

# Cellular criticality

Sergio A. Cannas



I F E G

Statistical Mechanics for Complexity 2023  
80<sup>th</sup> birthday of C. Tsallis celebration

## Collaborators

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Universidad Nacional de San Martín, Buenos Aires, Argentina (CONICET)

- Orlando Billoni

Universidad Nacional de Córdoba, Argentina (IFEG-CONICET)

- Nahuel Zamponi

Department of Medicine, Weill Cornell Medicine, USA

- Emiliano Zamponi

University of Colorado-Boulder, USA

# MITOCHONDRIA

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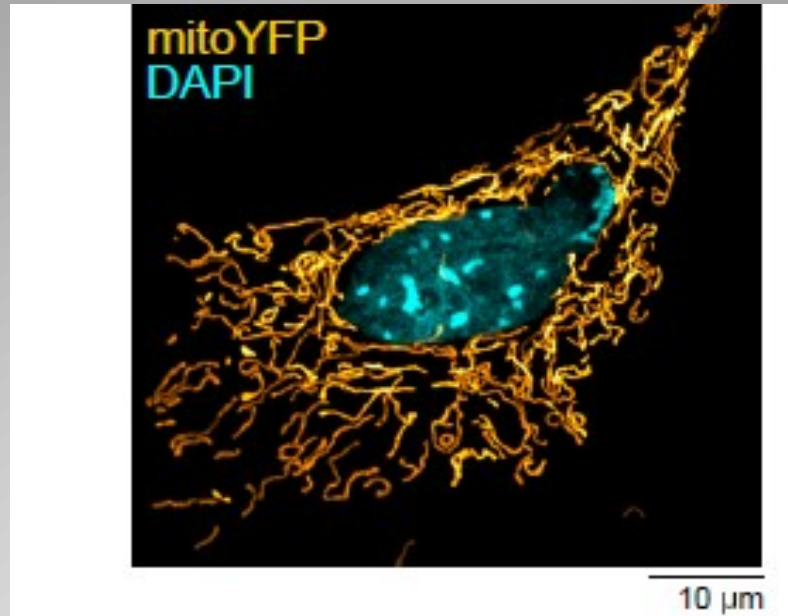
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- **Other functions:** participate of several cellular processes, such as apoptosis (programmed cell death), phospholipids synthesis, regulation of membrane potential, etc..

# MORPHOLOGY



# AGENTS BASED MODEL: MEAN FIELD

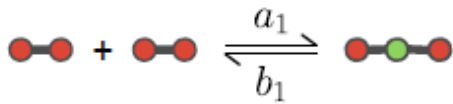
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PLOS COMPUTATIONAL BIOLOGY

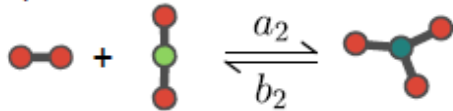
## Emergence of the Mitochondrial Reticulum from Fission and Fusion Dynamics

Valerii M. Sukhorukov<sup>1,2\*</sup>, Daniel Dikov<sup>3,4</sup>, Andreas S. Reichert<sup>3,4</sup>, Michael Meyer-Hermann<sup>1,5\*</sup>

tip-to-tip:



tip-to-side:





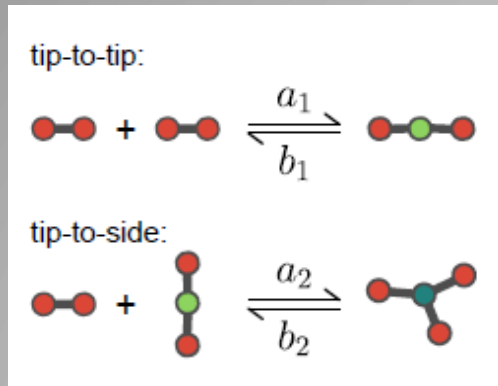
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$X_k$  : nodes with degree  $k = 1, 2, 3$

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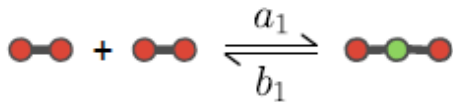
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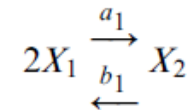


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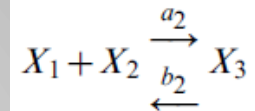


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tip-to-tip fusion ( $a_1$ ) and fission ( $b_1$ ):



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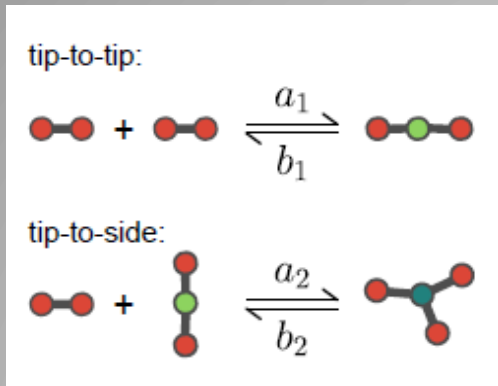
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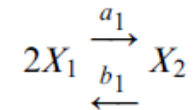
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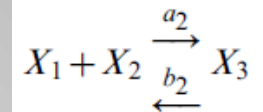


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tip-to-tip fusion ( $a_1$ ) and fission ( $b_1$ ):



tip-to-side fusion ( $a_2$ ) and fission ( $b_2$ ):



$$b_2 = (3/2)b_1 \equiv (3/2)b \quad c_1 \equiv a_1/b \text{ and } c_2 \equiv a_2/b$$

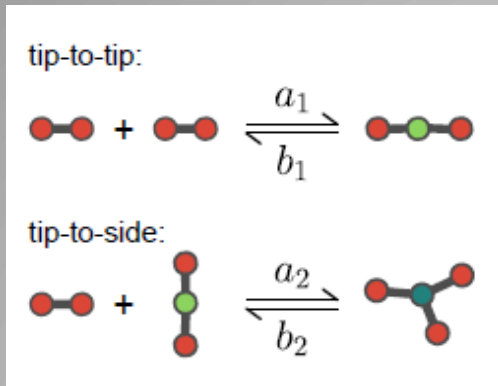
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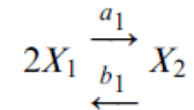
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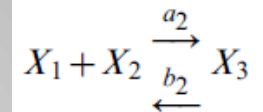


