Universidade Federal do Ceará
Departamento de Física

A Maximum Entropy Model for the Network of Commercial Transactions between Cities based on Data from Electronic Invoices

José Soares de Andrade Jr.

Departamento de Física - UFC

Fortaleza, Ceará, Brazil

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C A P E S

F U N C A P

It started with disordered superconductors…

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Flux Front Penetration in Disordered Superconductors

Stefano Zapperi,[1] André A. Moreira,[2] and José S. Andrade, Jr.[2]

[1]*INFM sezione di Roma 1, Dipartimento di Fisica, Università "La Sapienza," P.le A. Moro 2, 00185 Roma, Italy*

[2]*Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil*

(Received 7 November 2000)

We investigate flux front penetration in a disordered type-II superconductor by molecular dynamics simulations of interacting vortices and find scaling laws for the front position and the density profile. The scaling can be understood by performing a coarse graining of the system and writing a disordered nonlinear diffusion equation. Integrating numerically the equation, we observe a crossover from flat to fractal front penetration as the system parameters are varied. The value of the fractal dimension indicates that the invasion process is described by gradient percolation.

DOI: 10.1103/PhysRevLett.86.3622 PACS numbers: 74.60.Ge, 05.45.–a, 47.55.Mh

Collecting all the terms, we finally obtain a disordered nonlinear diffusion equation for the density of flux lines

$$\Gamma \frac{\partial \rho}{\partial t} = \vec{\nabla}(a\rho\vec{\nabla}\rho - \rho\vec{F}_c) + k_B T \nabla^2 \rho . \quad (5)$$

And the Tsallis thermostatistics was there!

PRL **105**, 260601 (2010) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2010

Thermostatistics of Overdamped Motion of Interacting Particles

J. S. Andrade, Jr.,[1,3] G. F. T. da Silva,[1] A. A. Moreira,[1] F. D. Nobre,[2,3] and E. M. F. Curado[2,3]
[1]*Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil*
[2]*Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil*
[3]*National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil*
(Received 8 August 2010; published 22 December 2010)

We show through a nonlinear Fokker-Planck formalism, and confirm by molecular dynamics simulations, that the overdamped motion of interacting particles at $T = 0$, where T is the temperature of a thermal bath connected to the system, can be directly associated with Tsallis thermostatistics. For sufficiently high values of T, the distribution of particles becomes Gaussian, so that the classical Boltzmann-Gibbs behavior is recovered. For intermediate temperatures of the thermal bath, the system displays a mixed behavior that follows a novel type of thermostatistics, where the entropy is given by a linear combination of Tsallis and Boltzmann-Gibbs entropies.

DOI: 10.1103/PhysRevLett.105.260601 PACS numbers: 05.10.Gg, 05.20.−y, 05.40.Fb, 05.45.−a

is a conveniently rescaled variable. This functional leads to the following entropic form:

$$S[P] = \frac{D}{\bar{\gamma}}\left[1 - \int_{-\infty}^{\infty} dx P^2(x,t)\right] - \frac{k_B T}{\bar{\gamma}} \int_{-\infty}^{\infty} dx P(x,t) \times \ln P(x,t). \qquad (17)$$

Equation (17) is precisely the sum of Tsallis entropy with $\nu = 2$, which appears as a consequence of many-body

$k_B T \gg a$, i.e., $\langle x^2 \rangle \propto t$. In the presence of a restoring external force and for $T > 0$, a stationary-state analytical solution for Eq. (13) can still be obtained,

$$\rho(x) = \frac{k_B T}{a} W\left\{\frac{a\rho(0)}{k_B T} \exp\left[\frac{a\rho(0)}{k_B T} - \frac{\alpha x^2}{2k_B T}\right]\right\}, \qquad (15)$$

where the W-Lambert function is defined implicitly through the equation $W(z)e^{W(z)} = z$ (see [22] and references therein). In order to test this prediction, extensive MD

