



# STATISTICAL MECHANICS FOR COMPLEXITY

A CELEBRATION OF THE 80TH BIRTHDAY OF CONSTANTINO TSALLIS

RIO DE JANEIRO, 6 TO 10 NOVEMBER 2023

## Characterizing Spatiotemporal Complex Patterns with GPA & Tsallis Permutation Entropy

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Laboratório Associado de Computação e Matemática Aplicada  
Coordenadoria de Pesquisa Aplicada e Desenvolvimento Tecnológico-COPDT  
Instituto Nacional de Pesquisas Espaciais-INPE

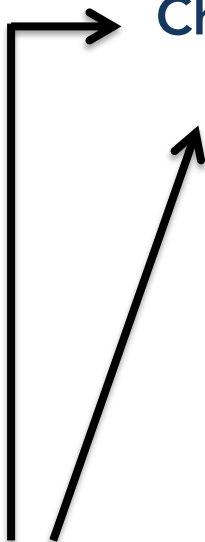
Rubens Sautter-CAP-INPE

Luan O. Baraúna – CAP-INPE

Erico L. Rempel – ITA-CTA

Juan A. Valdivia – DF-U.Chile

Alejandro Flery – Un. Wellington, NZ



*Timeline*

1998

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□ 1.ª Escola Brasileira de Ciências Emergentes | São José dos Campos - SP



Processo: 98/11624-3

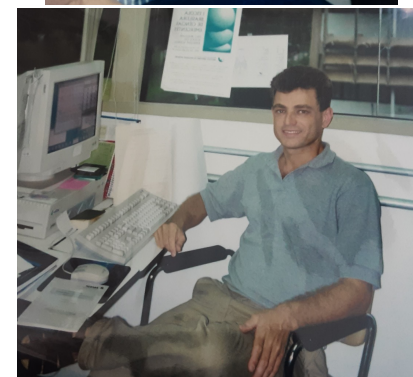
Linha de fomento: Auxílio Organização - Reunião Científica

Vigência: 22 de novembro de 1998 - 26 de novembro de 1998

Área do conhecimento: Ciências Exatas e da Terra - Física - Áreas Clássicas de Fenomenologia e suas Aplicações

Pesquisador responsável: Reinaldo Roberto Rosa  

Beneficiário: Reinaldo Roberto Rosa  



1998



1998



1998



# 21st IUPAP International Conference on Statistical Physics (STATPHYS 21)

15-21 July 2001. Cancun, Mexico (C01-07-15.1)



## Nonlinear Analysis: Theory, Methods & Applications

Volume 47, Issue 5, August 2001, Pages 3521-3530



### Nonextensive thermostatics description of intermittency in turbulence and financial markets

Fernando M. Ramos, Reinaldo R. Rosa, Camilo Rodrigues Neto,  
Mauricio J.A. Bolzan, Leonardo D. Abren Sá




## Physica A: Statistical Mechanics and its Applications

Volume 344, Issues 3-4, 15 December 2004, Pages 554-561



### Value-at-risk and Tsallis statistics: risk analysis of the aerospace sector

Adriana P. Mattedi<sup>a</sup>, Fernando M. Ramos<sup>a</sup>, Reinaldo R. Rosa<sup>a</sup>  ,  
Rosario N. Mantegna<sup>b</sup>

Tsallis



□ **Harry L. Swinney | University of Texas at Austin - Estados Unidos**

**Processo:** 01/03488-7

**Linha de fomento:** Auxílio à Pesquisa - Pesquisador Visitante - Internacional

**Vigência:** 31 de maio de 2001 - 17 de junho de 2001

**Área do conhecimento:** Ciências Exatas e da Terra - Física - Áreas Clássicas de Fenomenologia e suas Aplicações

**Pesquisador responsável:** Reinaldo Roberto Rosa   

**Beneficiário:** Reinaldo Roberto Rosa   

**Pesquisador visitante:** Harry L. Swinney

**Inst. do pesquisador visitante:** University of Texas at Austin (UT) (Estados Unidos)





## Non-extensive statistics and three-dimensional fully developed turbulence

Fernando M. Ramos <sup>a</sup>   [Reinaldo R. Rosa <sup>a</sup>](#), [Camilo Rodrigues Neto <sup>a</sup>](#), [Maurício J.A. Bolzan <sup>a</sup>](#), [Leonardo D. Abreu Sá <sup>a</sup>](#), [Haroldo F. Campos Velho <sup>a</sup>](#)




## Generalized thermostatistics description of probability densities of turbulent temperature fluctuations

Fernando M. Ramos <sup>a</sup>   [Reinaldo R. Rosa <sup>a</sup>](#), [Camilo Rodrigues Neto <sup>a</sup>](#), [Maurício J.A. Bolzan <sup>b</sup>](#), [Leonardo D. Abreu Sá <sup>b</sup>](#)

## JOURNAL OF GEOPHYSICAL RESEARCH **Atmospheres**

AN AGU JOURNAL

Climate and Dynamics |  **Free Access**

## Analysis of fine-scale canopy turbulence within and above an Amazon forest using Tsallis' generalized thermostatistics

Maurício J. A. Bolzan, Fernando M. Ramos, Leonardo D. A. Sá, Camilo Rodrigues Neto, Reinaldo R. Rosa

First published: 06 September 2002 | <https://doi.org/10.1029/2001JD000378> | Citations: 24



Honorable Mention  
for the pioneering work  
Applying Tsallis's Statistics  
in Turbulence



Preface

## Anomalous Distributions, Nonlinear Dynamics, and Nonextensivity

[Harry Swinney <sup>a</sup>](#)   [Constantino Tsallis <sup>b 1</sup>](#) 