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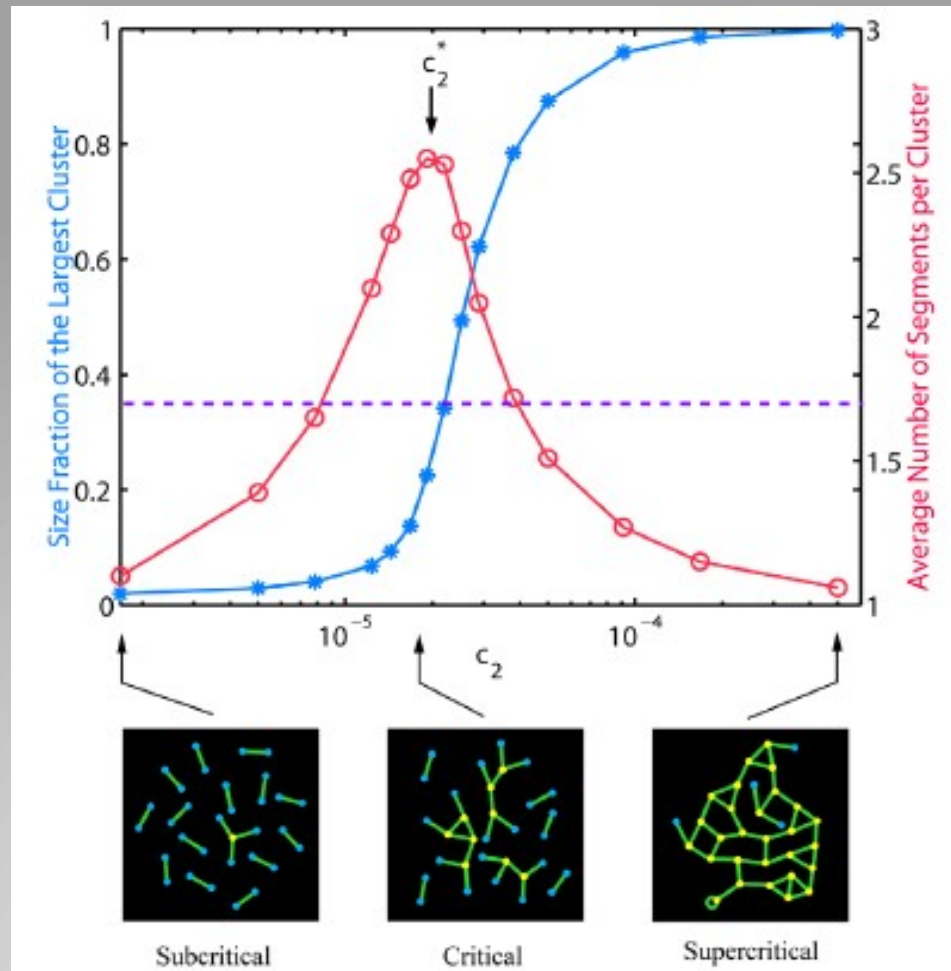


tip-to-side fusion ( $a_2$ ) and fission ( $b_2$ ):



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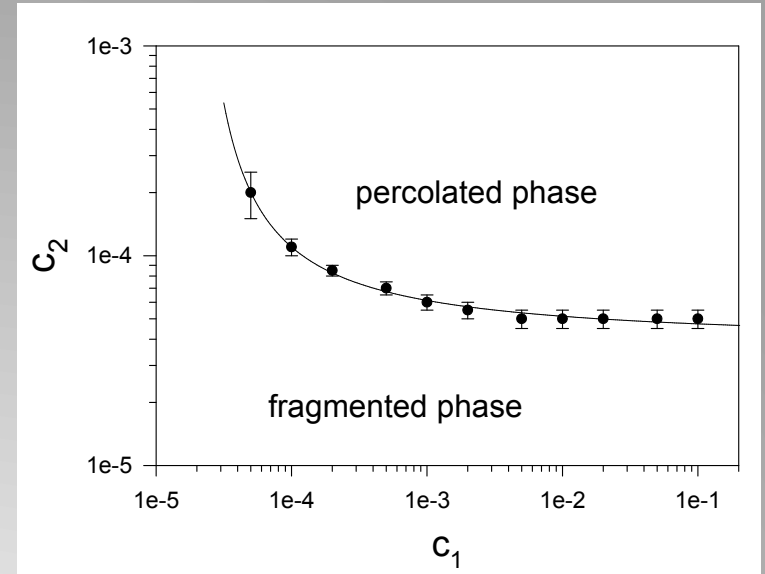
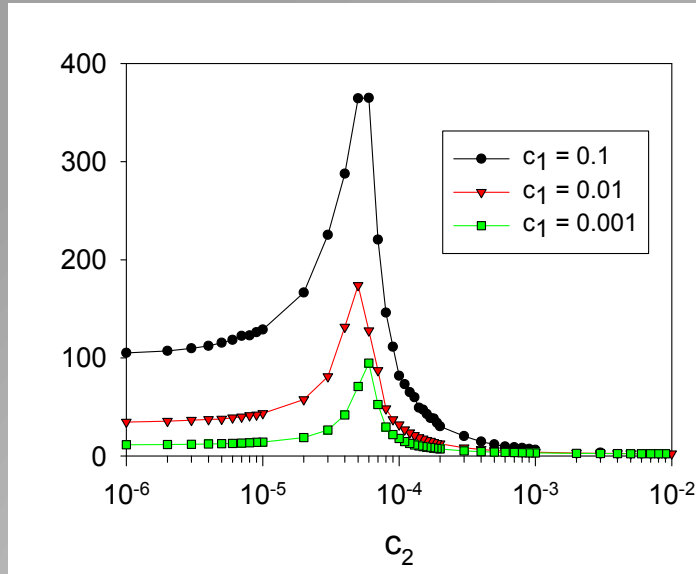
Dynamics: Gillespie algorithm with  $N_e$  dimers



