Emergent cooperative behavior in transient compartments

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L. Peliti, A. Fierro, JJA, A. Coniglio
Napoli 1996

As a consequence...

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- I built a large network of friends and collaborators and, on top of it, my scientific career.
- And learned a few tricks:

Cooperation and network reciprocity

- Cooperation may emerge from selfish behavior! **How?**
- Several proposed mechanisms: kin selection, direct or indirect reciprocity, network reciprocity, etc.
- Network reciprocity depends on the spatial correlations generated from continued interactions (persistent groups)
- the mutual protection from other cooperators in the bulk outweights the exploitation on the surface


Prisoner's Dilemma game (PD)

	C	D
C	$b-c$	$-c$
D	b	0

cost $(c>0)$

benefit $(b>c)$

- Replicator equation:

$$\dot{\rho}_i = \rho_i \Big(f_i - F \Big)$$

average fitness: $F = f_C \rho_C + f_D \rho_D$

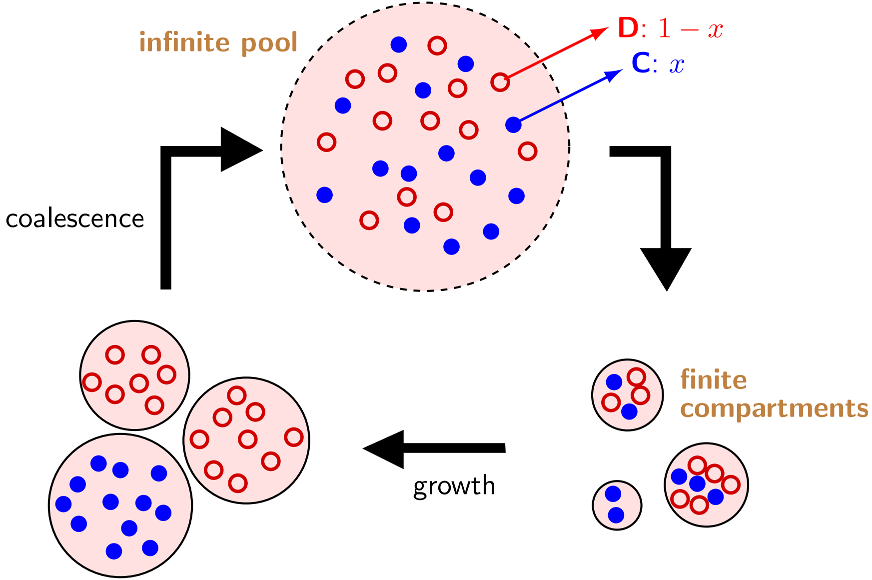
- Mean field (fully-mixed): **no cooperation!**

$$f_C = \frac{(N_C - 1)b - (N-1)c}{N-1} < F \implies \boxed{\rho_D \to 1}$$

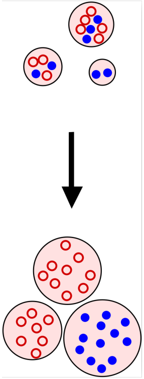
$$f_D = \frac{bN_C}{N-1} > F$$

Multilevel selection

- Natural selection may operate across different scales of organization, from the individual-level to higher orders involving groups of agents
- within-group and between-groups interactions:
  - structured populations
  - patchy environments and transient compartments
  - bottleneck processes
  - different social environments during lifetime (also due to mobility)
  - etc
- What is the role of the compartmental structure and its different levels of selection in scaffolding stable multicellular groups?


infinite pool
D: $1 - x$
C: x
coalescence
finite compartments
growth

Agent-level dynamics: growth phase



- Compartment: fully mixed and replicator equation

$$f_{\mathrm{C}} = b\rho - c$$

average **C** payoff

$$\Longrightarrow \qquad \dot{\rho} = \rho(f_{\mathrm{C}} - F) = -c\rho(1-\rho)$$

instantaneous average payoff

$$f_{\mathrm{D}} = b\rho$$

- Variable population:

$$\frac{dN}{dt} = F(\rho)\,N \qquad \Longrightarrow \qquad N_\infty = N_0 e^{\Phi(\rho_0)}$$

fitness = payoff

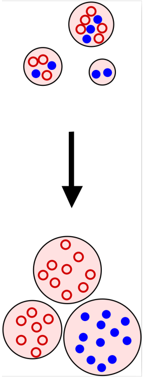
- The accumulated fitness

$$\Phi(\rho_0) = \int_0^\infty dt\; F(\rho) = -\int_{\rho_0}^{\rho_\infty} d\rho\, \frac{F(\rho)}{c\rho(1-\rho)}$$

$$= \frac{b-c}{c} \ln \frac{1-\rho_\infty}{1-\rho_0}$$

Agent-level dynamics: growth phase

- $\rho_0 = 0$: no growth ($N_\infty = N_0$)
- $\rho_0 < 1$: mixed initial compartments ($\rho_\infty = 0$)