<sup>a</sup> Department of Physics, Center for Nonlinear Dynamics, University of Texas at Austin, Austin, TX 78712, USA



<sup>b</sup> CBPF, Brazilian Center for Research in Physics, The Rio de Janeiro 22290-180, RJ, Brazil




## Physica D: Nonlinear Phenomena


Volume 193, Issues 1-4, 15 June 2004, Pages 278-291

## Atmospheric turbulence within and above an Amazon forest

[Fernando Manuel Ramos <sup>a</sup>](#)   [Maurício José Alves Bolzan <sup>b</sup>](#), [Leonardo Deane Abreu Sá <sup>a c</sup>](#), [Reinaldo Roberto Rosa <sup>a</sup>](#)



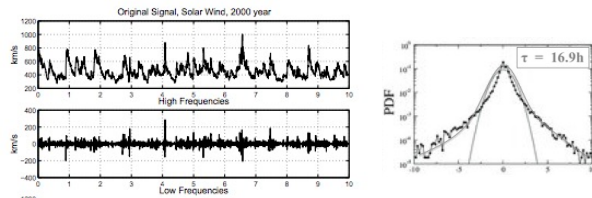
Advances in Space Research  
Volume 32, Issue 6, September 2003, Pages 1175-1180




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## Nonlinear distribution of the sunspot magnetic field during the solar maximum

Reinaldo R. Rosa<sup>1</sup> ✉, Encarnacion A.M. Gonzalez-Machado<sup>2</sup>,  
Heloisa M. Boechat-Roberty<sup>2</sup>, Nandamudi L. Vijaykumar<sup>1</sup>,  
Hanumant S. Sawant<sup>3</sup>





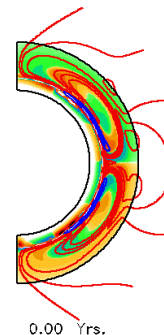
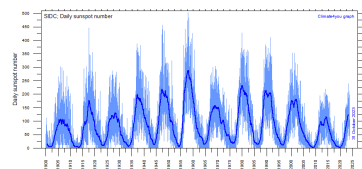
Journal of Atmospheric and Solar-  
Terrestrial Physics  
Volume 67, Issues 17-18, December 2005, Pages 1843-1851



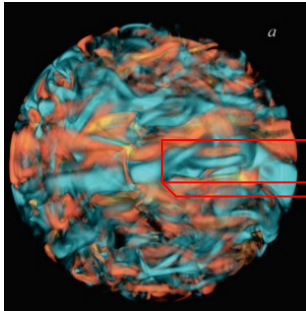
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## Generalized thermostatics and wavelet analysis of solar wind and proton density variability

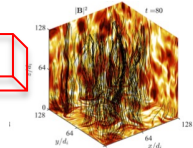
Maurício José Alves Bolzan<sup>a</sup> ✉, Reinaldo Roberto Rosa<sup>b</sup>,  
Fernando Manuel Ramos<sup>b</sup>, Paulo Roberto Faundes<sup>a</sup>, Yogeshwar Sahai<sup>a</sup>



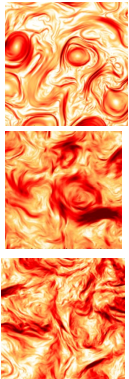
Ap. J 853, 2018



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3D+1





# Observational Cosmology → Cosmological Large Scale Structure Formation (Millennium Simulation – Virgo Consortium)



Physica D: Nonlinear Phenomena

Volumes 168–169, 1 August 2002, Pages 404–409



## Multiscaling and nonextensivity of large-scale structures in the Universe

F.M. Ramos<sup>a</sup>, C.A. Wuensche<sup>b</sup>, A.L.B. Ribeiro<sup>c</sup>, R.R. Rosa<sup>a</sup>



Physica A: Statistical Mechanics and its Applications

Volume 344, Issues 3–4, 15 December 2004, Pages 743–749



## Nonextensivity and galaxy clustering in the Universe

C.A. Wuensche<sup>a</sup>, A.L.B. Ribeiro<sup>b</sup>, F.M. Ramos<sup>c</sup>, R.R. Rosa<sup>c</sup>

Show more

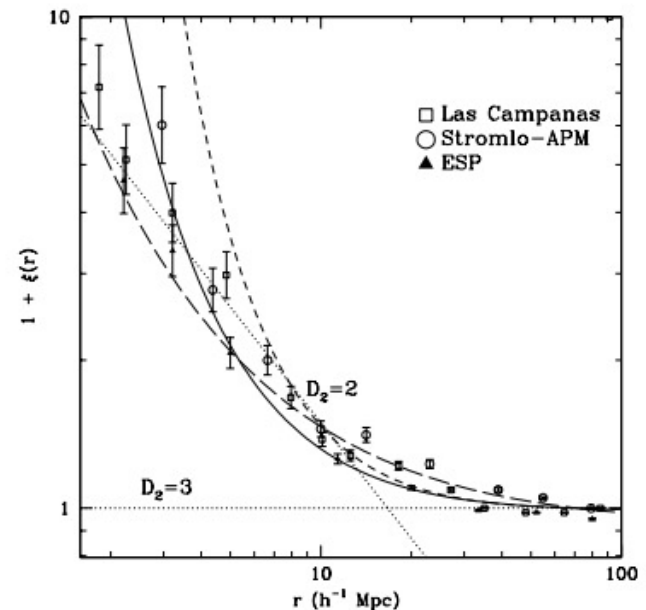
$$S_q = k \frac{1 - \sum_i^W p_i^q}{q-1} = \frac{k}{q-1} \sum_i^W (p_i - p_i^q)$$

