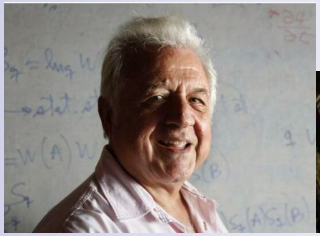
MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

# Non-additive particle filter for drone autonomous navigation


Haroldo F. de Campos Velho (INPE)

INPE – National Institute for Space Research – Brazil

# Summary

- Tsallis' thermostatistics applications
- Drone autonomous navigation:
  - Visual odometry
  - Computer vision
  - Data fusion by Non-extensive particle filter (PF)
- New applications for NEx-PF
  - Cancer dynamics
  - Data assimilation
- Final Remarks

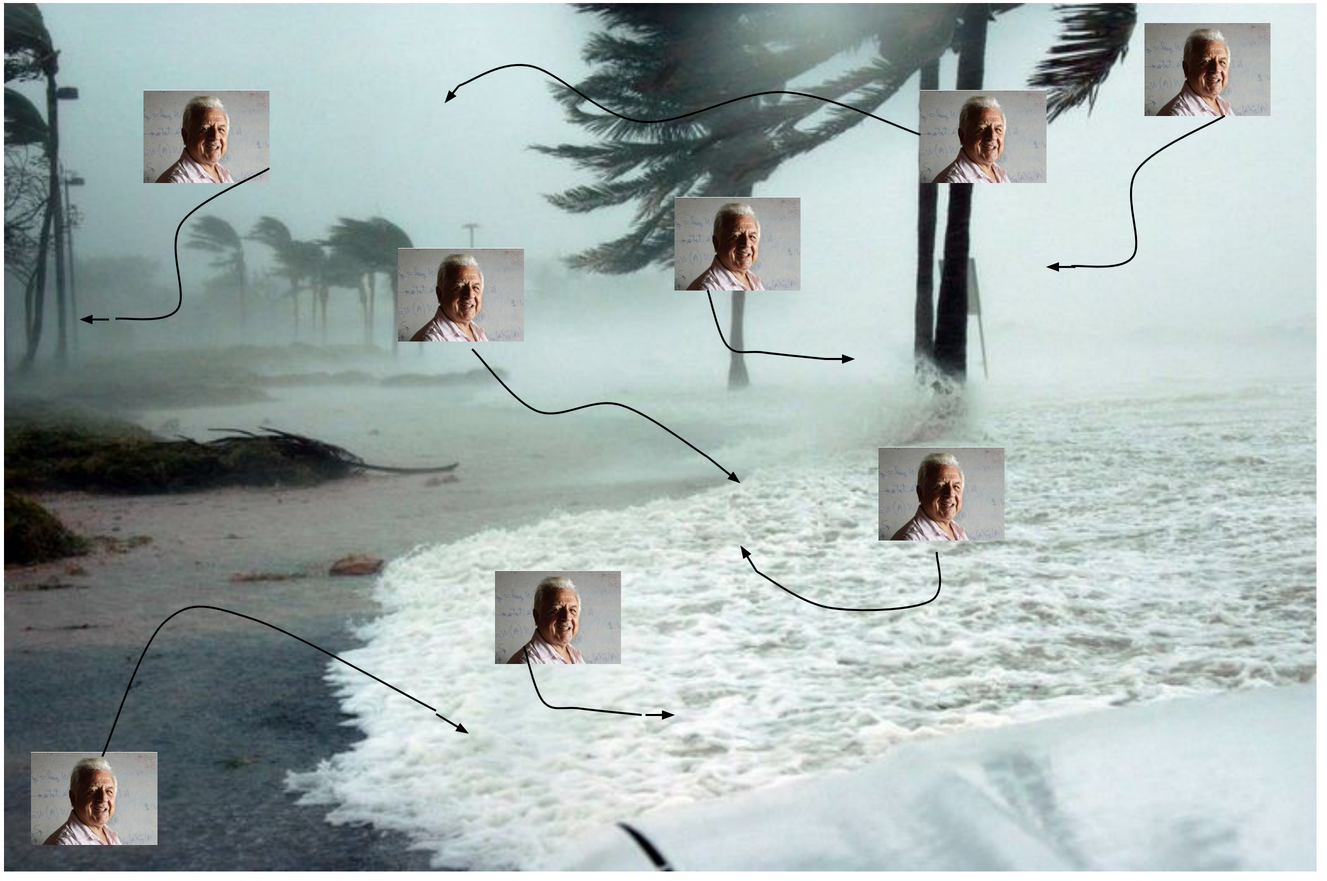
# Tsallis' statistics applications

- Applications to atmospheric turbulence
- Description of structures in the Universe
- Application to inverse problems
- Particle filter: new approach
  - Theoretical features
  - Aerial drone autonomous navigation
  - Future applications:

    a) cancer dynamics
    b) data assimilation

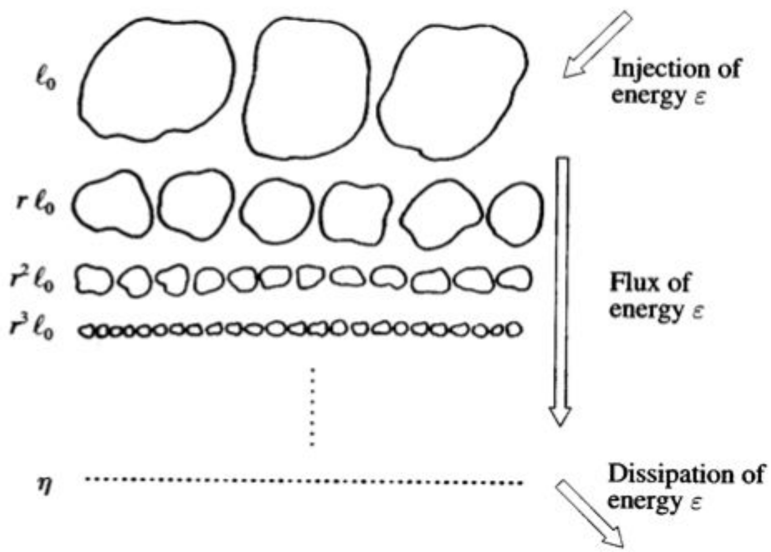
# Tsallis' statistics applications

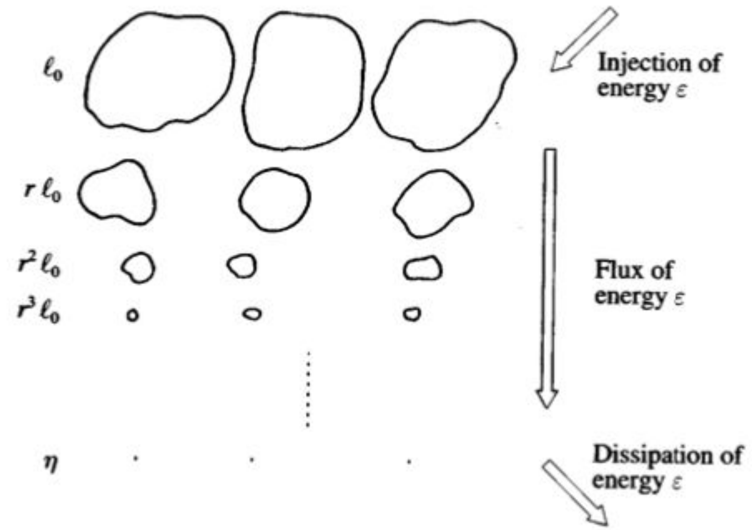
- INPE: School of Emergent Sciences
  (Constantino Tsallis' storm)



# Tsallis' statistics applications

- Applications to atmospheric turbulence




**The cascade according K41 theory: Notice that at each step the eddies are space-filling.**

**The cascade according β-model: Notice that with each step the eddies become less and less space-filling.**

# Tsallis' statistics applications

- Applications to atmospheric turbulence

ELSEVIER

Physica A 295 (2001) 250–253

PHYSICA A

www.elsevier.com/locate/physa

Non-extensive statistics and three-dimensional fully developed turbulence

Fernando M. Ramos[a,*], Reinaldo R. Rosa[a], Camilo Rodrigues Neto[a], Mauricio J.A. Bolzan[a], Leonardo D. Abreu Sá[a], Haroldo F. Campos Velho[a]

# Tsallis' statistics applications

- Applications to atmospheric turbulence

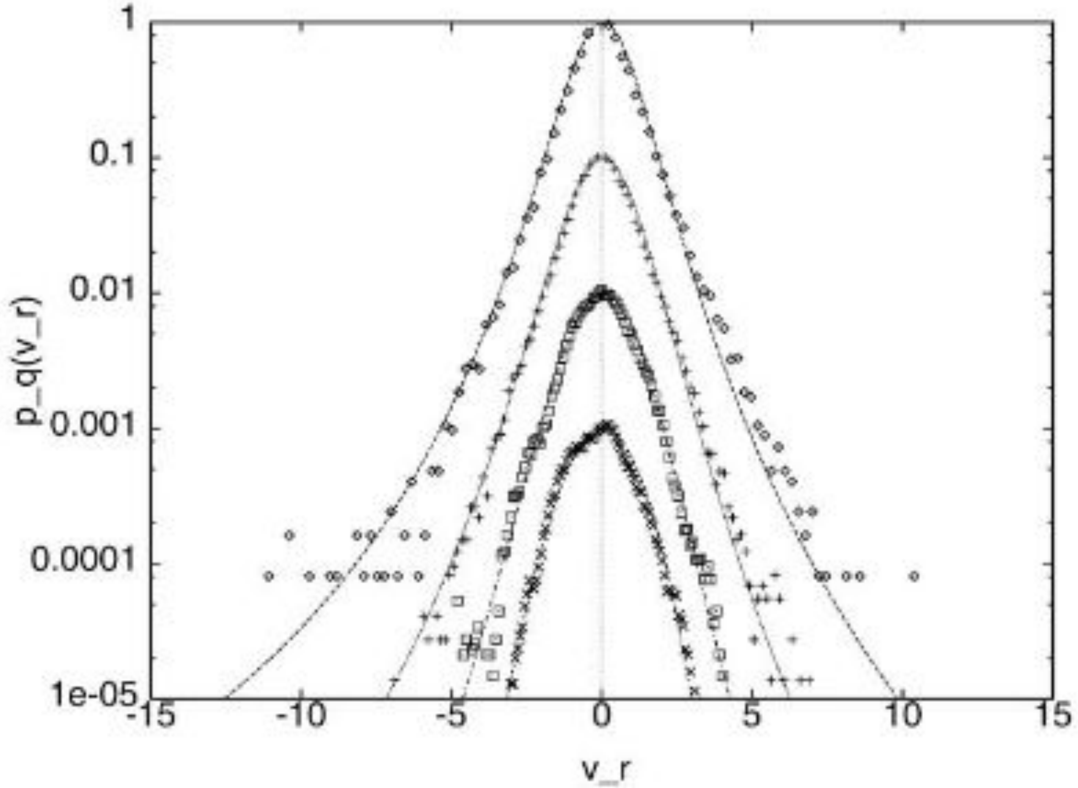



Fig. 1. Standardized probability distributions $p_q(v_r)$ of velocity differences $v_r(x) = v(x) - v(x + r)$ for spatial scales from $r = 0.07$ (top) to 70 m (bottom).