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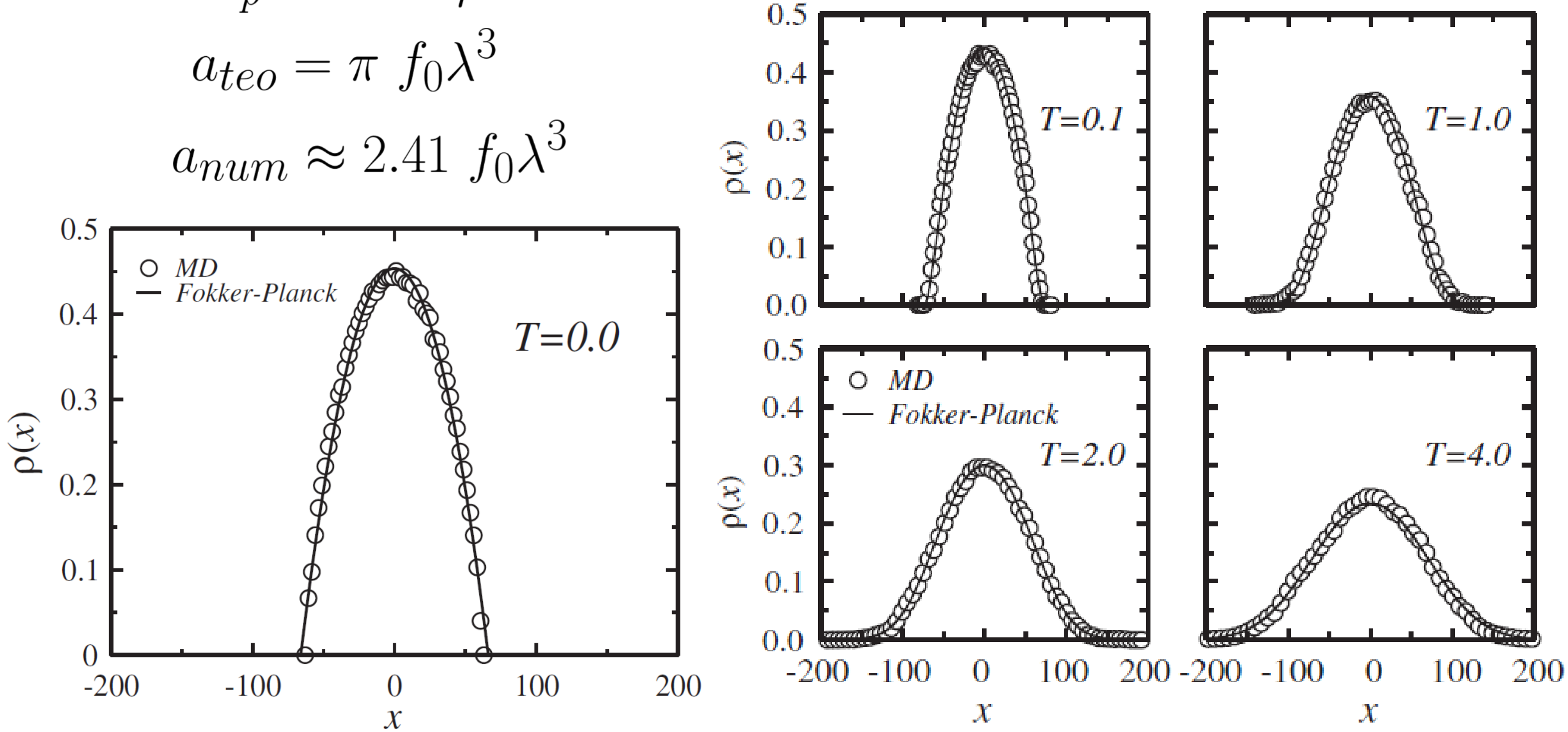
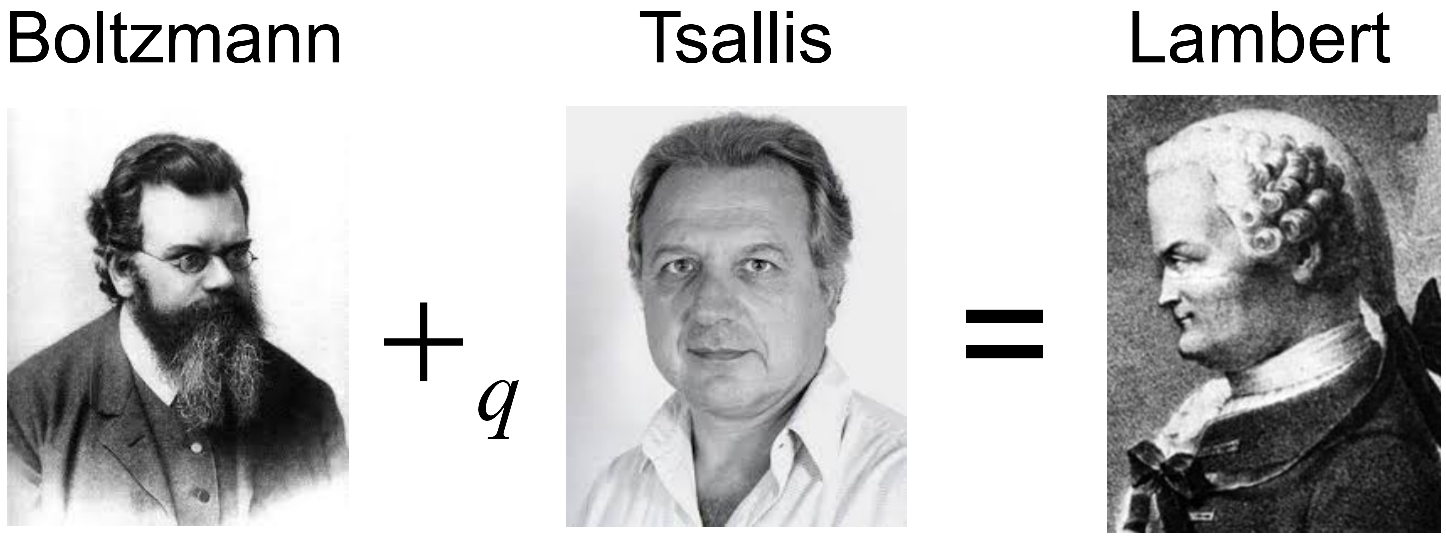
$$\mathbf{F}_p = -2a\nabla\rho$$

$$a_{teo} = \pi\ f_0\lambda^3$$

$$a_{num} \approx 2.41\ f_0\lambda^3$$




Boltzmann
Tsallis
Lambert
$+_q$
=

Nonextensive Statistics and Complex Networks

✓ **Dynamical and Growth Models:**

1) D. Soares, C. Tsallis, A. Mariz, and L. da Silva, Preferential attachment growth model and nonextensive statistical mechanics, Europhys. Lett. **70**, 70 (2005).
2) S. Brito, L. da Silva, and C. Tsallis, Role of dimensionality in complex networks, Sci. Rep. **6**, 27992 (2016).
3) S. Brito, T. C. Nunes, L. R. da Silva, and C. Tsallis, Scaling properties of d-dimensional complex networks, Phys. Rev. E **99**, 012305 (2019).

Prof. Luciano da Silva
A Real Force of Nature!

✓ **Random Network Models:**

PHYSICAL REVIEW RESEARCH **5**, 033088 (2023)