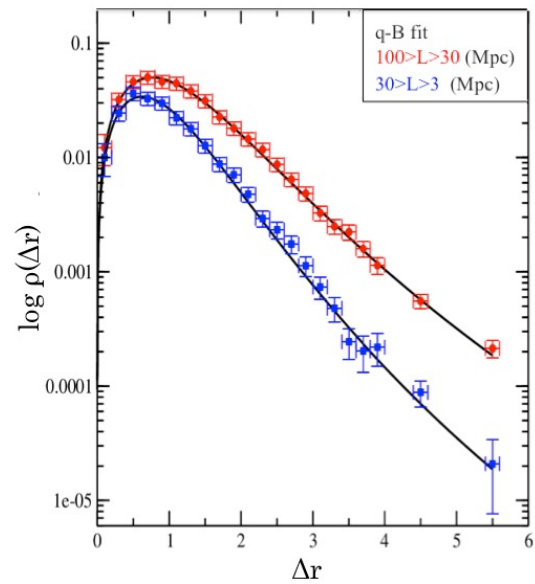
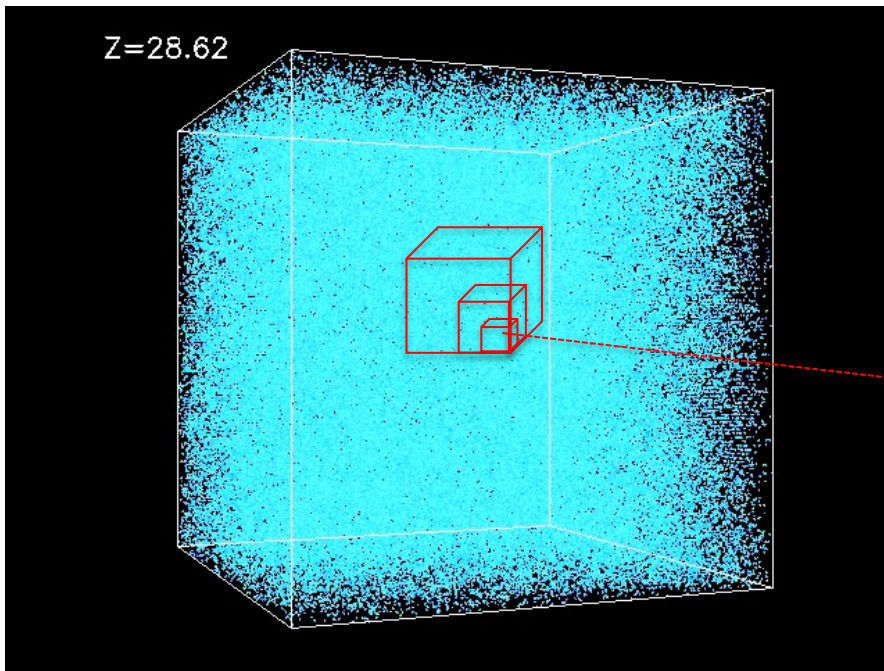
$$r \sim 1/(q-1)^\beta$$

$$(q-1) \sim (1/r)^{1/\beta}, \text{ with } \beta > 0.$$

$$D_2(r) = 3 \frac{\log(2 + a(1 - q(r)))}{\log 2} \quad a = 2S_q/k$$

$$1 + \xi(r) = 13 D_2 r^{(D_2 - 3)}$$





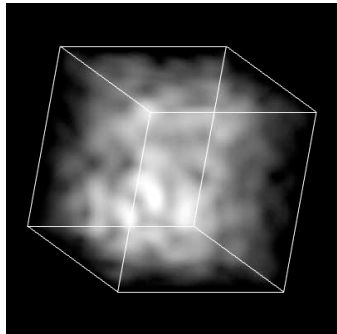


# Characterizing **Spatiotemporal Complex Patterns** with **GPA & Tsallis Spectral-Permutation Entropy**

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Rubens Sautter-CAP-INPE  
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 +55 12 98129-8988



# Gradient pattern analysis of Swift–Hohenberg dynamics: phase disorder characterization

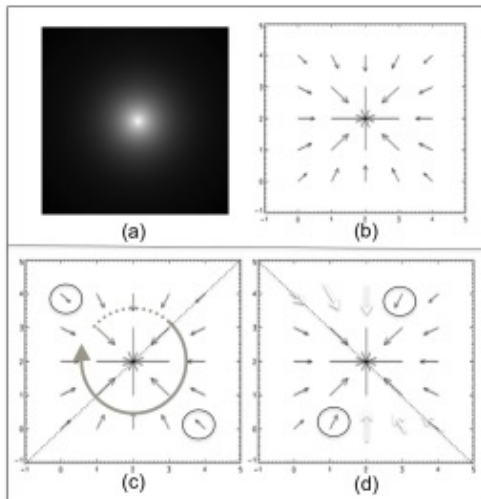
R.R. Rosa<sup>a</sup>, J. Pontes<sup>b</sup>, C.I. Christov<sup>1,d</sup>, F.M. Ramos<sup>a</sup>, C. Rodrigues Neto<sup>a</sup>, E.L. Rempel<sup>a</sup>, D. Walgraef<sup>c</sup>

JOURNAL ARTICLE

## Gradient pattern analysis applied to galaxy morphology FREE

R.R. Rosa ✉, R.R. de Carvalho ✉, R.A. Sautter, P.H. Barchi ✉, D.H. Stalder, T.C. Moura, S.B. Rembold, D.R.F. Morell, N.C. Ferreira