# Tsallis' statistics applications

- Applications to atmospheric turbulence

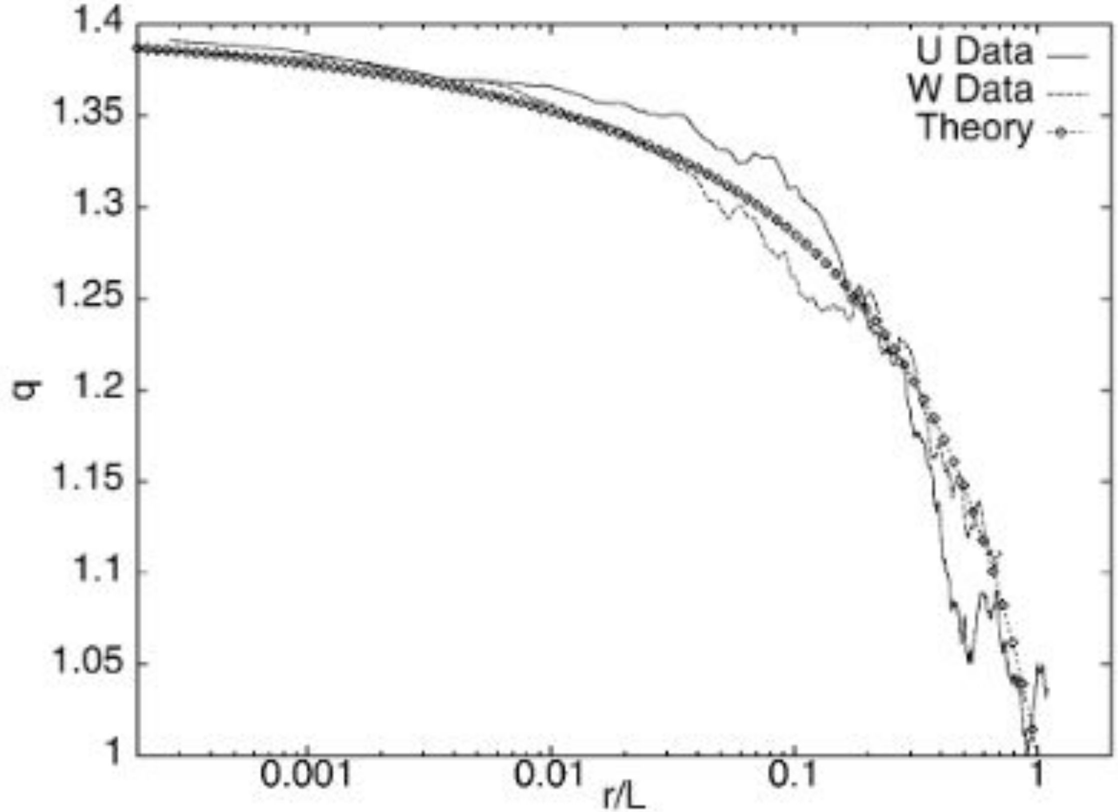
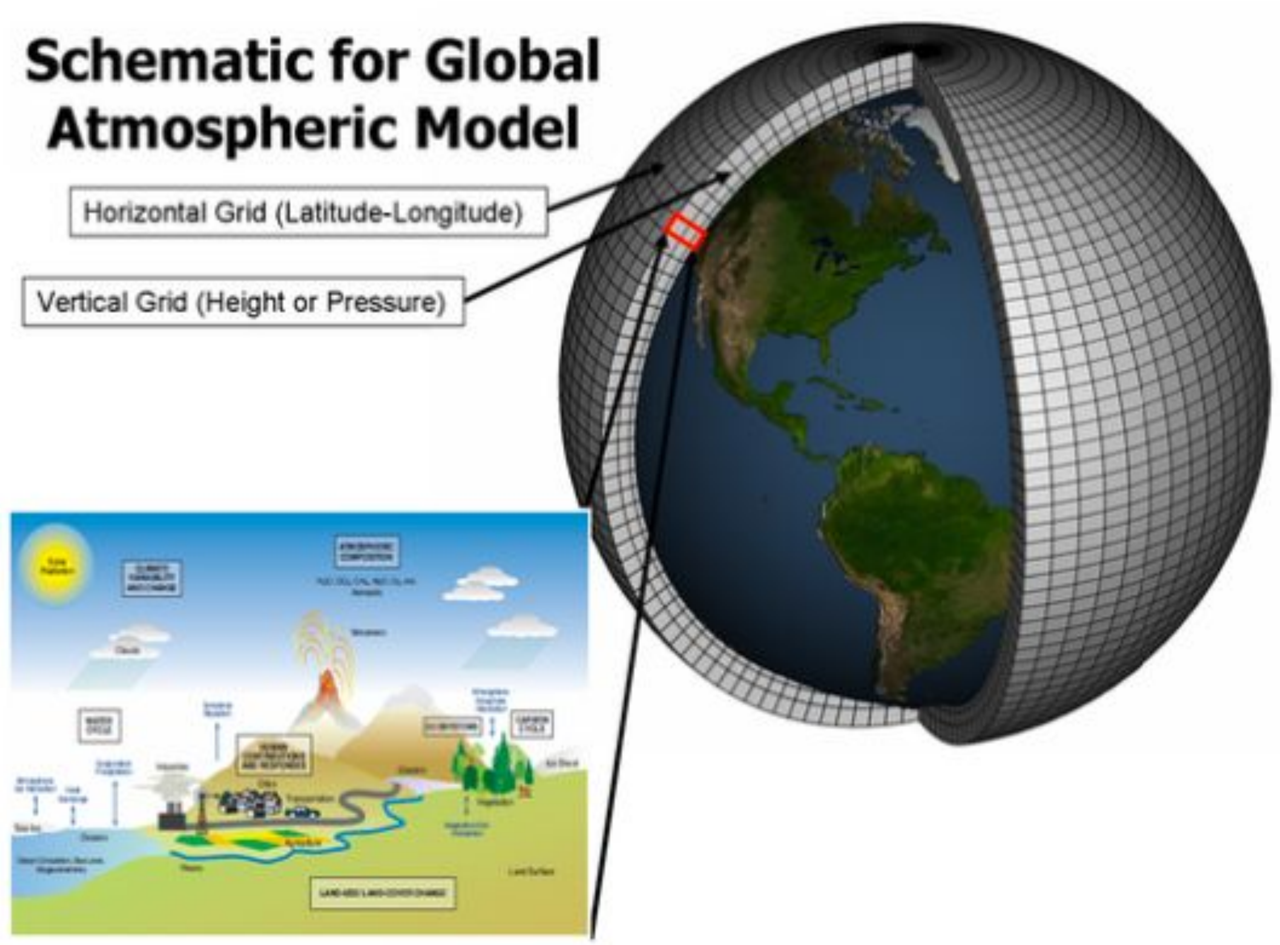