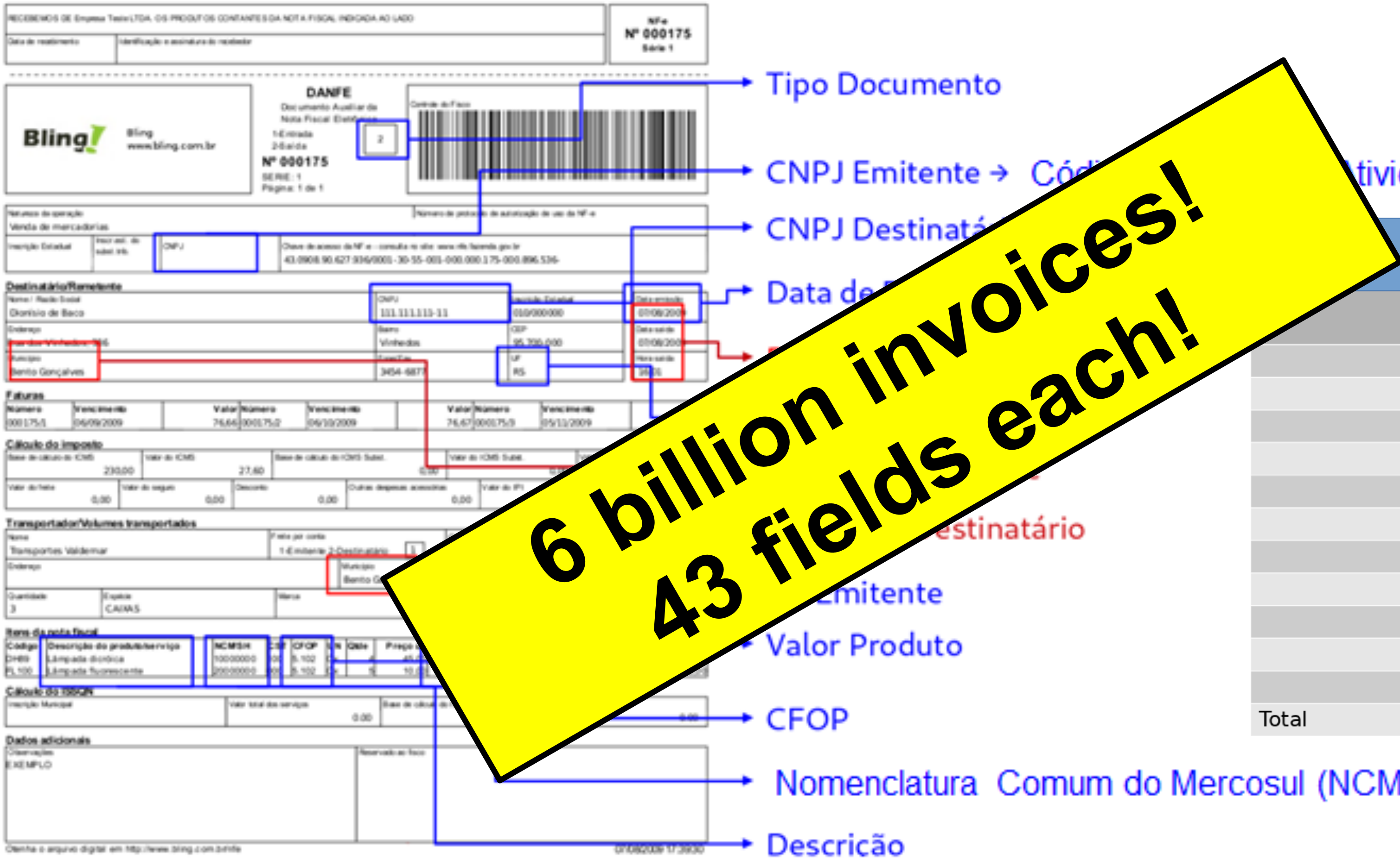
Random networks with q-exponential degree distribution

Cesar I. N. Sampaio Filho,[1] Marcio M. Bastos,[1] Hans J. Herrmann,[1,2] André A. Moreira,[1] and José S. Andrade, Jr.[1]
[1]*Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Brazil*
[2]*PMMH, ESPCI, CNRS UMR 7636, 7 quai St. Bernard, 75005 Paris, France*

1) **Using the Configurational Model, we can generate random unbiased complex networks exhibiting *q*-exponential degree distributions with arbitrary parameter values.**

2) **With an additional degree of freedom, these networks generalize the scale-free ones, therefore having great flexibility with respect to topological and transport properties, like assortativity, small-world behavior, and resilience to random and malicious attacks.**

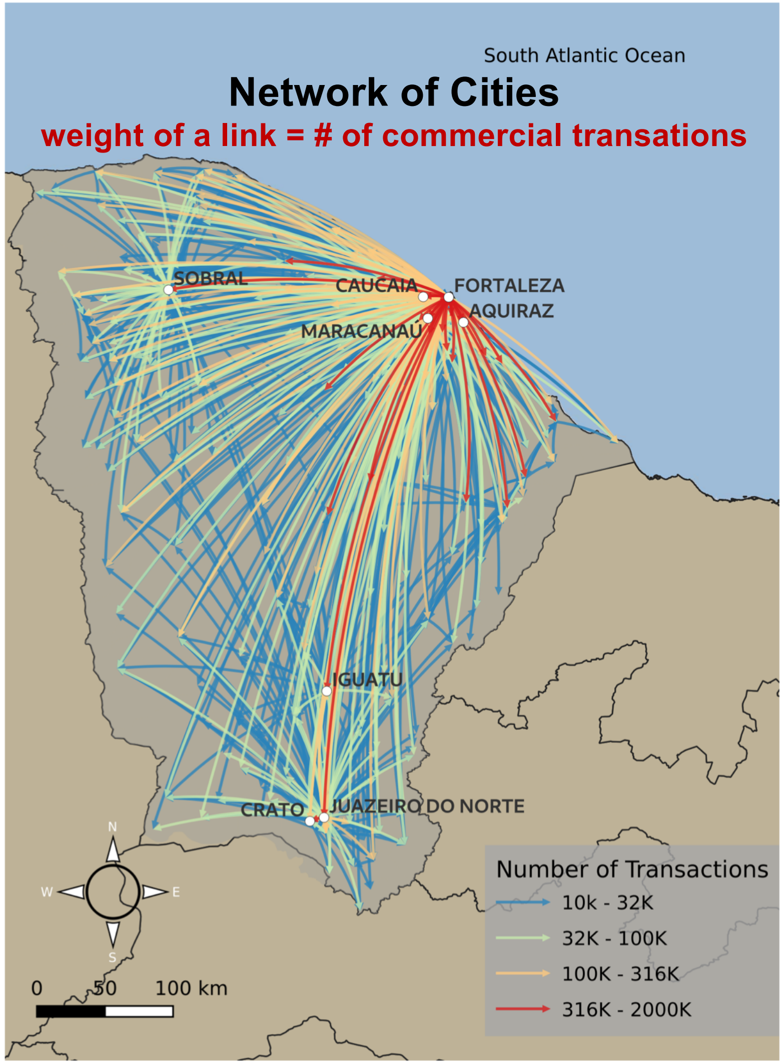
Projeto Cientista Chefe de Dados – Economia
Nota Fiscal Eletrônica



Jan/2010 à Jul/2020 1.6TB - 43 campos	
	No. de Registros (milhões)
2010	333
2011	471
2012	542
2013	610
2014	614
2015	594
2016	604
2017	634
2018	657
2019	667
2020	352
Total	6078