Monthly Notices of the Royal Astronomical Society: Letters, Volume 477, Issue 1, June 2018, Pages L101–L105, <https://doi.org/10.1093/mnrasl/sly054>

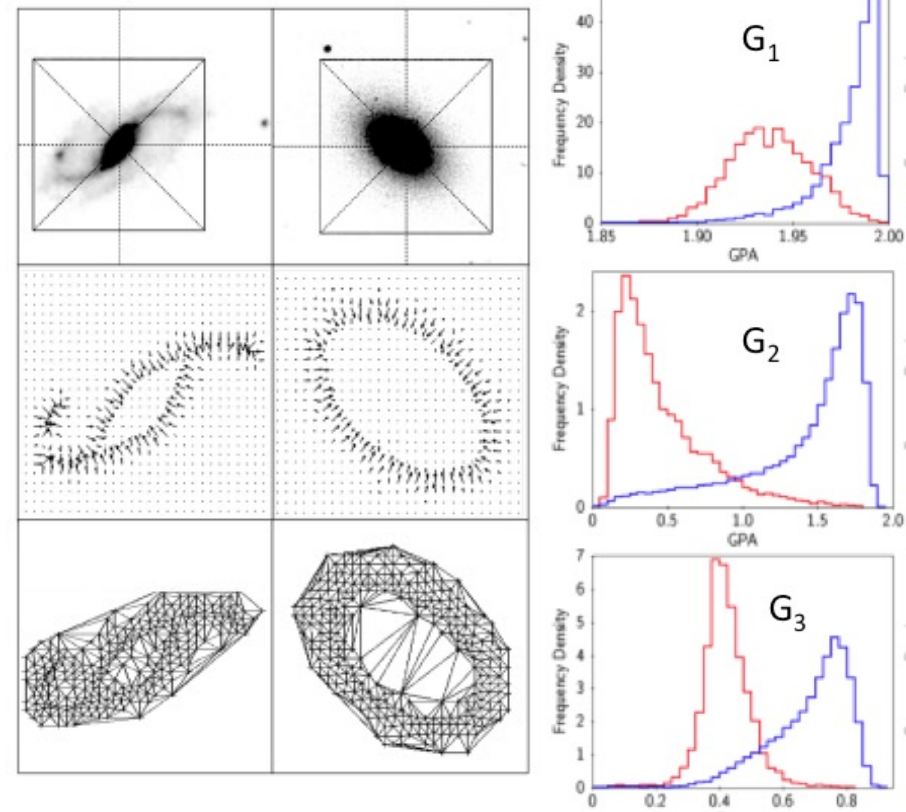


$$G_1 = \frac{T_A - V_A}{V_A}$$

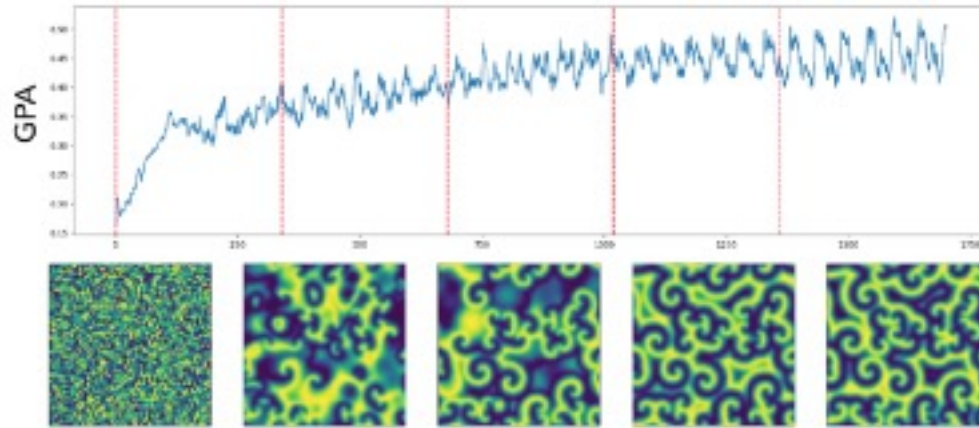
$$G_2 = \frac{V_A}{V} \left( 1 - \frac{|\sum_{i=0}^{V_A} v_i|}{2 \sum_{i=0}^{V_A} |v_i|} \right)$$