Fig. 2. Variation with scale of parameter $q$; experimental values corresponding vertical (w) and longitudinal (u) velocities measurements.

# Tsallis' statistics applications

- Applications to atmospheric turbulence



# Weather and climate prediction

- Dynamical core (Navie-Stokes equation "solver")

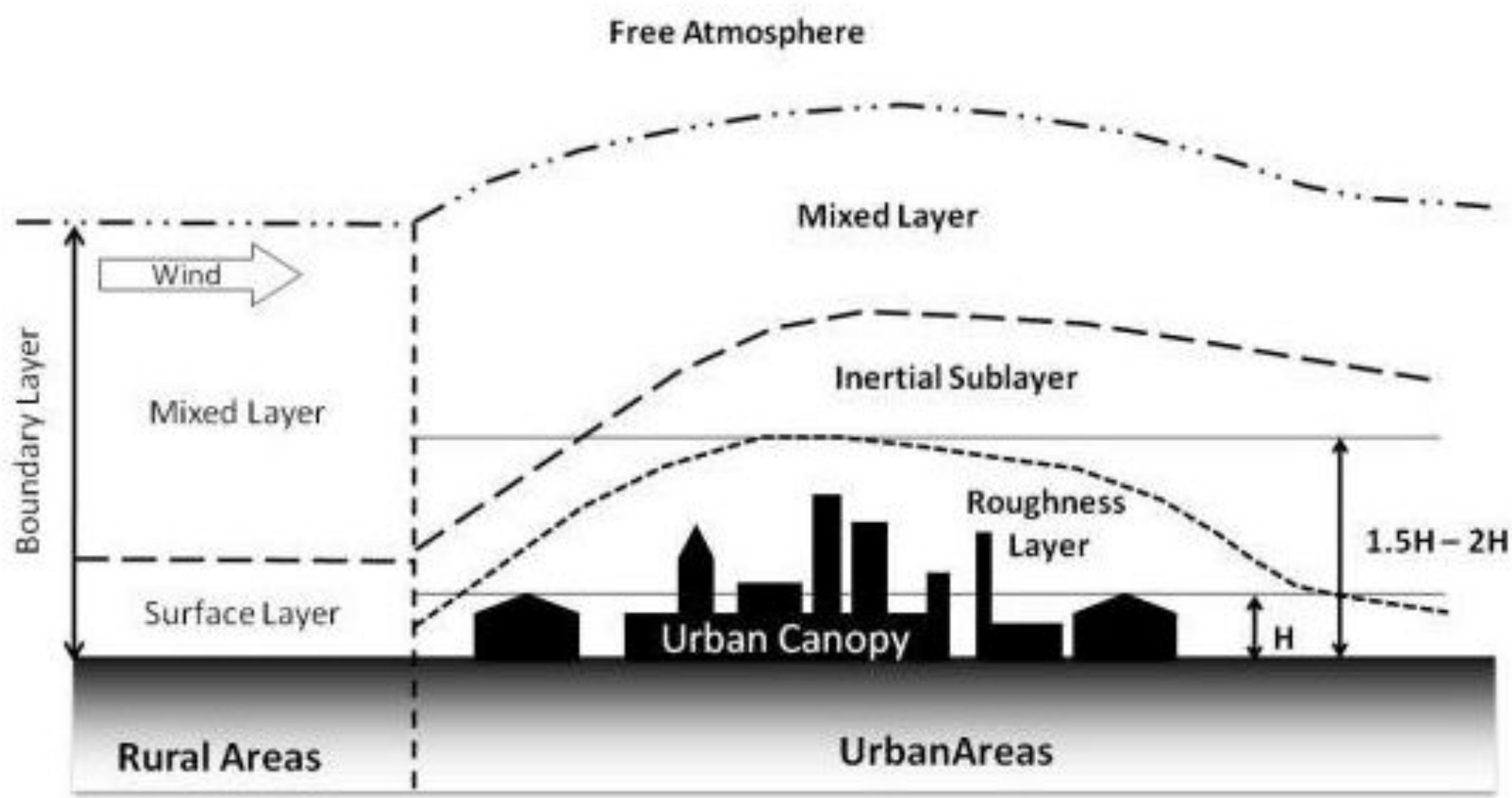
$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} &= -\nabla \cdot (\zeta + f)\vec{v}_H - \vec{k} \cdot \nabla \times \left( RT\nabla q + \dot{\sigma}\frac{\partial \vec{c}_H}{\partial \sigma} - \vec{f} \right) \\
\frac{\partial D}{\partial t} &= \vec{k} \cdot \nabla \times (\zeta + f)\vec{v}_H - \nabla \cdot \left( RT\nabla q + \dot{\sigma}\frac{\partial \vec{c}_H}{\partial \sigma} - \vec{f} \right) - \nabla^2 \left( \phi + \frac{\vec{v}_H \cdot \vec{v}_h}{2} \right) \\
\frac{\partial T}{\partial t} &= -\nabla \cdot (T\vec{v}_H) + TD + \dot{\sigma}\gamma - \frac{RT}{c_p}\left( D + \frac{\partial \dot{\sigma}}{\partial \sigma} + H_T \right) \\
\frac{\partial q}{\partial t} &= -\vec{v}_H \cdot \nabla q - D - \frac{\partial \dot{\sigma}}{\partial \sigma} \\
\sigma \frac{\partial \phi}{\partial \sigma} &= -RT \\
\frac{\partial r}{\partial t} &= -\nabla \cdot (r\vec{v}_H) + rD - \dot{\sigma}\frac{\partial r}{\partial \sigma} + M
\end{aligned}
$$