Definition of Macroregions of Financial Activity in Ceará State

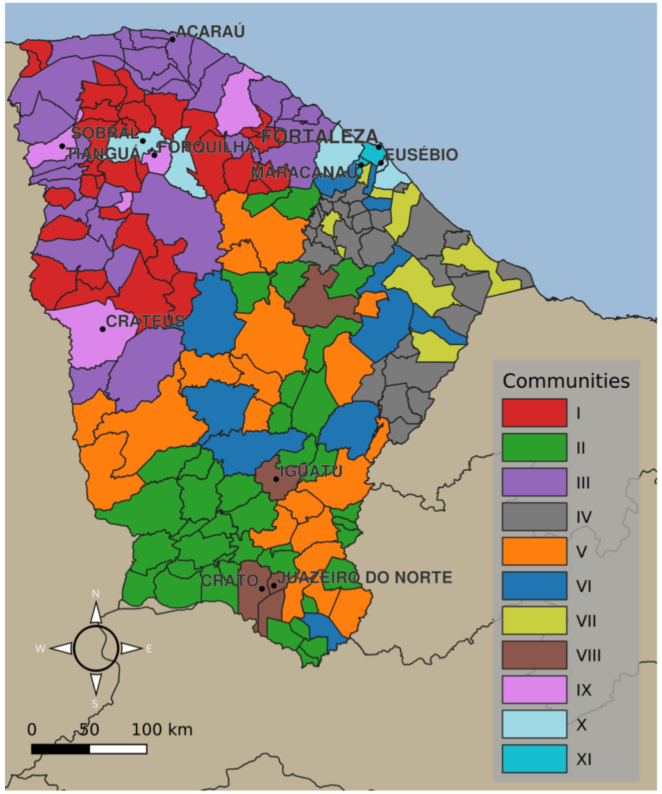
Commercial Transactions Network



Generalized Modularity Algorithm

Stochastic Block Model [Peixoto, *PRE* (2018)]

The algorithm identifies modules based on the correlations between pairs of sites and on the network generation process.



Definition of Macroregions of Financial Activity in Ceará State

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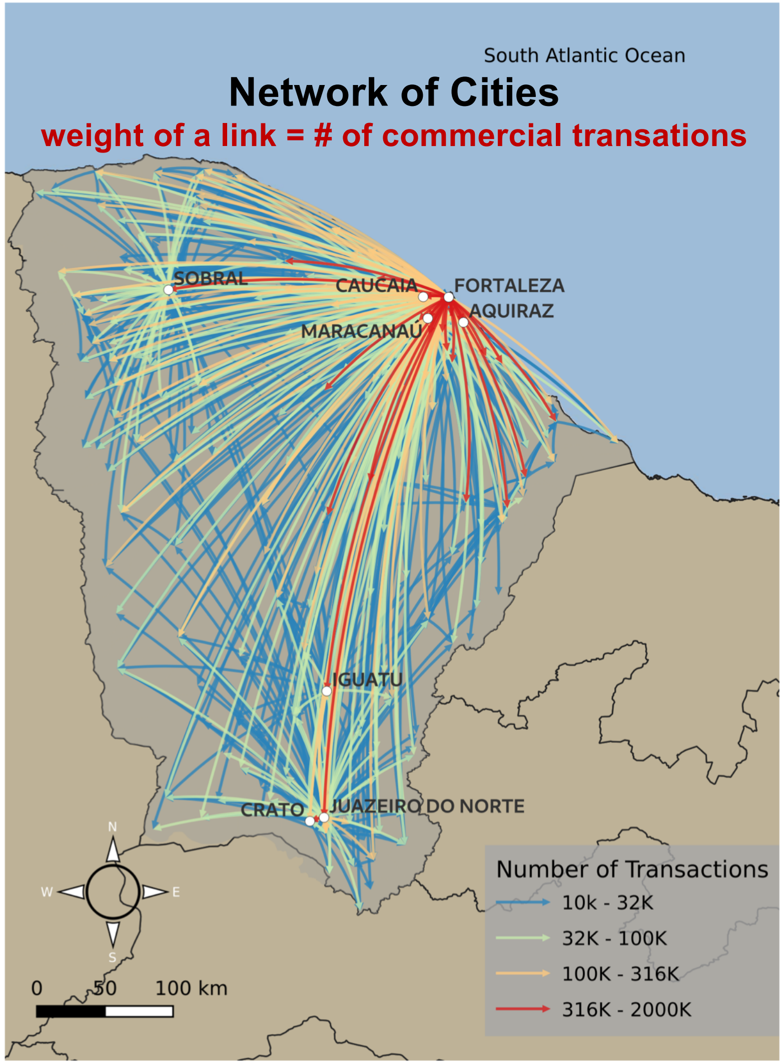
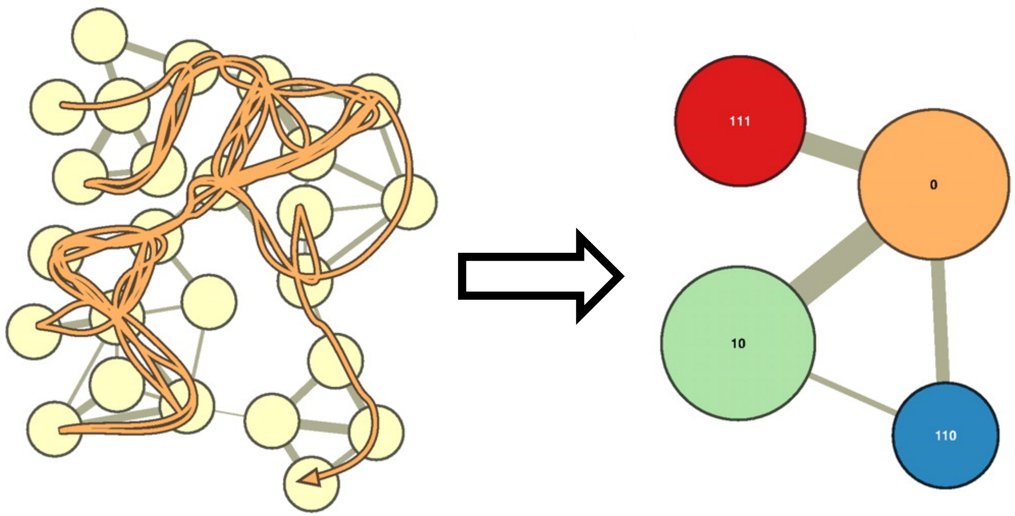


Infomap Algorithm

Rosvall & Bergstrom, *PNAS* (2008)

- ✓ The algorithm finds modules in a network by minimizing the lengths of a random walker's movements.
- ✓ The network will be compacted if regions are identified where the walker tends to remain for a long time.
- ✓ It captures the network's optimal community structure in terms of its associated flow dynamics.