## Acknowledgements

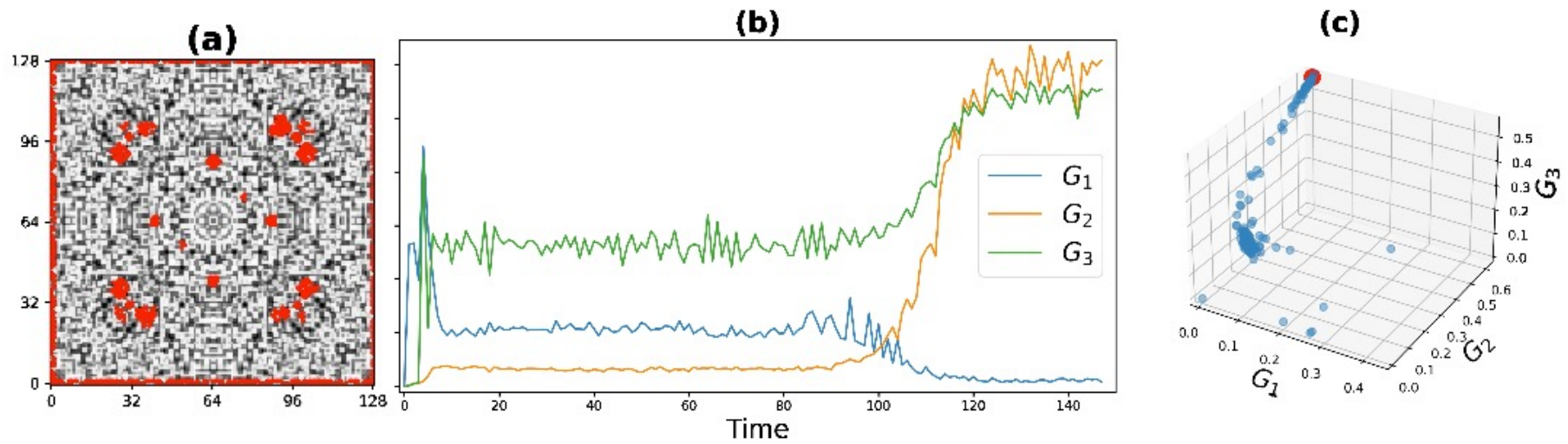
A.F.S. acknowledges support from the Brazilian Agency CNPq. R.R.R. is very grateful to the Brazilian Agency FAPESP under PROC 97/13374-1 and PROC 98/03104-0, and to F.M. Ramos, C. Rodrigues-Neto and C. Tsallis for discussions and valuable suggestions. L.S.R. acknowledges financial support from Luppia Engenharia e Sistemas and the Swedish Research Council for Engineering Science. E.V. is very grateful to support from the Danish Natural Science Research Council, the Carlsberg Foundation, Director...



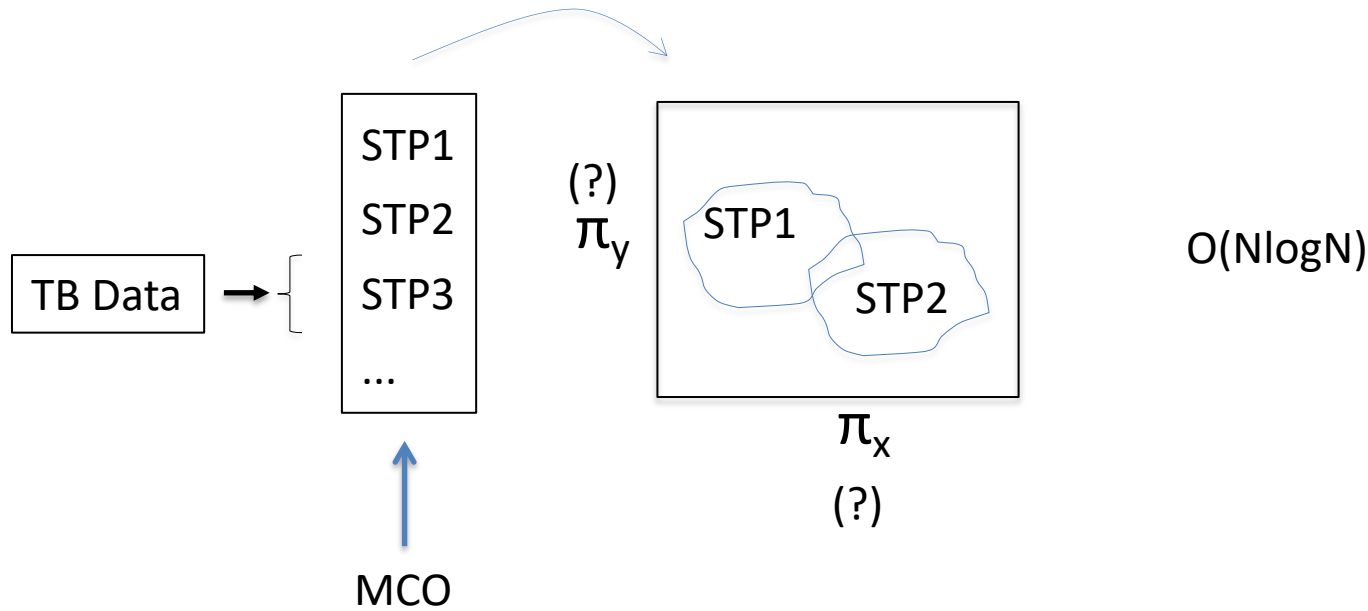
# STP1



# STP2



# The Analytical Challenge



Physica A: Statistical Mechanics and its Applications

Volume 523, 1 June 2019, Pages 10-20



Multiscale Tsallis permutation entropy analysis for complex physiological time series

Chao Li Pengjian Shang

RESEARCH ARTICLE | JUNE 07 2021

**ordpy: A Python package for data analysis with permutation entropy and ordinal network methods**

Arthur A. B. Pessa ; Haroldo V. Ribeiro

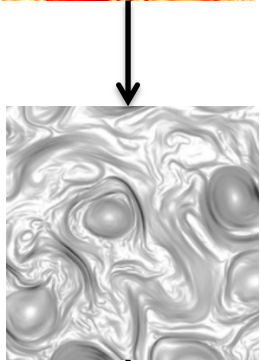
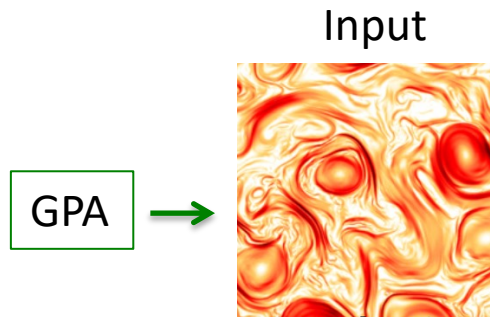
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+ Author & Article Information

Chaos 31, 063110 (2021)