$$c_1 = 0.1 \quad - \quad N_e = 3 \times 10^4$$

# PHASE DIAGRAM

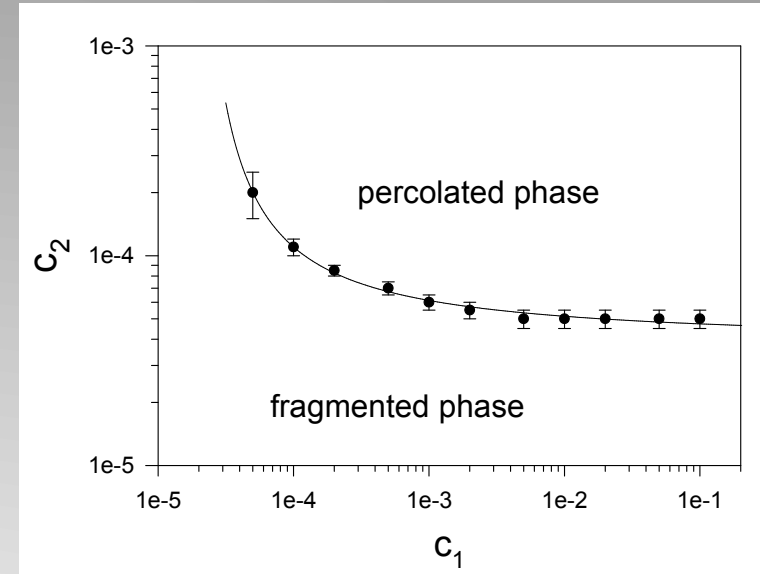
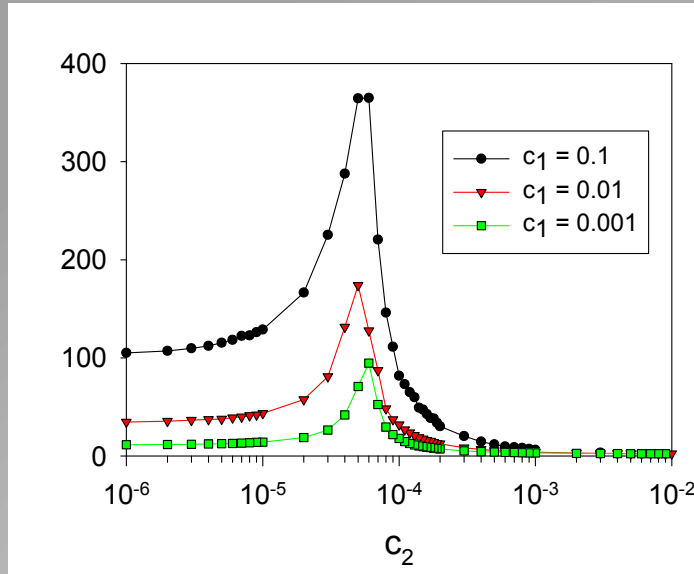
$$N_e = 15000$$



$$\langle s \rangle = \frac{\sum'_s N_s s^2}{\sum'_s N_s s}$$

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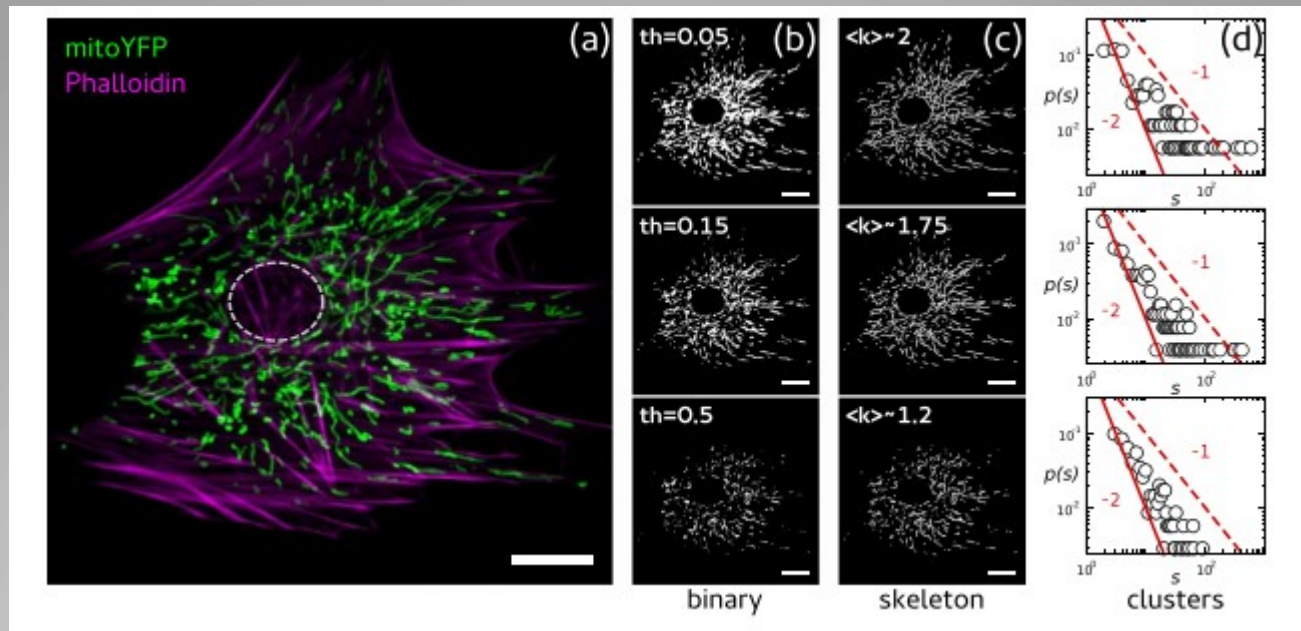


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In which part of this phase diagram might real mitochondria be located?