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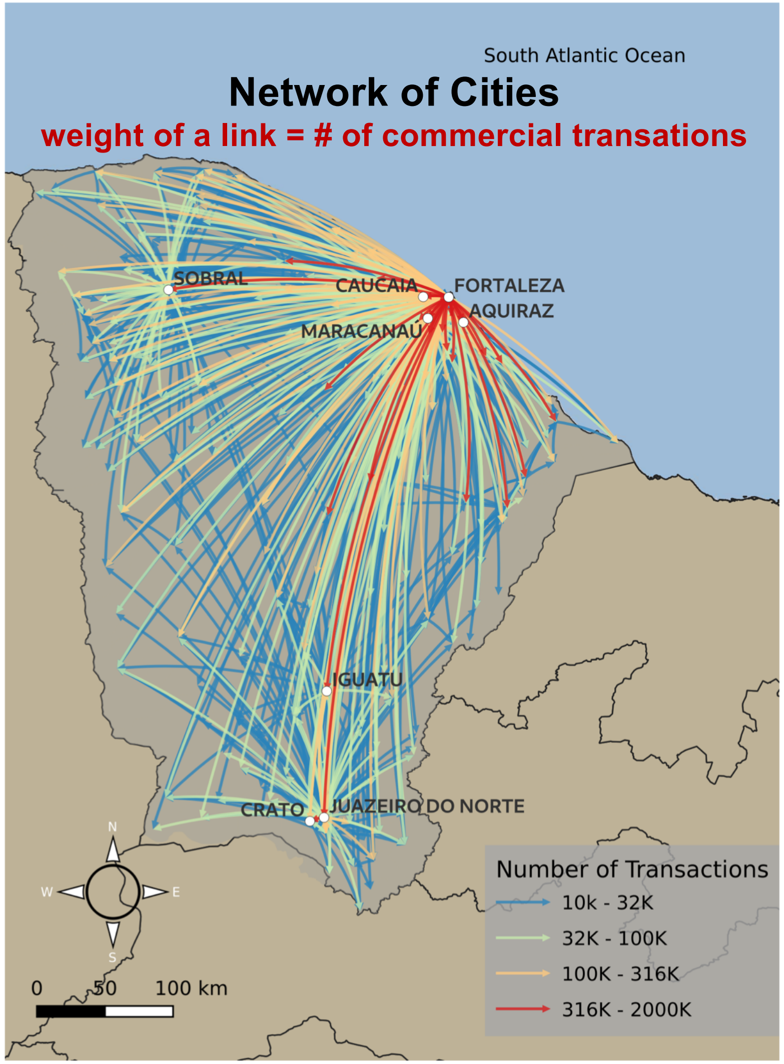