<https://doi.org/10.1063/5.0049901> Article history

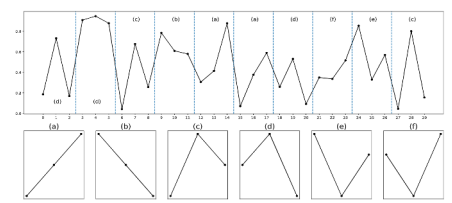
<http://github.com/arthurpessa/ordpy>



1D Vector (4096)

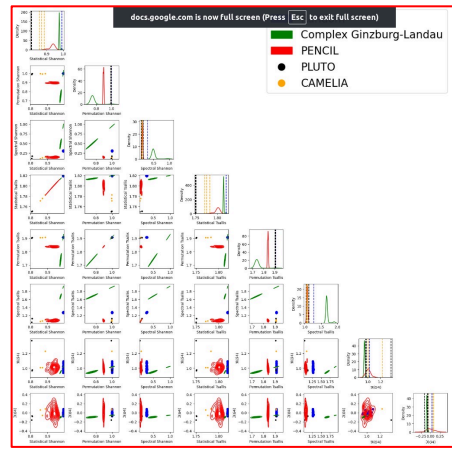
ORDpy (sautter)

$$G_4 = \sum_i P(v_i) \cdot \log\left(\frac{1}{P(\varphi_i)}\right)$$



**D!**  
(D=3, D=9)

Output



Bandt & Pompe

$$S_H = -k \sum_{i=1}^W p_i \log p_i$$

Shannon Permutation

$$S_q = k \left( \frac{1 - \sum_{i=1}^W p_i^q}{q - 1} \right)$$

Tsallis Permutation

$$S_H = -k \sum_{i=1}^W p_i \log p_i$$

Shannon Spectral

$$S_q = k \left( \frac{1 - \sum_{i=1}^W p_i^q}{q - 1} \right)$$

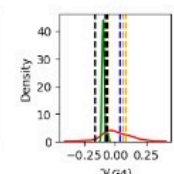
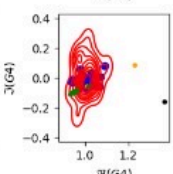
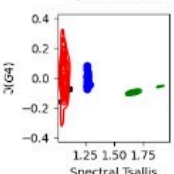
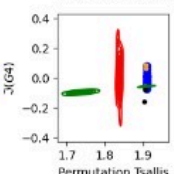
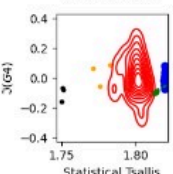
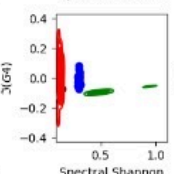
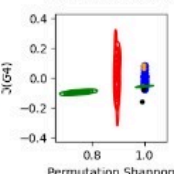
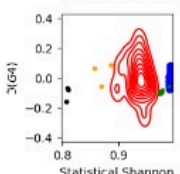
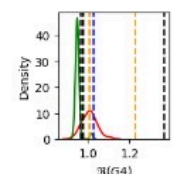
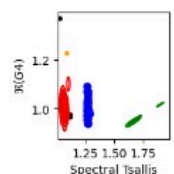
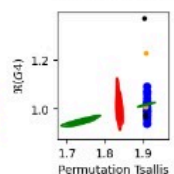
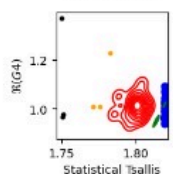
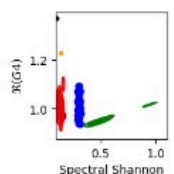
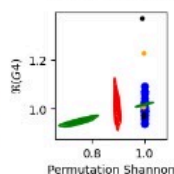
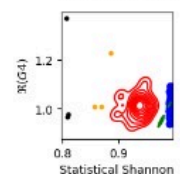
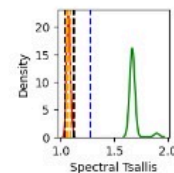
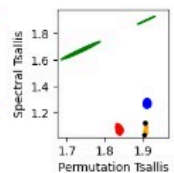
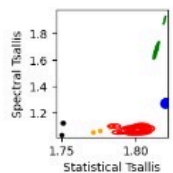
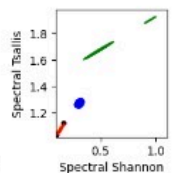
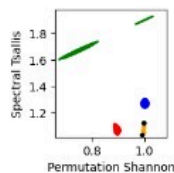
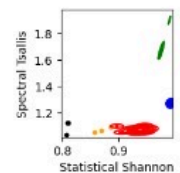
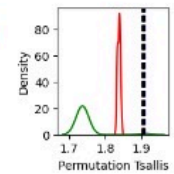
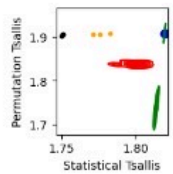
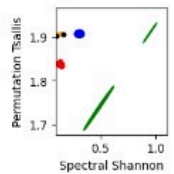
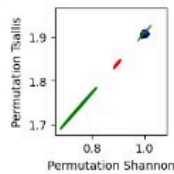
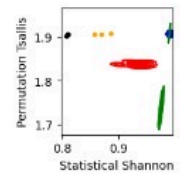
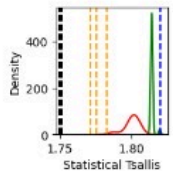
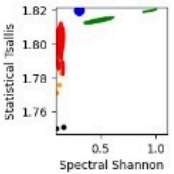
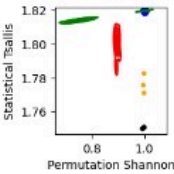
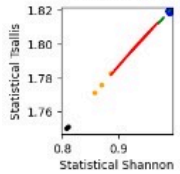
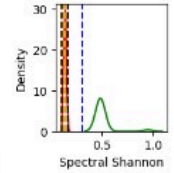
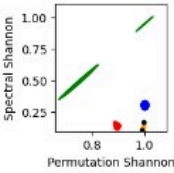
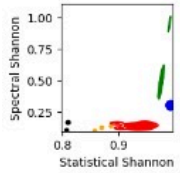
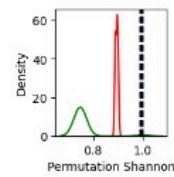
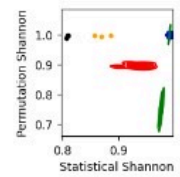
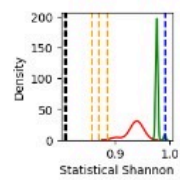
Tsallis Spectral