## EXPERIMENTS

- Imaging using confocal microscopy on genetically modified cells: mouse embryonic fibroblasts

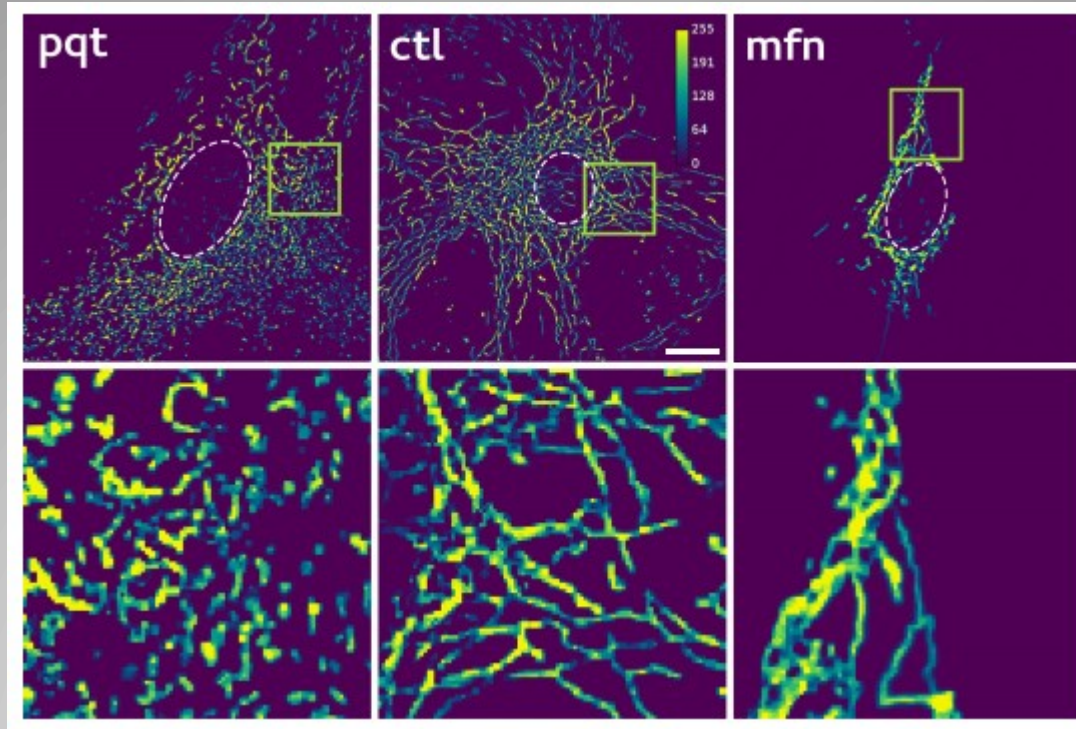


N. Zamponi, E. Zamponi, S.A. Cannas, O.V. Billoni, P. Helguera, D. R. Chialvo, Scientific Reports **8**, 363 (2018)



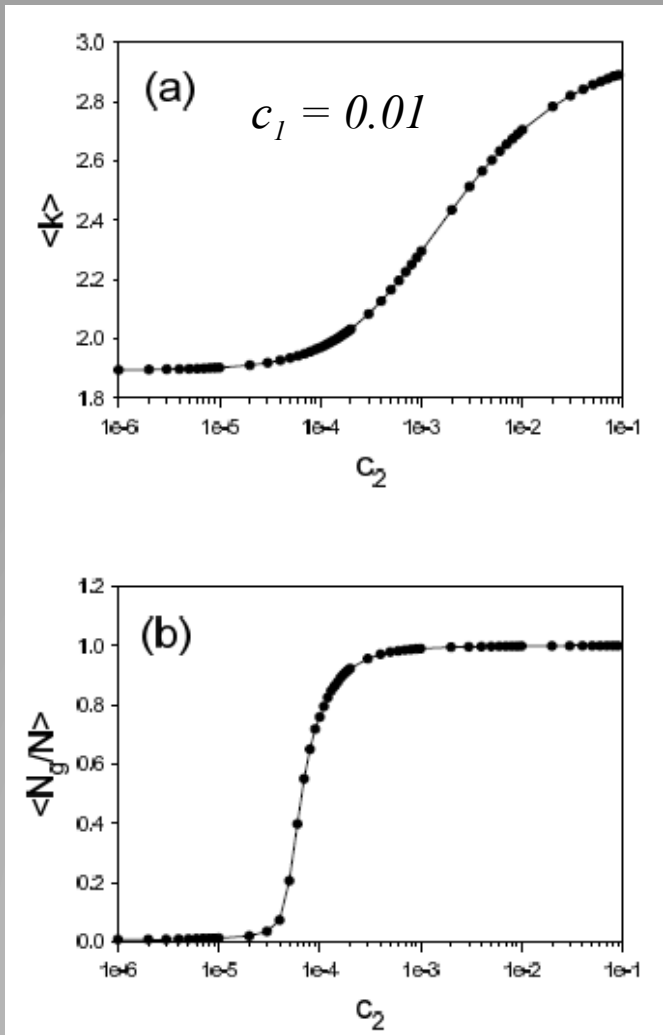
## Morphology manipulation: treatments

- **Paraquat (pqt):** promotes fission