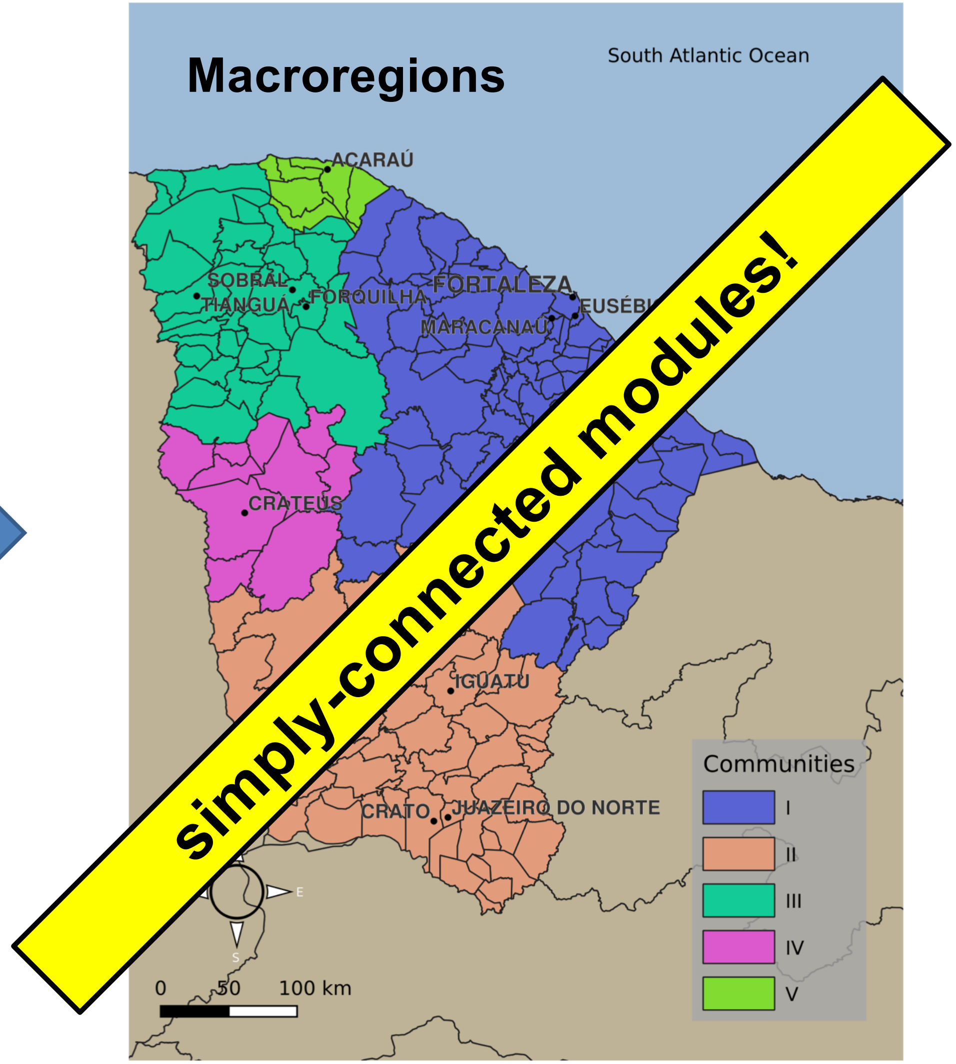

Definition of Macroregions of Financial Activity in Ceará State

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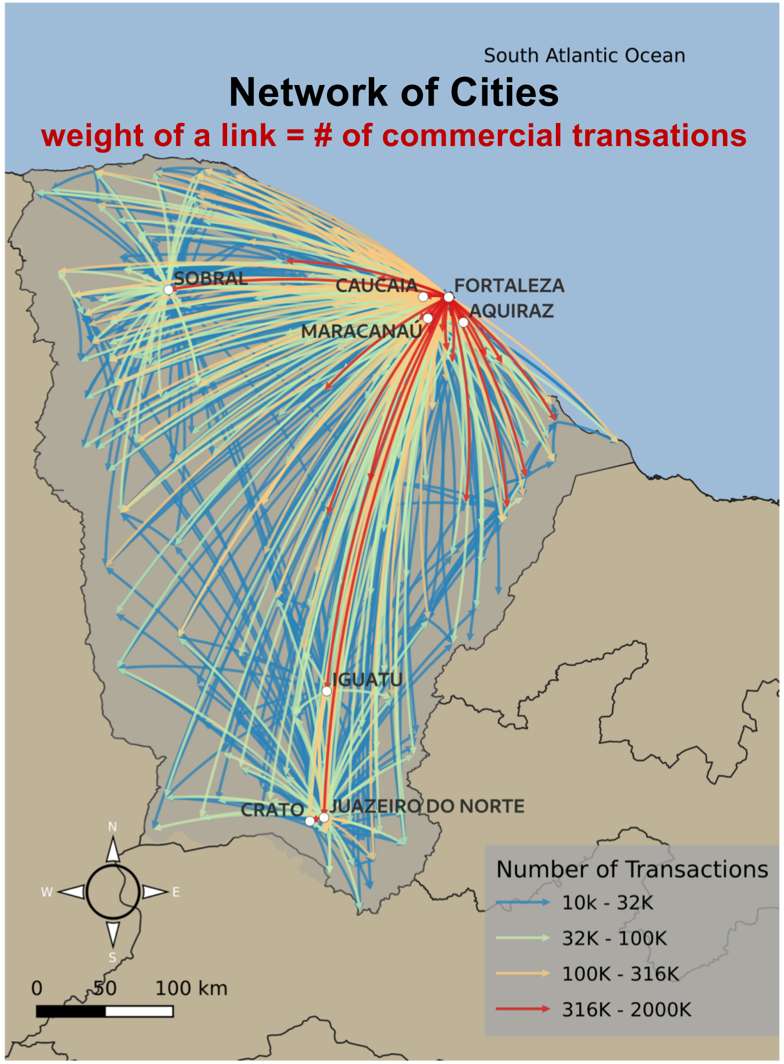
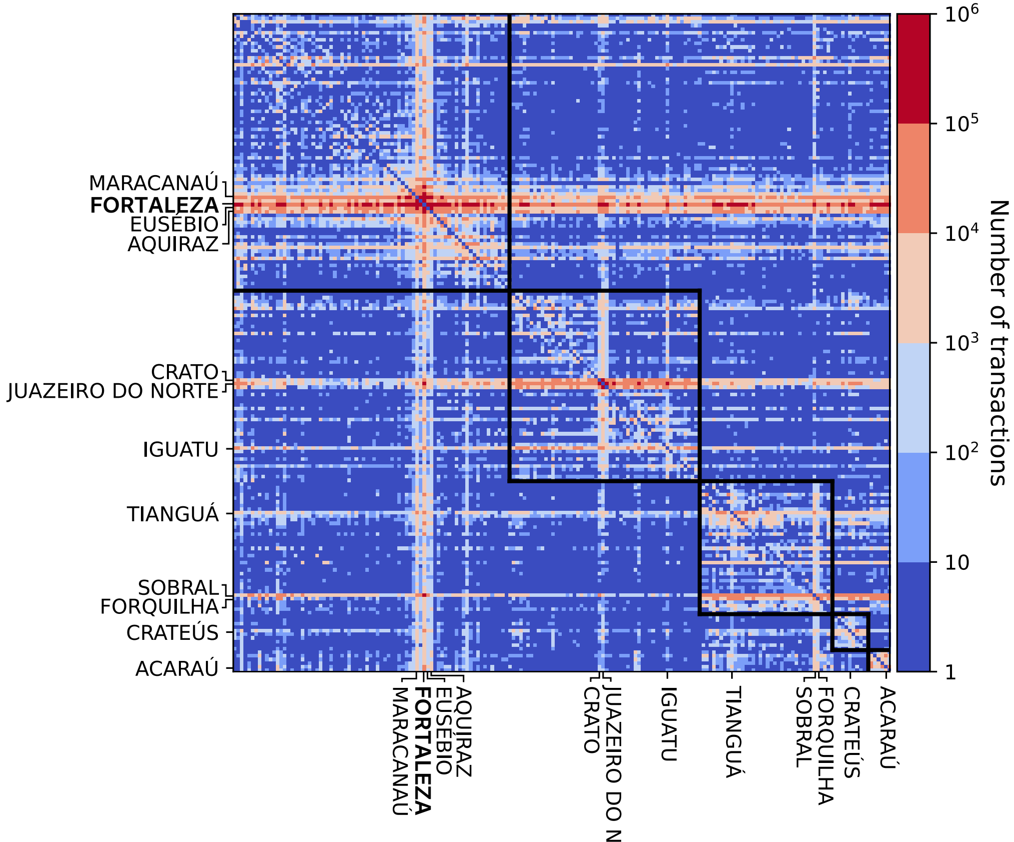


Matrix of Commercial Transactions



strong correlations within the communities!

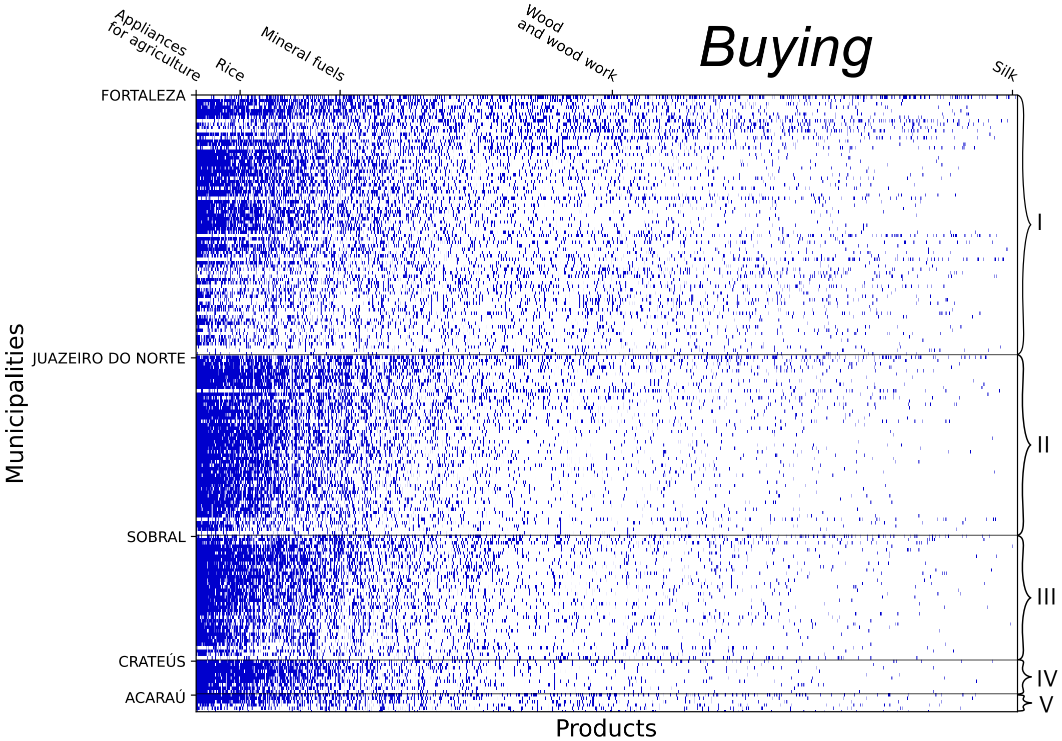
Bipartite Networks of Cities and Traded Products