PSD

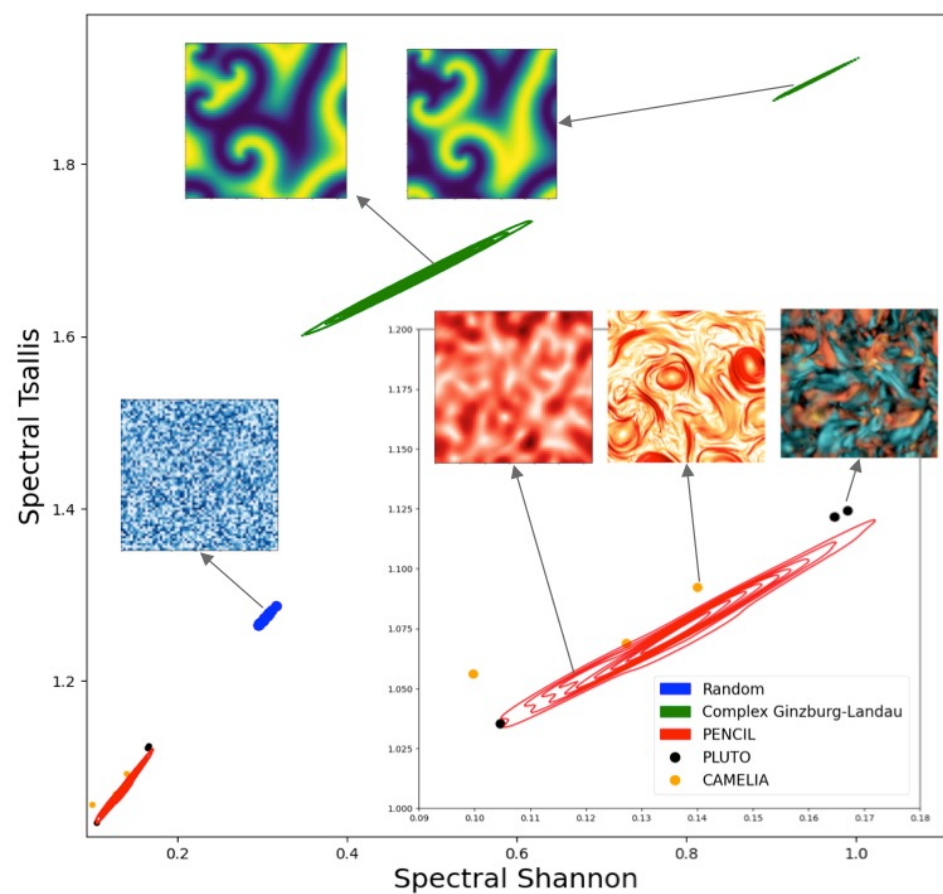
$p_i$

$p_i$

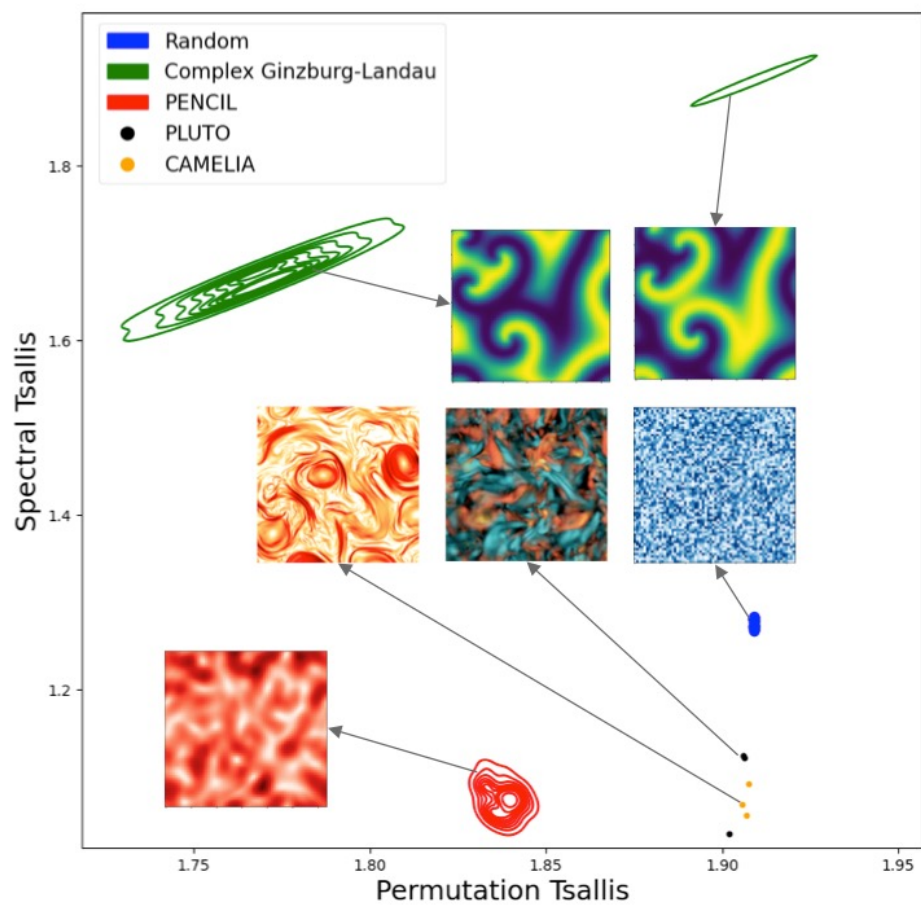
$64^2$







RTX4090 (<O>)  
 1GB/ $3.2 \times 10^{-3}$   
 1GB/ $6.5 \times 10^{-1}$  (GPA)  
 1GB/ $1.2 \times 10^{-3}$



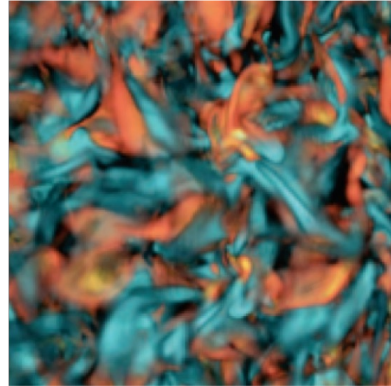
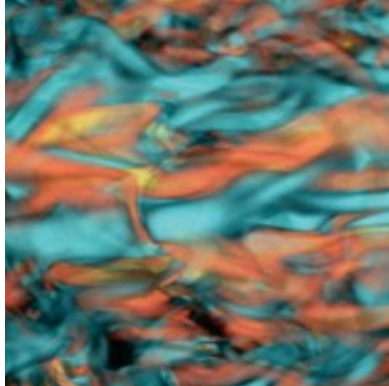
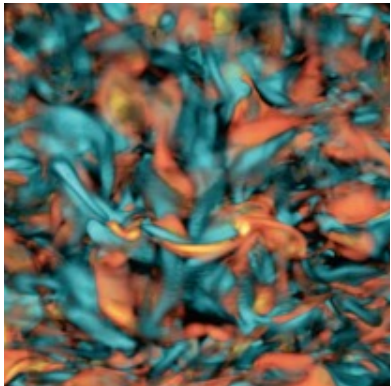
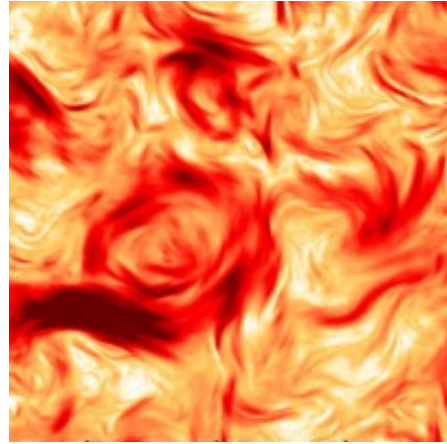
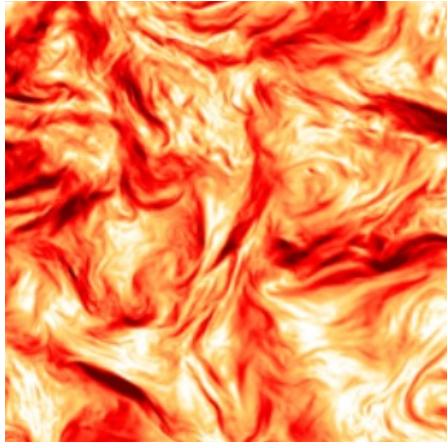
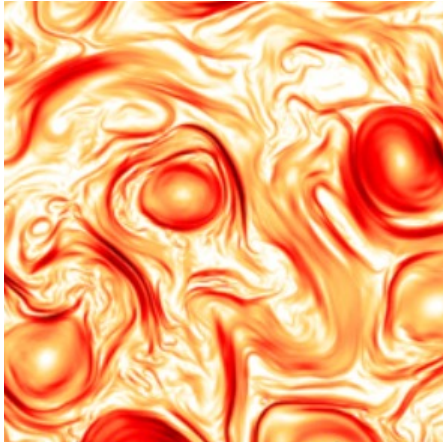
## Future Work

- Massive Application Criteria
- Explicitly Marking Dynamics
- Labels for Deep Learning

I have learning on [Non-additive Tsallis Fundamental Physics](#): Tear and Shear Processes




Constantino!!



Article

# Characterizing Spatiotemporal Complex Patterns with Tsallis Permutation Entropy

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**Abstract:** Complexity measures are important for understanding and analysing time series and one-dimensional profiles. However, the extension of these methods to two-dimensional data is still lacking. In the scope of physics, the problem of classifying complex 2D patterns is fundamental to some theoretical and applied fields, from quantum mechanics to cosmology. To achieve this objective, different tools can be used depending on the knowledge of the system and the quality and quantity of accessible data. In this work we selected different classes of structural patterns arising from 2D and 3D turbulent and chaotic processes to test the performance of Tsallis permutation entropy (TPE) as a classifier of complex textures. The results show that TPE is a valuable technique for analysing complex 2D patterns and that it is computationally faster than other techniques such as BPPE and GPA.

**Keywords:** spatiotemporal patterns; complexity measure; Tsallis permutation entropy; gradient pattern analysis