- **Mitofusin (mfn):** promotes fusion

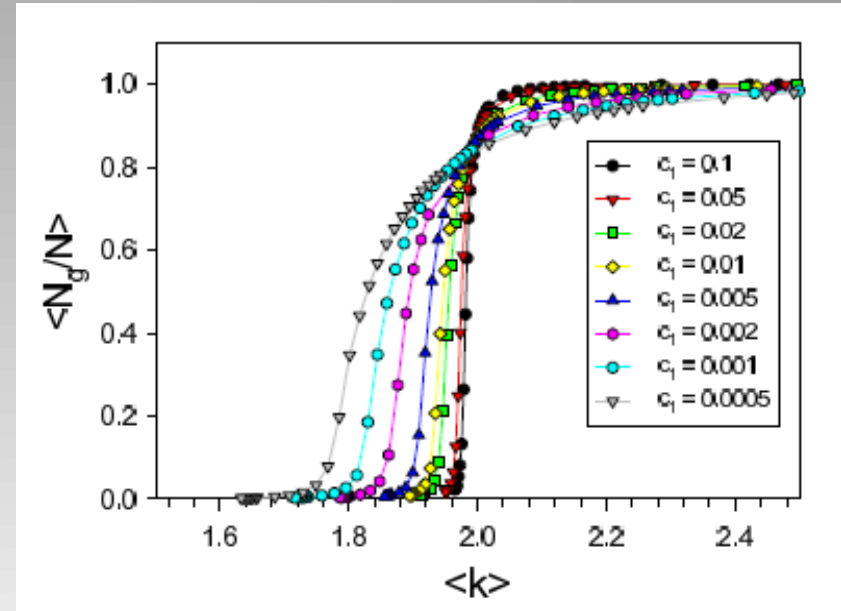
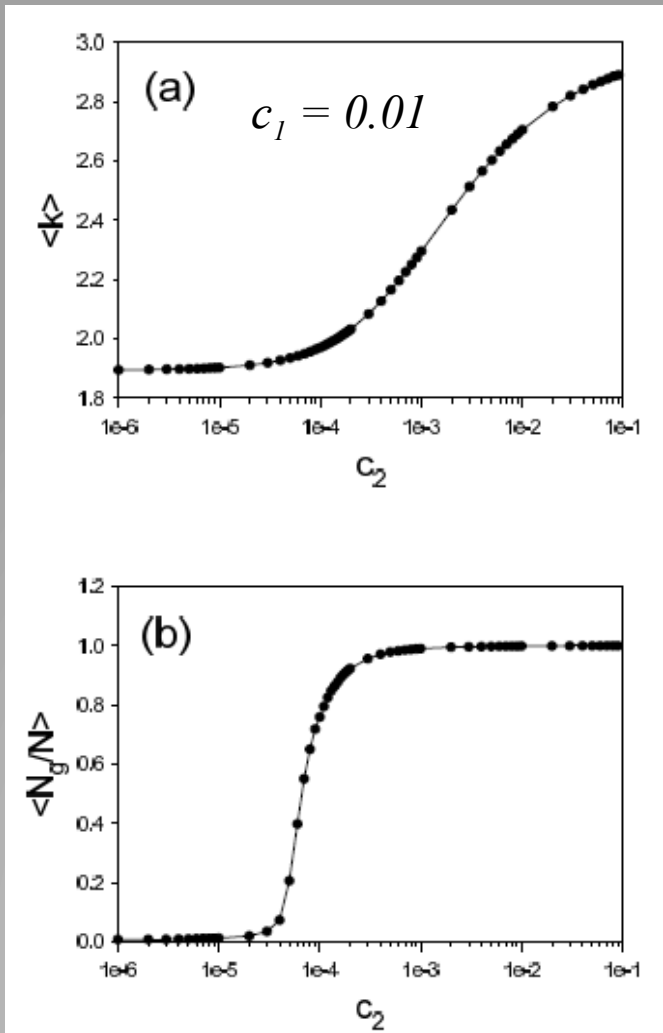


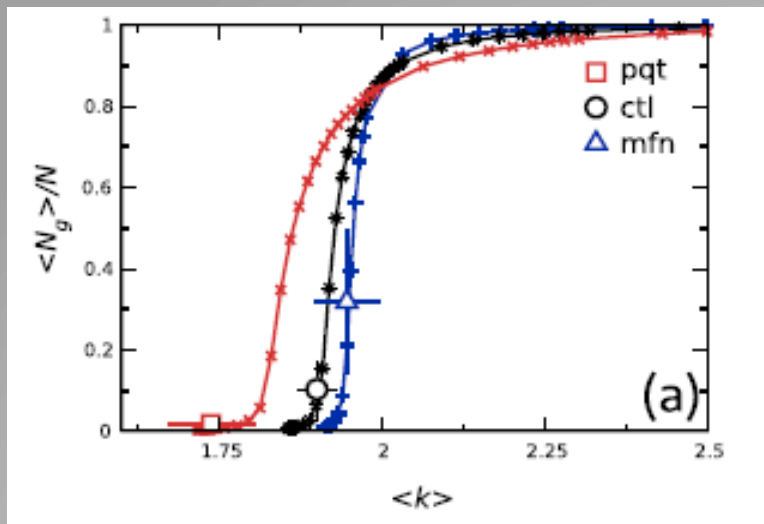
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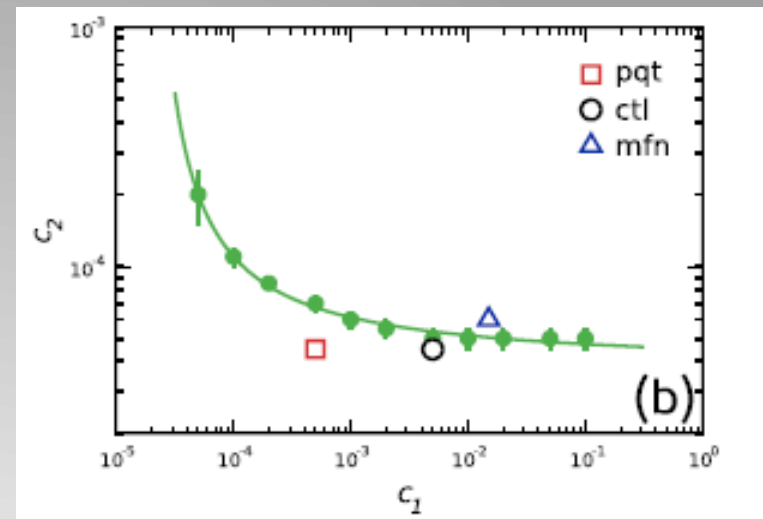
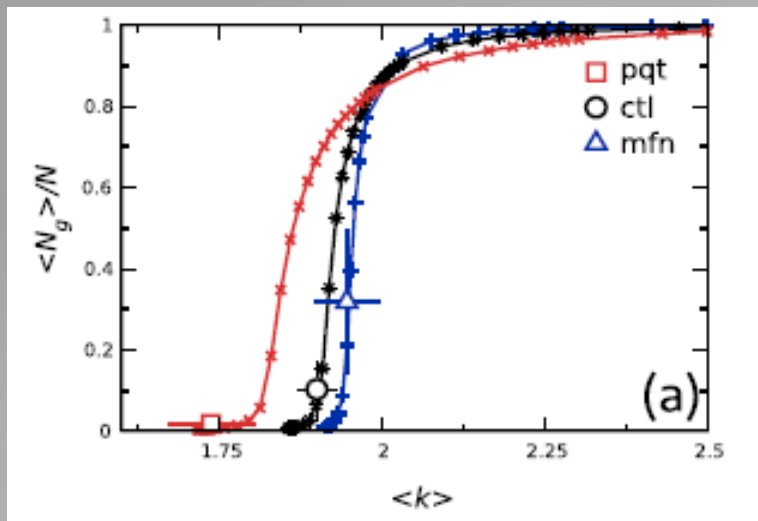
## MEAN FIELD MODEL