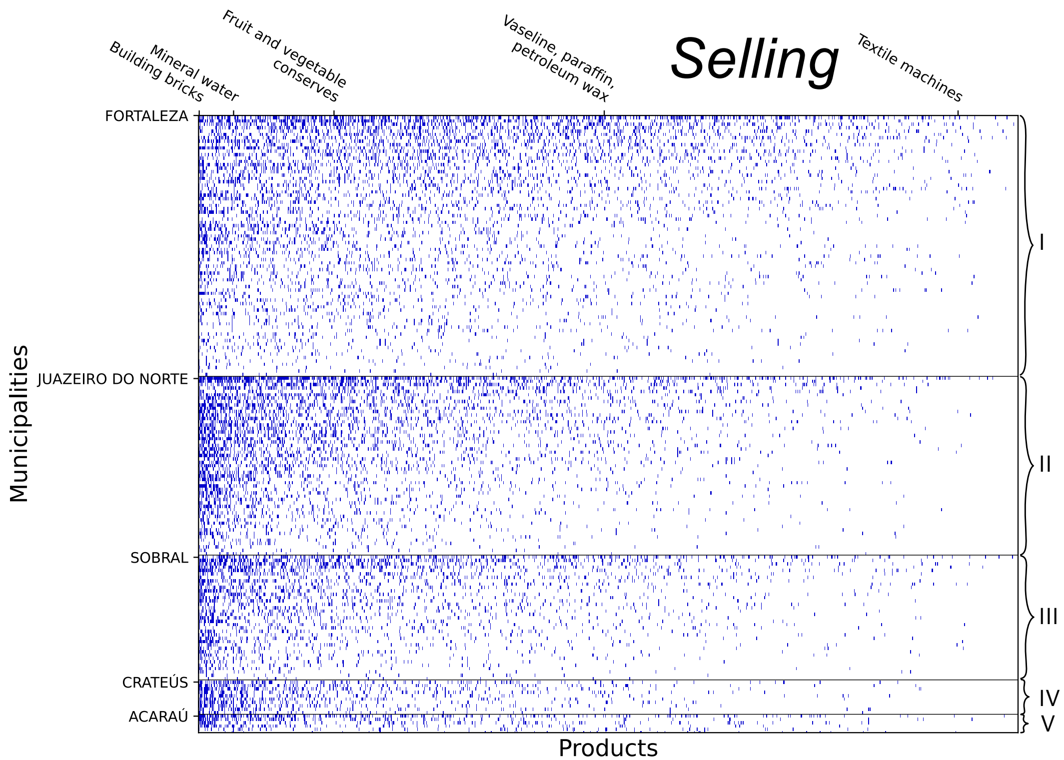
Revealed Comparative Advantage (RCA) Index

Hidalgo *et al.*, *Science* (2007)

$$RCA_m^p = \frac{q_m^p}{\sum_p q_m^p} \Big/ \ldots$$

$q_m^p \rightarrow$ monetary value traded by the city m of the product p.





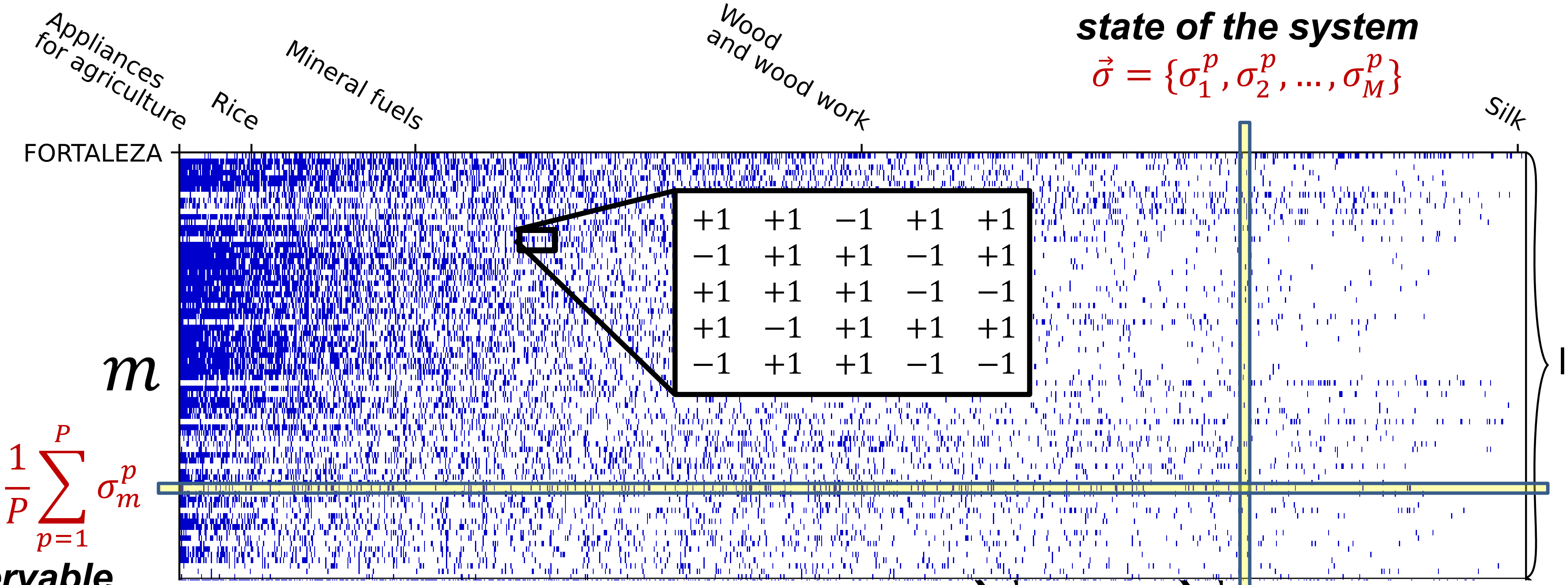
From Correlations to "Interactions" - Maximum-Entropy Model