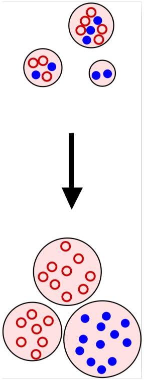
$$N_\infty = \frac{N_0}{(1-\rho_0)^{b/c-1}} \quad \Longrightarrow \quad N_{n,m} = \frac{n}{(1-m/n)^{b/c-1}}$$

The final average number of defectors per compartment is

$$N_{\rm D}(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{n-1} p_{n,m} N_{n,m}$$

- $\rho_0 = 1$: cooperators only survive if $m = n$

$$N_{\rm C}(x) = \sum_{n=1}^{\infty} p_{n,n} N_{\max}$$



- After the growth phase:

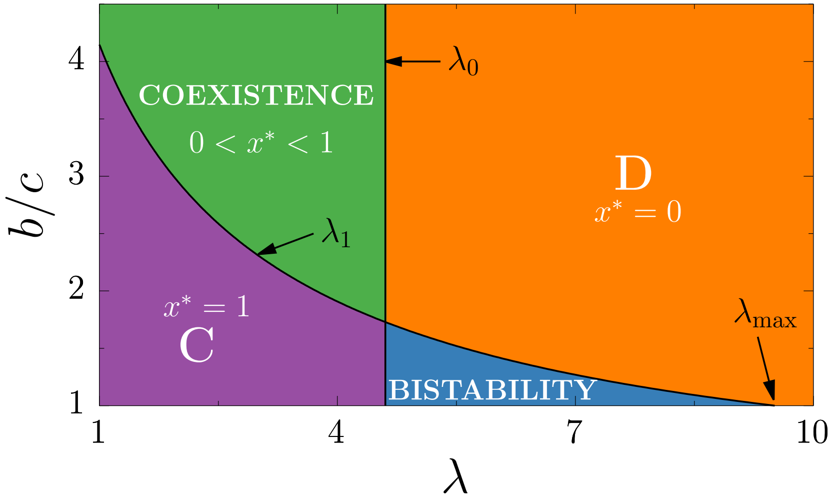
$$N_{\rm D}(x) = \sum_n \frac{1}{n!}\frac{\lambda^n n^{b/c}}{e^\lambda - 1}\sum_{m<n}\binom{n}{m}\frac{(1-x)^{n-m}x^m}{(n-m)^{b/c-1}}$$

$$N_{\rm C}(x) = N_{\rm max}\frac{e^{\lambda x}-1}{e^\lambda - 1}$$

- After the coalescence phase:

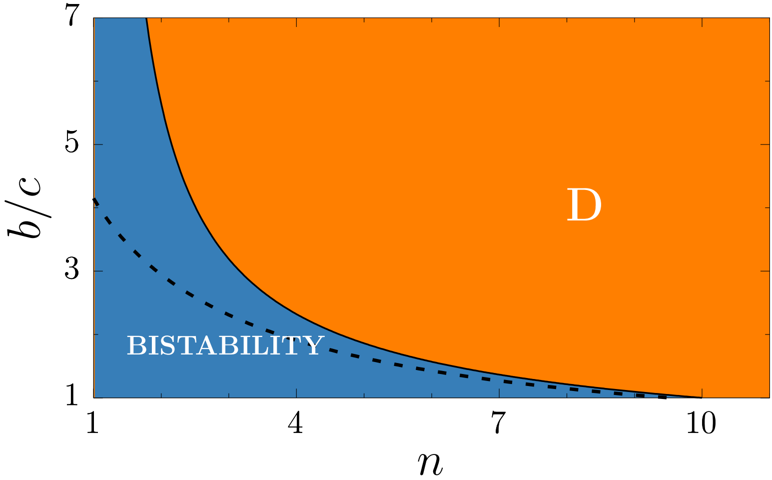
$$x' = \frac{N_{\rm C}(x)}{N_{\rm C}(x)+N_{\rm D}(x)}$$

- Fixed points ($x = x' = x^*$) and stability: $\left.\frac{dx'}{dx}\right|_{x^*} = 1$



b/c
4
3
2
1
1
4
7
10
λ
COEXISTENCE
$0 < x^* < 1$
λ_0
λ_1
D
$x^* = 0$
$x^* = 1$
C
BISTABILITY
$\lambda_{\max}$

Phase diagram: uniform compartment size



Conclusions

- Minimal model for a two-level selection mechanism:
  - fragmentation, growth and coalescence
- importance of finite size compartments and composition fluctuations for cooperation
- cooperation is possible in transient groups with mean-field internal interactions
- size diversity is important: pure **C** and coexistence phase

Happy birthday, Constantino!