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N. Zamponi, E. Zamponi, S.A. Cannas, O.V. Billoni, P. Helguera, D. R. Chialvo, Scientific Reports **8**, 363 (2018)

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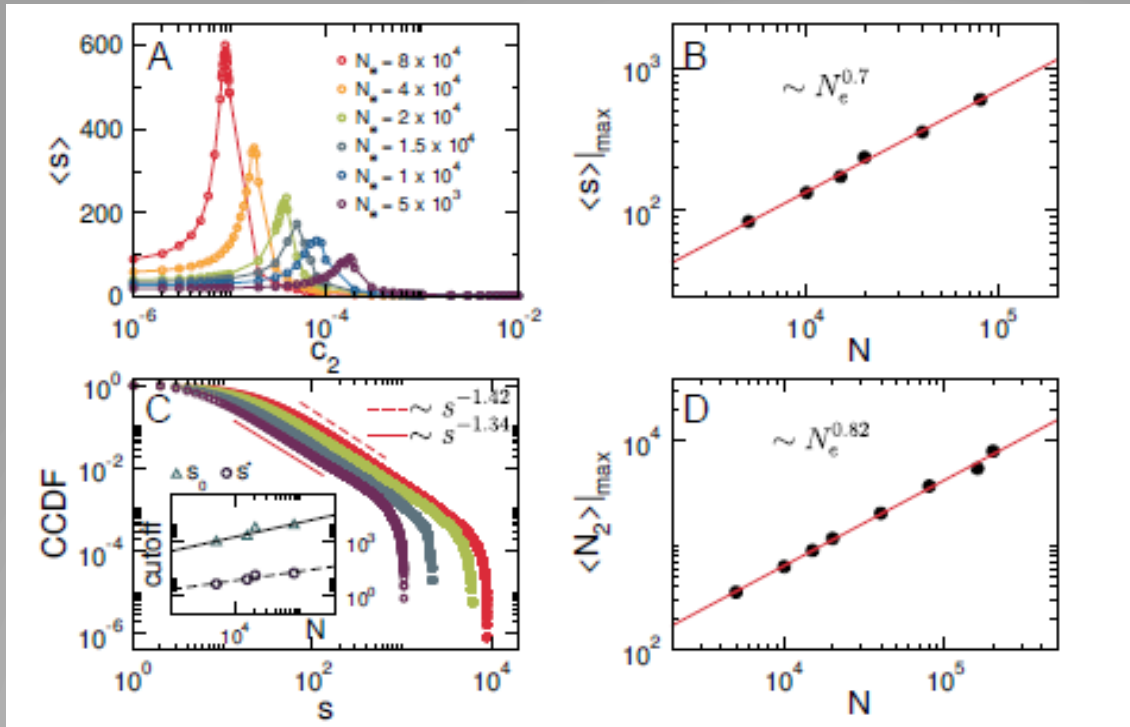
## QUESTIONS:

- Do the fusion/fission mechanism really generate criticality? **Finite size scaling?**
- Universality class?
- What happens in finite dimension?