{with: $q = \log(p_0)$}

{with: $\phi = gh$ ; and: $\sigma = p/p_0$}

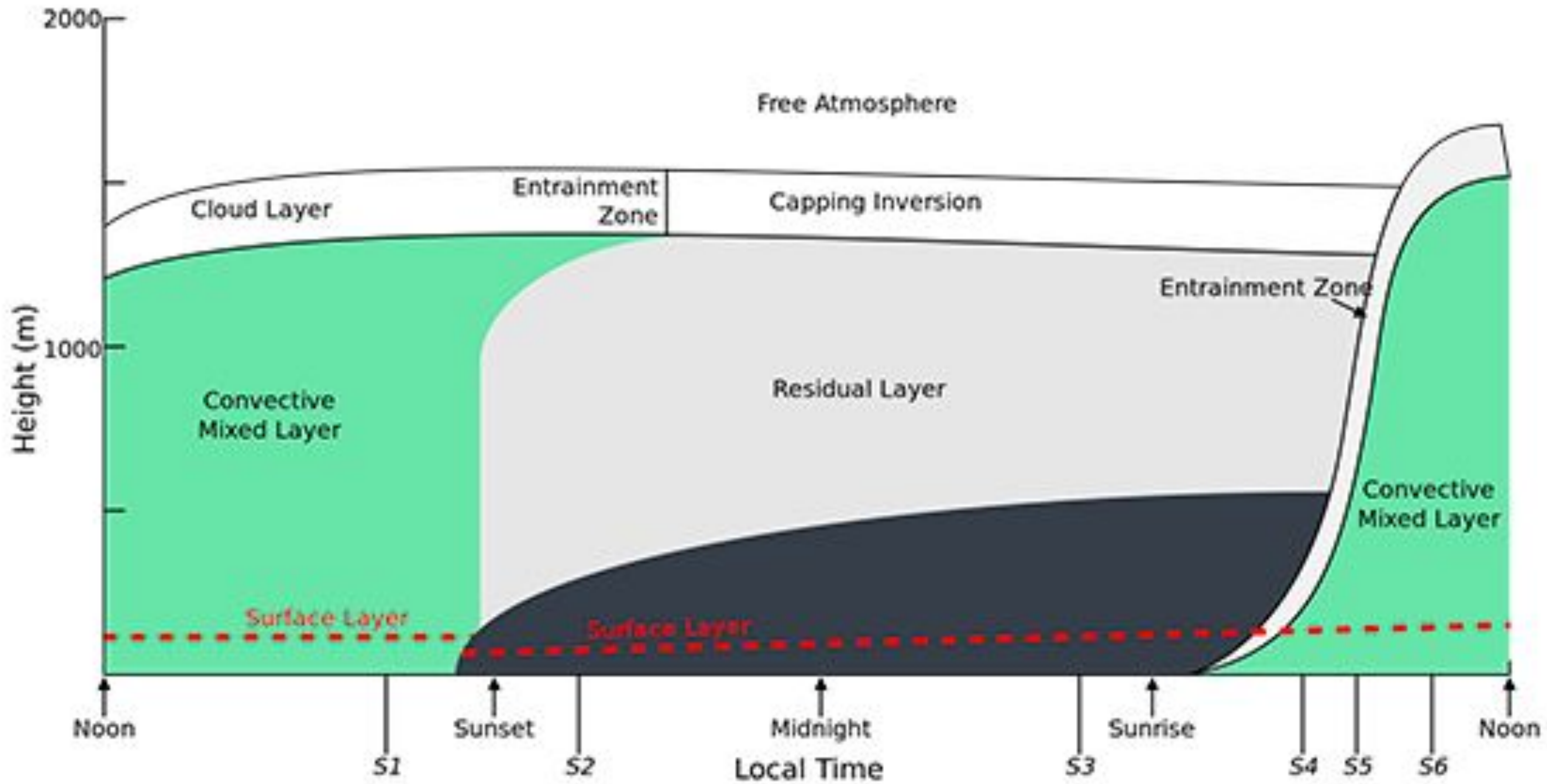
# Tsallis' statistics applications

- Applications to atmospheric turbulence



# Weather and climate prediction

## 3.2 Model physics: Turbulence model



# Tsallis' statistics applications

- Applications to atmospheric turbulence

**and a Eulerian form for eddy diffusivity follows:**

$$\boxed{K_{\alpha\alpha} = \frac{\sigma_i^2 \beta_i^2}{2\pi} \int_0^\infty F_i(n) \frac{\sin(2\pi\, nt/\beta_i)}{n^2} dn}$$