Schneidman, Berry II, Segev & Bialek, *Nature* (2006)

- Our **observables** correspond to the average activity and pairwise

state of the system

$\vec{\sigma} = \{\sigma_1^p, \sigma_2^p, \ldots, \sigma_M^p\}$



$$\langle \sigma_m \rangle = \frac{1}{P} \sum_{p=1}^{P} \sigma_m^p$$

observable

$$P(\vec{\sigma}) = Z^{-1} \exp[-\beta \mathcal{H}] \quad \text{with} \quad \mathcal{H}(\vec{\sigma}) = -\sum_m h_m \sigma_m - \sum_{m<n} J_{mn} \sigma_m \sigma_n$$

$$Z = \sum_{\{\vec{\sigma}\}} \exp[-\beta \mathcal{H}(\vec{\sigma})]$$

Ising model from real data!

From Correlations to "Interactions"
Maximum-Entropy Model for **Cities and their Traded Products**

- Assuming that $\beta = 1$ and rewriting the equations as,

$P(\vec{\sigma}) = Z^{-1}\exp(\sum_m h_m\sigma_m + \sum_{m<n} J_{mn}\sigma_m\sigma_n)$ and $Z = \sum_{\{\vec{\sigma}\}}\exp(\sum_m h_m\sigma_m + \sum_{m<n} J_{mn}\sigma_m\sigma_n)$

the fields $\{h_m\}$ and couplings $\{J_{mn}\}$ can be obtained by solving,

$$\langle\sigma_m\rangle = \frac{\partial}{\partial h_m} ln\, Z = \sum_{\{\vec{\sigma}\}} \sigma_m P(\vec{\sigma}) \quad \text{and} \quad \langle\sigma_m\sigma_n\rangle = \frac{\partial}{\partial J_{mn}} ln\, Z = \sum_{\{\vec{\sigma}\}} \sigma_m\sigma_n P(\vec{\sigma})$$

Boltzmann Machine Learning

- In practical terms, we search for a solution of this inverse-Ising problem using a Monte Carlo (*MC*) algorithm as a core solver with the following updating rules:

$$h_m(l+1) = h_m(l) - \eta(l)[\langle\sigma_m\rangle_{MC} - \langle\sigma_m\rangle_{obs}] \qquad (1)$$

$$J_{mn}(l+1) = J_{mn}(l) - \eta(l)[\langle\sigma_m\sigma_n\rangle_{MC} - \langle\sigma_m\sigma_n\rangle_{obs}] \qquad (2)$$

Once we infer all the parameters $\{h_m\}$ and $\{J_{mn}\}$ that better reproduce the sets $\{\langle\sigma_m\rangle_{obs}\}$ and $\{\langle\sigma_m\sigma_n\rangle_{obs}\}$, while maximizing the entropy, the Boltzmann distribution characterizes the statistics of the product activities of the cities composing a given community.

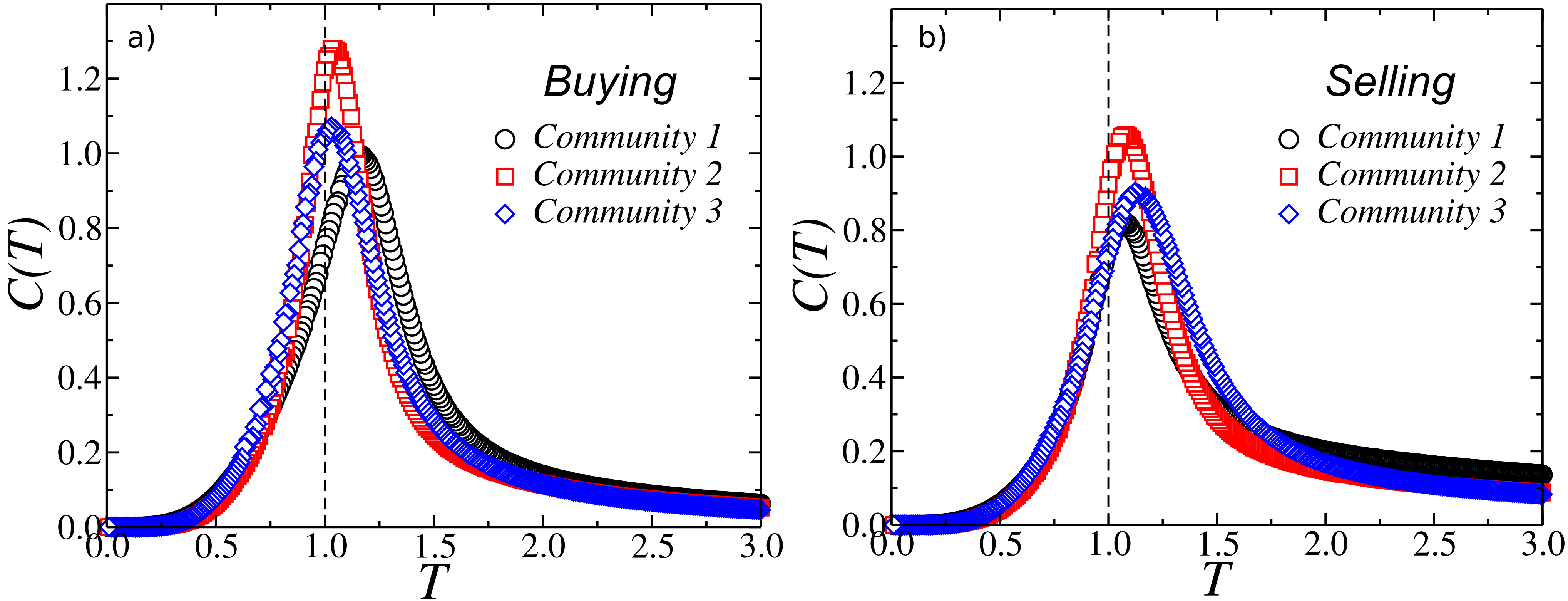
From Correlations to "Interactions"

Boltzmann Machine applied to the Microdynamics of Ceará's Economy

$$C(T) = \frac{1}{T^2}\left(\langle E(T)^2\rangle - \langle E(T)\rangle^2\right)$$

"Specific Heat"

Subcritical!



Muito obrigado!