'(t) &= \frac{1}{2}\kappa + rS(t) - 4h_2\mu S(t)^2, \\ y'(t) &= -4\mu h_2(y(t)-x_p)S(t), \\ A'(t) &= \kappa\frac{A(t)}{2S(t)} - rA(t) + \mu h_1 + \mu h_2(y(t)-x_p)^2. \end{aligned}$$

Perception dynamics



- If $h_2 < 0$, the solution is bad because $y(t) \to 0$.
- $y(t) < 0$ does not represent any economic situation.
- $h_2 > 0$ generates well-behaved solutions.

Conclusions

Conclusions

- The connection between several physics and social problems can illuminate new knowledge in every field.
- A power-law ansatz (Tsallis distribution) helps to solve the nonlinear Fisher-Kolmogorov equation to support applications in several scenarios.
- The ansatz maximizes the general form of the Tsallis entropy.
- An additional solution using traveling wave approximation can be helpful in nonlinear optics applications.

Consequences: public polices

Our results lead to propose public policies, taking into account the following:

1. The interval between the minimal and maximal incomes is a relevant parameter that introduces concepts such as ethical income. This measure can decrease inequality.
2. However, the maximal and minimal income are irrelevant if the shape is arbitrary.
3. Employee income distribution needs to be defined according to upward concavity in the interval between the minimum and maximum.
4. A single peak in c(x) (absolute equality of income), $h_2 \to 0$, represents a rapidly collapsing economy and adverse effects on the per capita income.
5. Equity and Ethics are concepts that we can draw here as a strategy to decrease inequality.

Future

- Study as the parameters time variation modify the results.
- Consider other macroeconomic parameters as the debt.
- Couple at least two Fisher-Kolmogorov equations.
- Study other problems with nonlinear diffusion equations.

Contributions to the EFKE

P. Troncoso, O. Fierro, S. Curilef, A.R. Plastino,	Physica **A 375**, 457–466 (2007)
S. Curilef,	AIP Conference Proceedings **1558**, 1771 (2013)
C. Valenzuela, L.A. del Pino, S. Curilef,	Physica **A 416**, 439–451 (2014)
S. Curilef, D. González, C. Calderón,	PLoS ONE **16** (8): e0256037 (2021)
S Curilef,	CHAOS **32**, 113133 (2022)
S. Curilef, A. R. Plastino and R. S. Wedemann	CHAOS **32**, 113134 (2022)

Collaborations

1. Recent
   - Edward Larroza (Graduate)
   - Nicolás Angel (Graduate)
   - René Moreira (Graduate)
   - Benny Nogales (Graduate)
   - Diego González (Post-doc)
   - Carlos Calderón (Interdisciplinary)
2. Other
   - Francisco Calderón
   - Dianela Herrera
   - Boris Atenas
   - Angel R. Plastino