$F_i(n)$

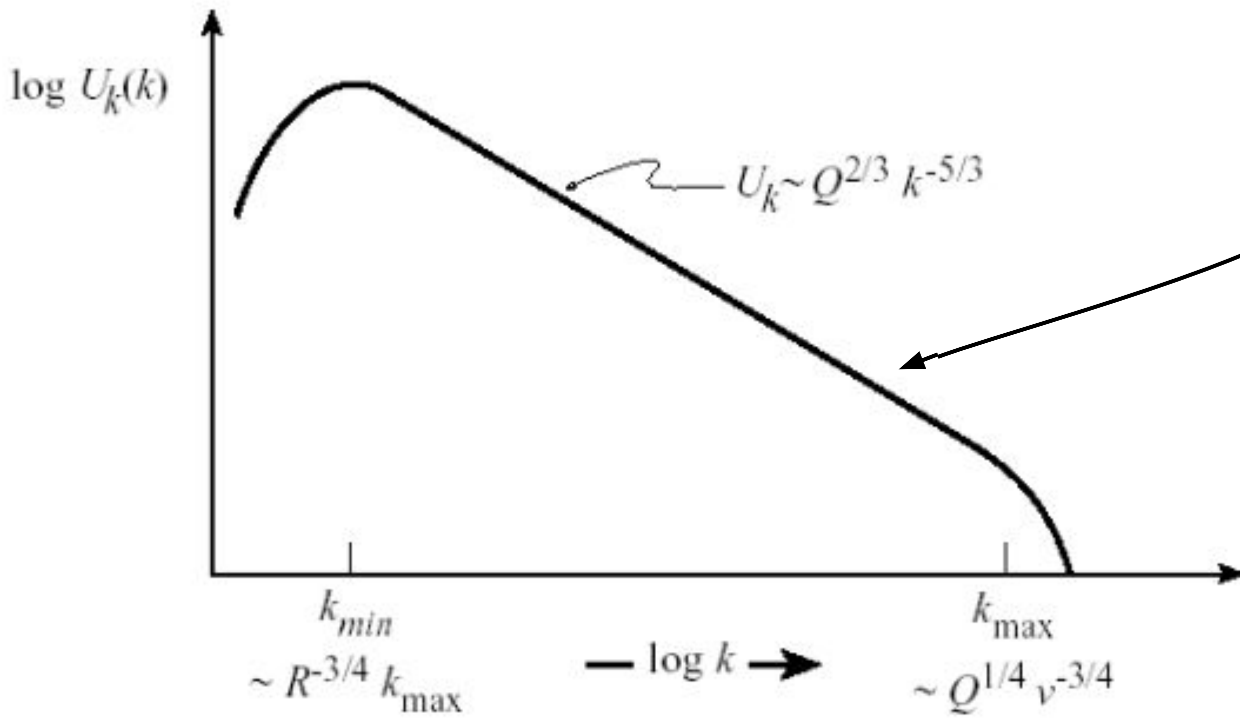
**An asymptotic form can also be derived (long travel times, $t \to \infty$):**

$$\boxed{K_{\alpha\alpha} = \frac{\sigma_i^2 \beta_i F_i(0)}{4}}$$

# Weather and climate prediction

## 3.2 Model physics: Turbulence model

$$\frac{nS_i(n)}{u_*^2} = \frac{Af^{c_3}}{\left(1 + Bf^{c_1}\right)^{c_2}}$$



# Weather and climate prediction

## 3.2 Model physics: Turbulence model

$$\frac{nS_i(n)}{u_*^2} = \frac{Af^{c_3}}{\left(1 + Bf^{c_1}\right)^{c_2}}$$

1. $S(k) \approx AB^{-m_1 m_2} k^{m_3 - m_1 m_2 - 1}$ for $k \to \infty$

2. $E_{\mathrm{model}}(k) \to E_{\mathrm{G\text{-}Kolmogorov}}$ for $k \in$ inertial subrange

$$E(k) = d_2 \varepsilon^{2/3} k^{-(1+\zeta_2)} \left(k/L_\eta\right)^{2/3-\zeta_2}$$

# Tsallis' statistics applications

- Applications to atmospheric turbulence

ELSEVIER

Physica A 295 (2001) 219–223

PHYSICA A

www.elsevier.com/locate/physa

Multifractal model for eddy diffusivity and counter-gradient term in atmospheric turbulence

Haroldo F. Campos Velho[a,*], Reinaldo R. Rosa[a], Fernando M. Ramos[a], Roger A. Pielke[b], Gervásio A. Degrazia[c], Camilo Rodrigues Neto[a], Ademilson Zanandrea[a]