# Mean field model: finite size scaling

$$c_1 = 0.01$$



$$\max \langle s \rangle \sim N^{\gamma/d}$$

$$\max \langle N_2 \rangle \sim N^{df/d}$$

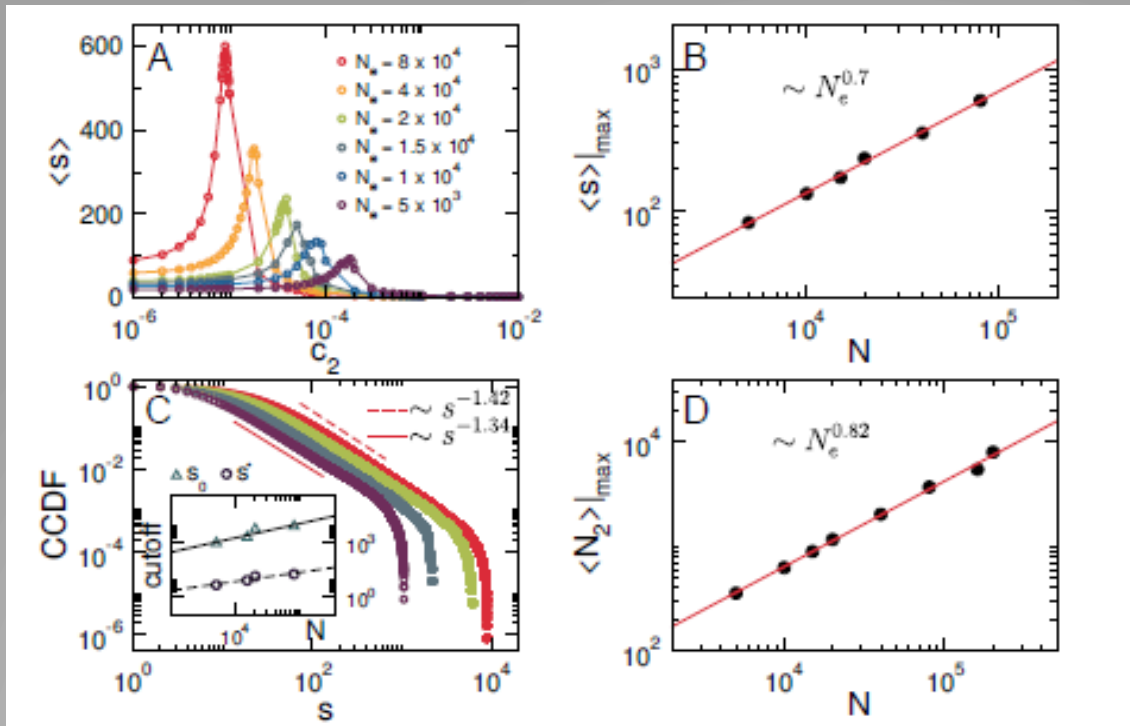
$$n_s \sim s^{-\tau} \exp(-s/s^*)$$

$$\text{CCDF}(s) = \sum_{s' \geq s} n_{s'}$$

$$\text{CCDF} \sim s^{-(\tau-1)} \exp(-s/s^*)$$

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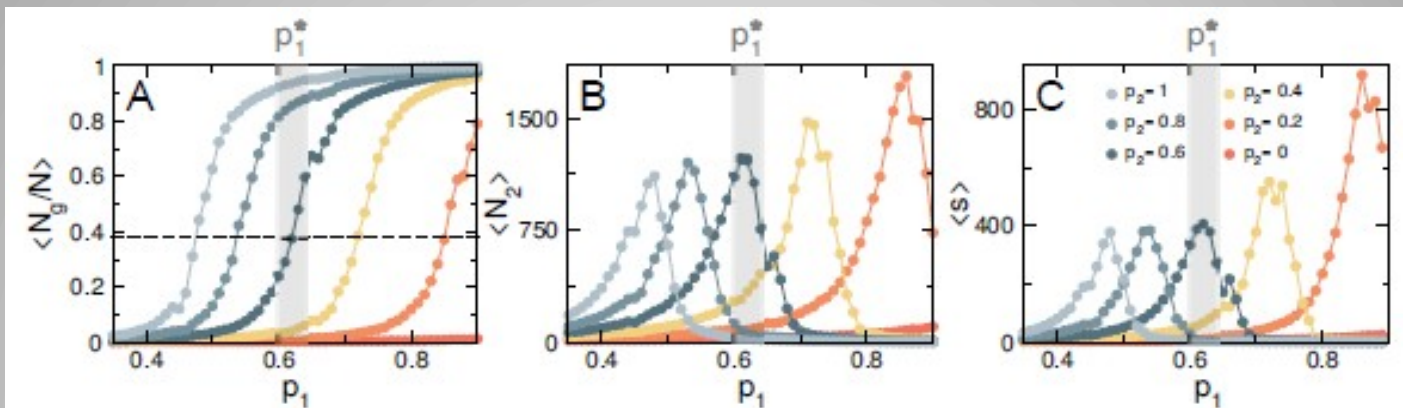
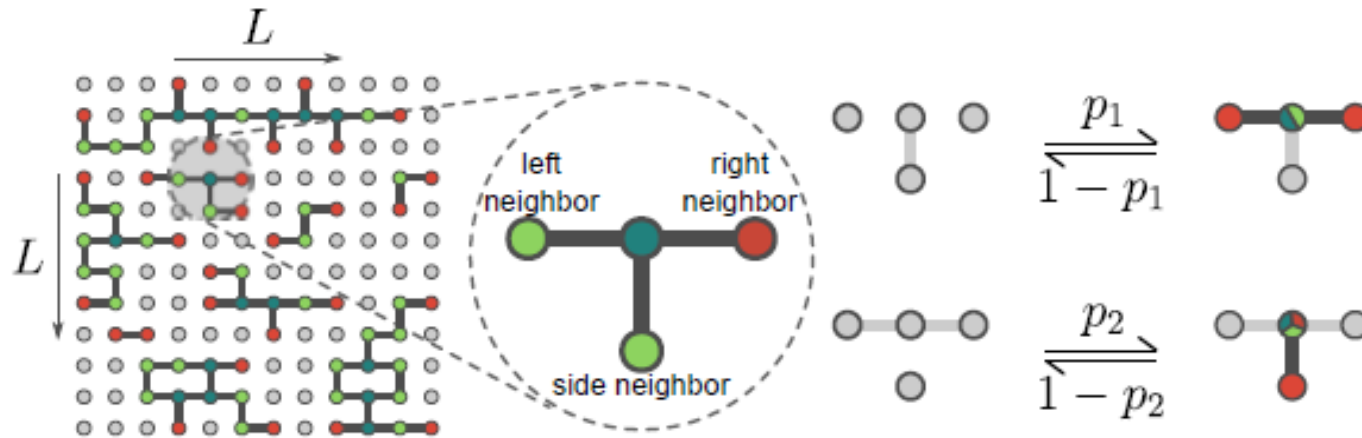
$$\tau = 2.38 \pm 0.04 \text{ (MF standard perc.: 2.5)}$$

$$\gamma/vd = 0.70 \pm 0.01 \text{ (MF standard perc.: 1/3)}$$

$$d_f/d = 0.82 \pm 0.01 \text{ (MF standard perc.: 2/3)}$$

$$\text{CCDF}(s) \approx 1 + \theta(s - s_0) s^{-\tau+1} e^{-s/s^*}$$

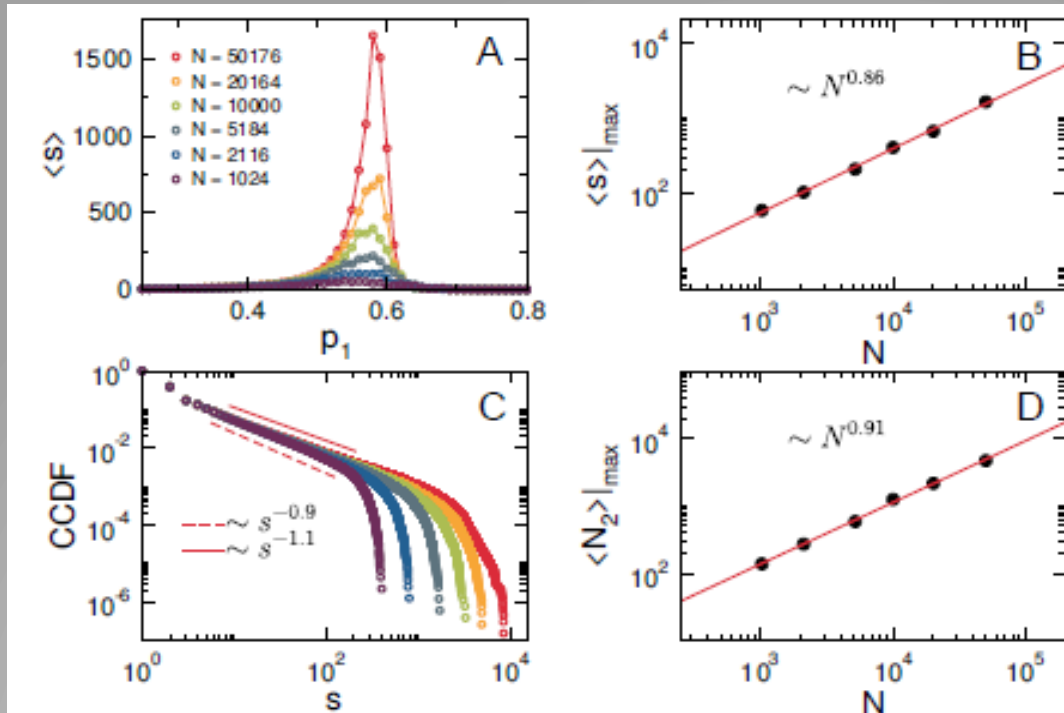
## Spatially explicit model (2D)