Acknowledgements

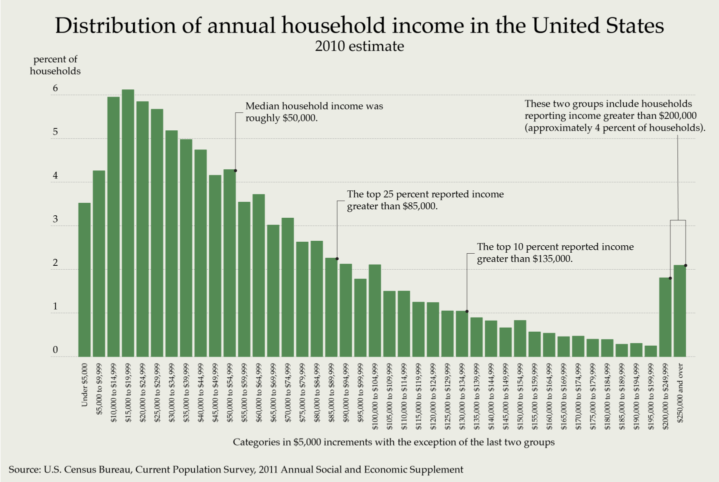
Research Groups

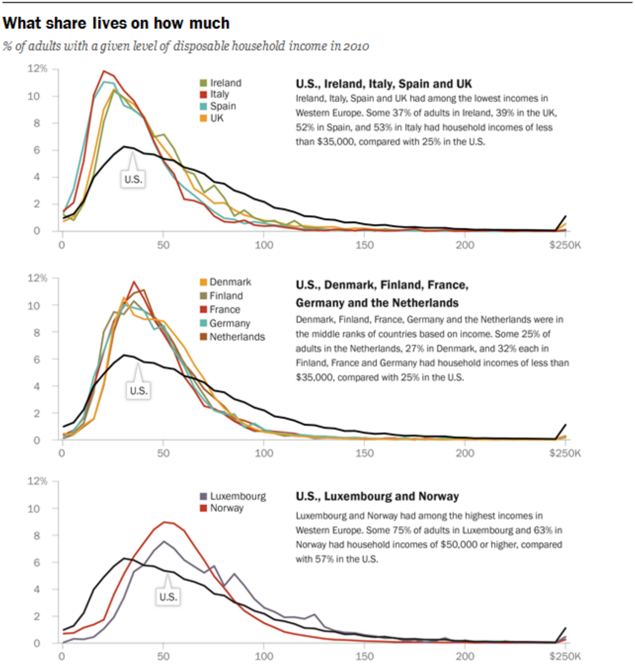
- Núcleo No.2-UCN-VRIDT 042/2020, Sistemas Complejos en Ciencia e Ingeniería
- Núcleo No.7-UCN-VRIDT 076/2020, Núcleo de Simulación y Modelación Científica

Grants

- Proyecto INNOVA en el Aula, SCIENCE UP 2023, "Including Nonlinear Differential Equations of Statistical Physics to Complex Systems"
- Programa ANID-VIU23P0093, "Smartflow: Macroeconomic decision making based on the Fisher-Kolmogorov equation"
- DGPRE-UCN No. 109/2021, “A predictive model of student retention-desertion quantitative theoretical toolsné-based”

Thank-q


Distribution of annual household income in the United States
2010 estimate
percent of households
6
5
4
3
2
1
0
Median household income was roughly $50,000.
The top 25 percent reported income greater than $85,000.
The top 10 percent reported income greater than $135,000.
These two groups include households reporting income greater than $200,000 (approximately 4 percent of households).
Under $5,000
$5,000 to $9,999
$10,000 to $14,999
$15,000 to $19,999
$20,000 to $24,999
$25,000 to $29,999
$30,000 to $34,999
$35,000 to $39,999
$40,000 to $44,999
$45,000 to $49,999
$50,000 to $54,999
$55,000 to $59,999
$60,000 to $64,999
$65,000 to $69,999
$70,000 to $74,999
$75,000 to $79,999
$80,000 to $84,999
$85,000 to $89,999
$90,000 to $94,999
$95,000 to $99,999
$100,000 to $104,999
$105,000 to $109,999
$110,000 to $114,999
$115,000 to $119,999
$120,000 to $124,999
$125,000 to $129,999
$130,000 to $134,999
$135,000 to $139,999
$140,000 to $144,999
$145,000 to $149,999
$150,000 to $154,999
$155,000 to $159,999
$160,000 to $164,999
$165,000 to $169,999
$170,000 to $174,999
$175,000 to $179,999
$180,000 to $184,999
$185,000 to $189,999
$190,000 to $194,999
$195,000 to $199,999
$200,000 to $249,999
$250,000 and over
Categories in $5,000 increments with the exception of the last two groups
Source: U.S. Census Bureau, Current Population Survey, 2011 Annual Social and Economic Supplement


What share lives on how much
% of adults with a given level of disposable household income in 2010
12%
10
8
6
4
2
0
Ireland
Italy
Spain
UK
U.S., Ireland, Italy, Spain and UK
Ireland, Italy, Spain and UK had among the lowest incomes in Western Europe. Some 37% of adults in Ireland, 39% in the UK, 52% in Spain, and 53% in Italy had household incomes of less than $35,000, compared with 25% in the U.S.
U.S.
0
50
100
150
200
$250K
Denmark
Finland
France
Germany
Netherlands
U.S., Denmark, Finland, France, Germany and the Netherlands
Denmark, Finland, France, Germany and the Netherlands were in the middle ranks of countries based on income. Some 25% of adults in the Netherlands, 27% in Denmark, and 32% each in Finland, France and Germany had household incomes of less than $35,000, compared with 25% in the U.S.
Luxembourg
Norway
U.S., Luxembourg and Norway
Luxembourg and Norway had among the highest incomes in Western Europe. Some 75% of adults in Luxembourg and 63% in Norway had household incomes of $50,000 or higher, compared with 57% in the U.S.

Estado estacionario

- El límite $t \to \infty$ está bien definido si $r^2 + 8h_2\kappa\mu \geq 0$.
- Si $h_2 > 0$, todos los parámeteros son positivos, la condición previa se satisface.
- Si $h_2 < 0$, la condición previa se reduce a $\kappa < r^2/8\left|h_2\right|\mu$.
- If $\kappa \to 0$, no hay proceso de difusión process, la solución es $W(x) \to r/\mu c(x)$
- Si el parámetro de difusión $\kappa \neq 0$ acelera la dinámica de fenómenos económicos.

In relation to the quantitative perception, this is $y(t)$.

In relation to the quantitative perception, this is $y(t)$.