# Tsallis' statistics applications

- Applications to atmospheric turbulence

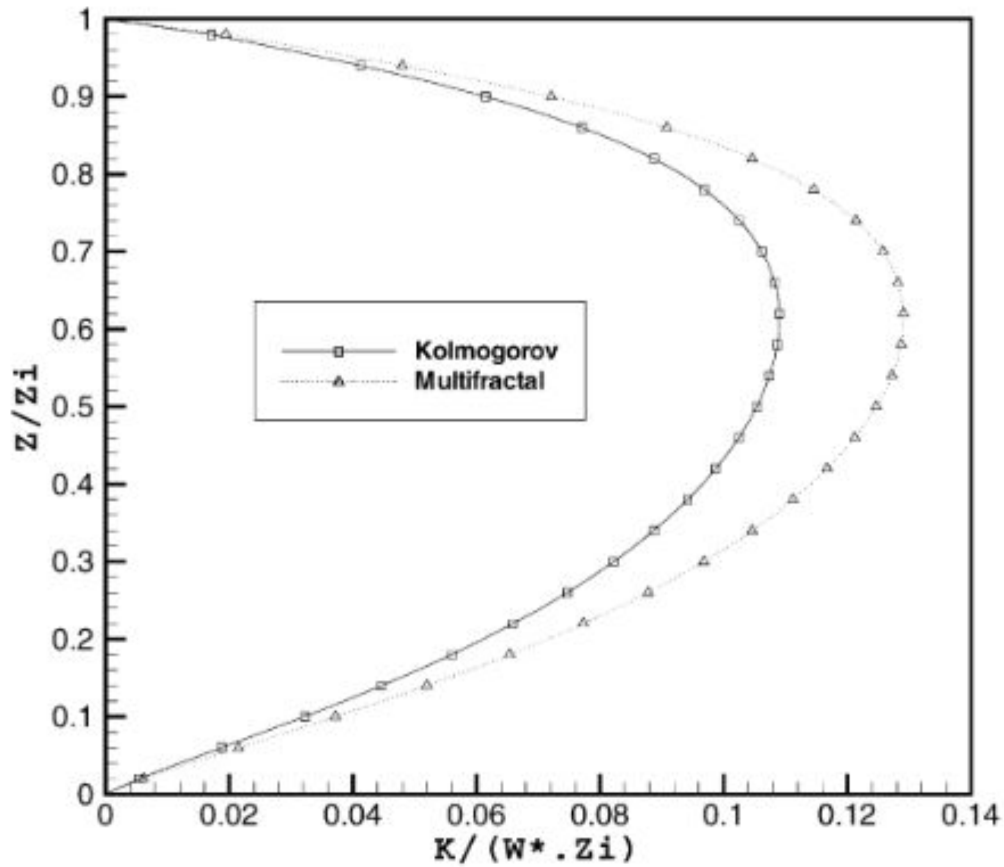



Fig. 1. Vertical eddy diffusivity for Kolmogorov's and multifractal approach.

# Tsallis' statistics applications

- Applications to atmospheric turbulence

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Physica A 354 (2005) 88–94

PHYSICA A

www.elsevier.com/locate/physa

## Representing intermittency in turbulent fluxes: An application to the stable atmospheric boundary layer

Haroldo F. Campos Velho[a,*], Reinaldo R. Rosa[a], Fernando M. Ramos[a], Roger A. Pielke Sr[b], Gervásio A. Degrazia[c]

# Tsallis' statistics applications

- Applications to atmospheric turbulence

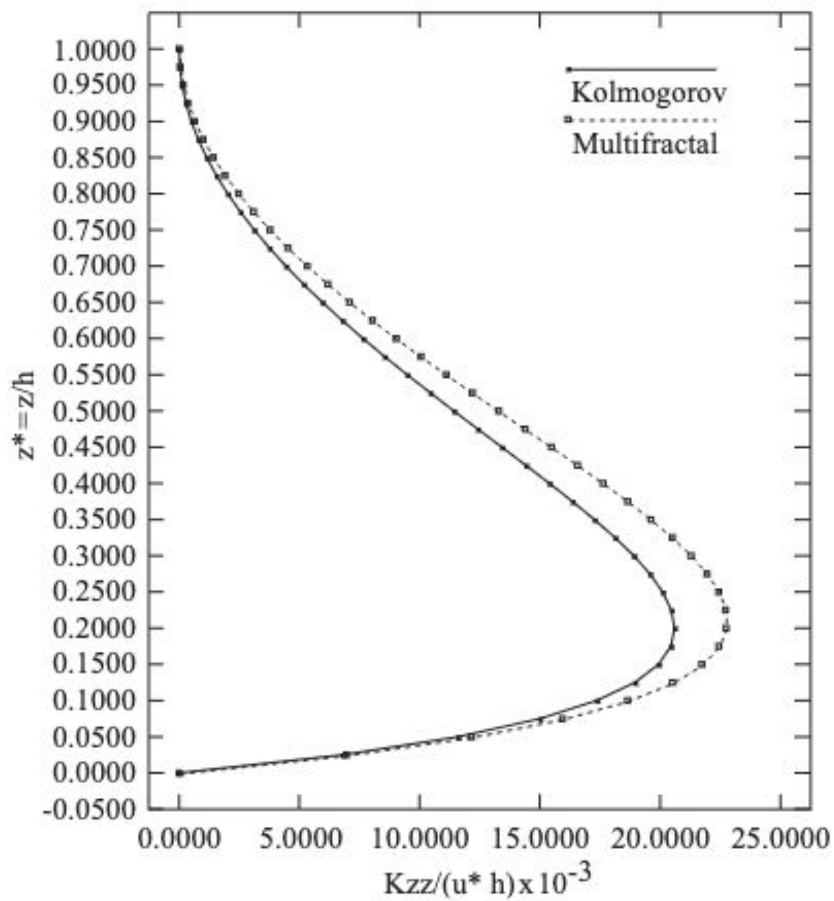


Fig. 1. Vertical eddy diffusivity for Kolmogorov's and multifractal approaches for SBL.

# Tsallis' statistics applications

- INPE: School of Emergent Sciences
  (Constantino Tsallis' storm)

# Tsallis' statistics applications

- Description of structures in the Universe

ELSEVIER

Physica D 168–169 (2002) 404–409

PHYSICA D

www.elsevier.com/locate/physd

## Multiscaling and nonextensivity of large-scale structures in the Universe

F.M. Ramos [a,*], C.A. Wuensche [b], A.L.B. Ribeiro [c], R.R. Rosa [a]