$$N = 10^4$$

## Spatially explicit model: finite size scaling

$$p_2 = 0.7$$



$$\max \langle S \rangle \sim N^{\gamma/vd}$$

$$\max \langle N_2 \rangle \sim N^{d_f/d}$$

$$n_s \sim s^{-\tau} \exp(-s/s^*)$$

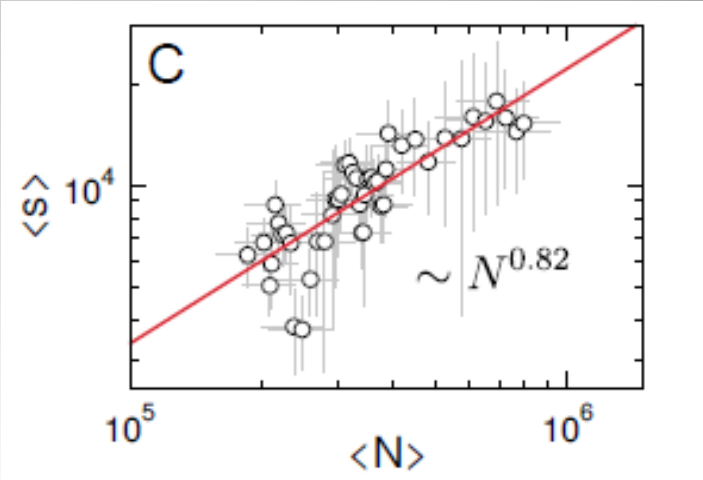
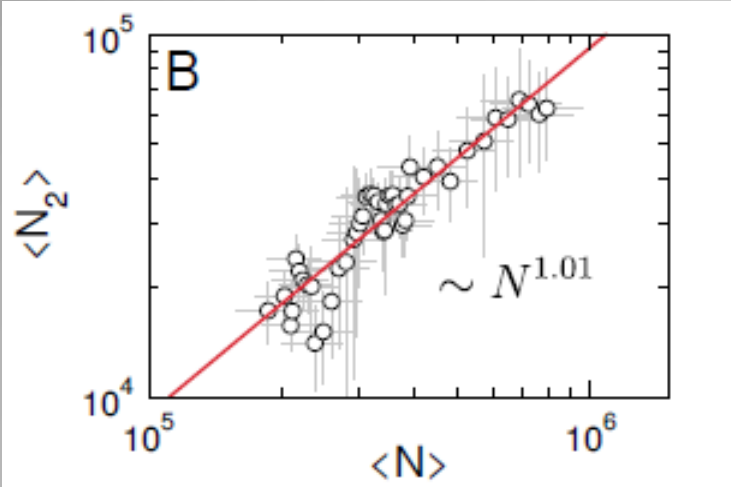
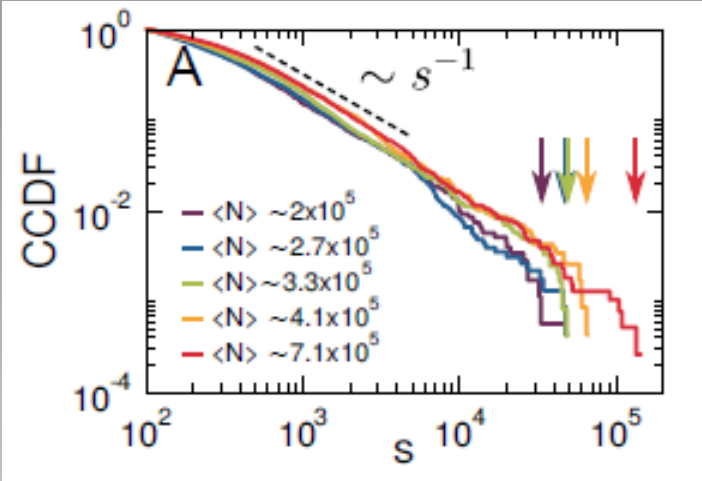
$$CCDF \sim s^{-(\tau-1)} \exp(-s/s^*)$$

$$\tau = 2.0 \pm 0.1 \text{ (2D standard perc.: 2.055)}$$

$$\gamma/vd = 0.86 \pm 0.02 \text{ (2D standard perc.: 0.896)}$$

$$d_f/d = 0.91 \pm 0.02 \text{ (2D standard perc.: 0.948)}$$

# Real mitochondria: finite size scaling



	$\tau$	$\gamma/vd$	$d_f/d$
Mean field standard perc.	$5/2 = 2.5$	$1/3 \approx 0.33..$	$2/3 \approx 0.66..$
Mean field directed perc.	3	$1/2$	$1/2$
Mean field model	$2.38 \pm 0.04$	$0.7 \pm 0.01$	$0.82 \pm 0.01$
3D standard perc.	2.15	0.67	0.84
2D standard perc.	$187/91 \approx 2.055$	$43/48 \approx 0.896$	$91/96 \approx 0.948$
2D directed perc.	$\approx 2.66$	$\approx 1.07$	$\approx 0.60$
2D model	$2.0 \pm 0.1$	$0.86 \pm 0.02$	$0.91 \pm 0.02$
Experiments	$2.01 \pm 0.01$	$0.82 \pm 0.08$	$1.01 \pm 0.06$

# Conclusions

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- Moving away from criticality leads mitochondria (and therefore the cell) to a pathological state.
- Mitochondrial critical point belongs to the standard percolation universality class.

N. Zamponi, E. Zamponi, S.A. Cannas, O.V. Billoni, P. Helguera, D. R. Chialvo, *Mitochondrial network complexity emerges from fission/fusion dynamics*, Scientific Reports **8**, 363 (2018)

N. Zamponi, E. Zamponi, S.A. Cannas, D. R. Chialvo, *Universal dynamics of mitochondrial networks: a finite-size scaling analysis*, Scientific Reports **12**, 17074 (2022)

Happy Birthday  
Constantino!