# Tsallis' statistics applications

- Description of structures in the Universe

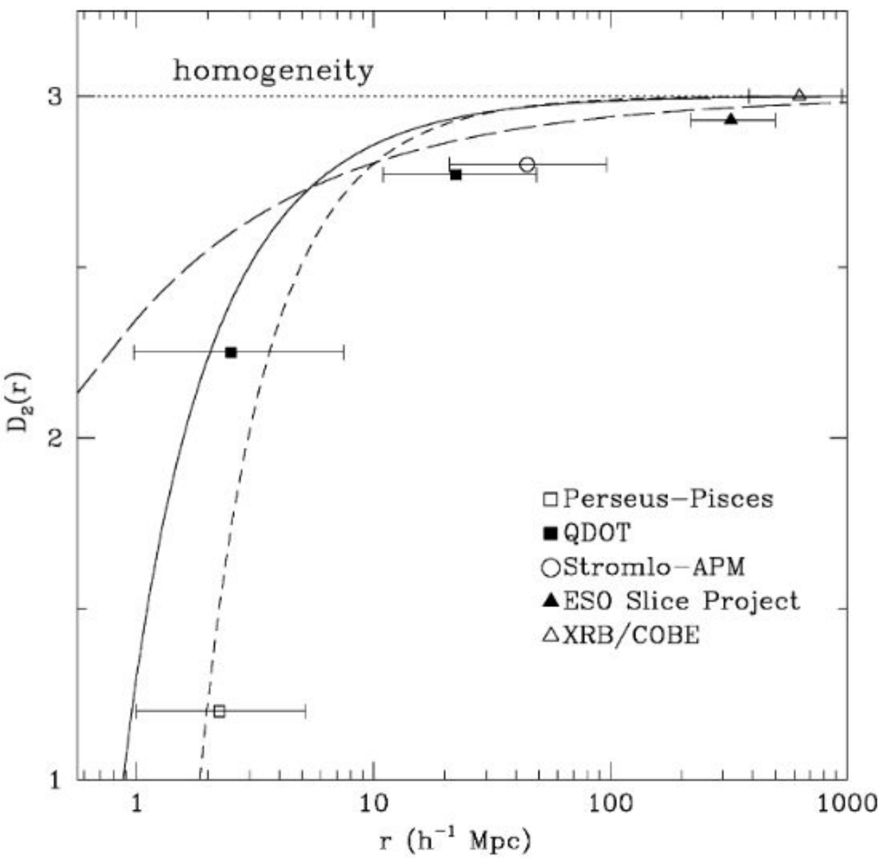


Fig. 2. The correlation dimension vs. scale, for various surveys [2], and for the present model, for $a = 0.65$ and $\beta = 1.0$ (solid line); $a = 1.60$ and $\beta = 0.8$ (dashed line); $a = 0.28$ and $\beta = 2.0$ (long dashed line) with $L = 100\,h^{-1}$ Mpc for all cases.

# Tsallis' statistics applications

- Description of structures in the Universe

Computer Physics Communications 180 (2009) 621–624

Contents lists available at ScienceDirect

Computer Physics Communications

www.elsevier.com/locate/cpc

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COMPUTER PHYSICS COMMUNICATIONS

## Extreme event dynamics in the formation of galaxy-sized dark matter structures

Reinaldo R. Rosa [a,*], Fernando M. Ramos [a], Cesar A. Caretta [b], Haroldo F. Campos Velho [a]

# Tsallis' statistics applications

- Description of structures in the Universe

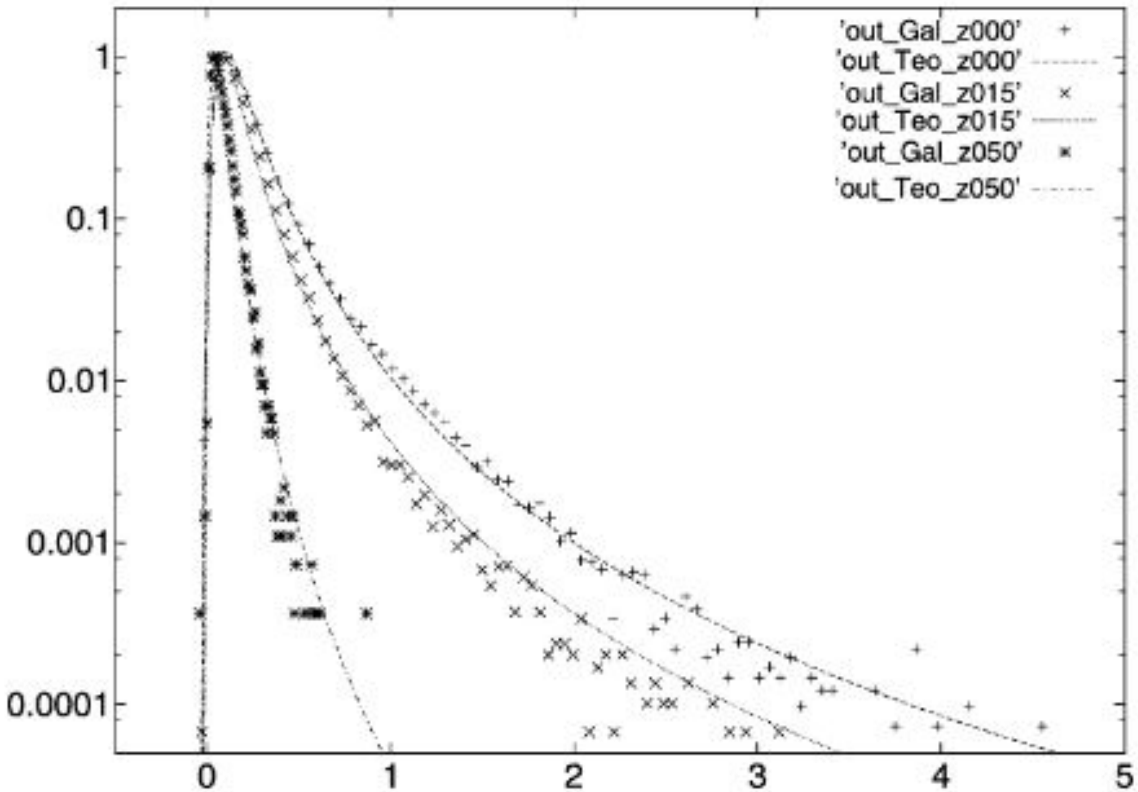



**Fig. 2.** Empirical and theoretical (GEV model) rescaled energy histograms, for red-shifts $z = 0$, 1.5 and 5.0.

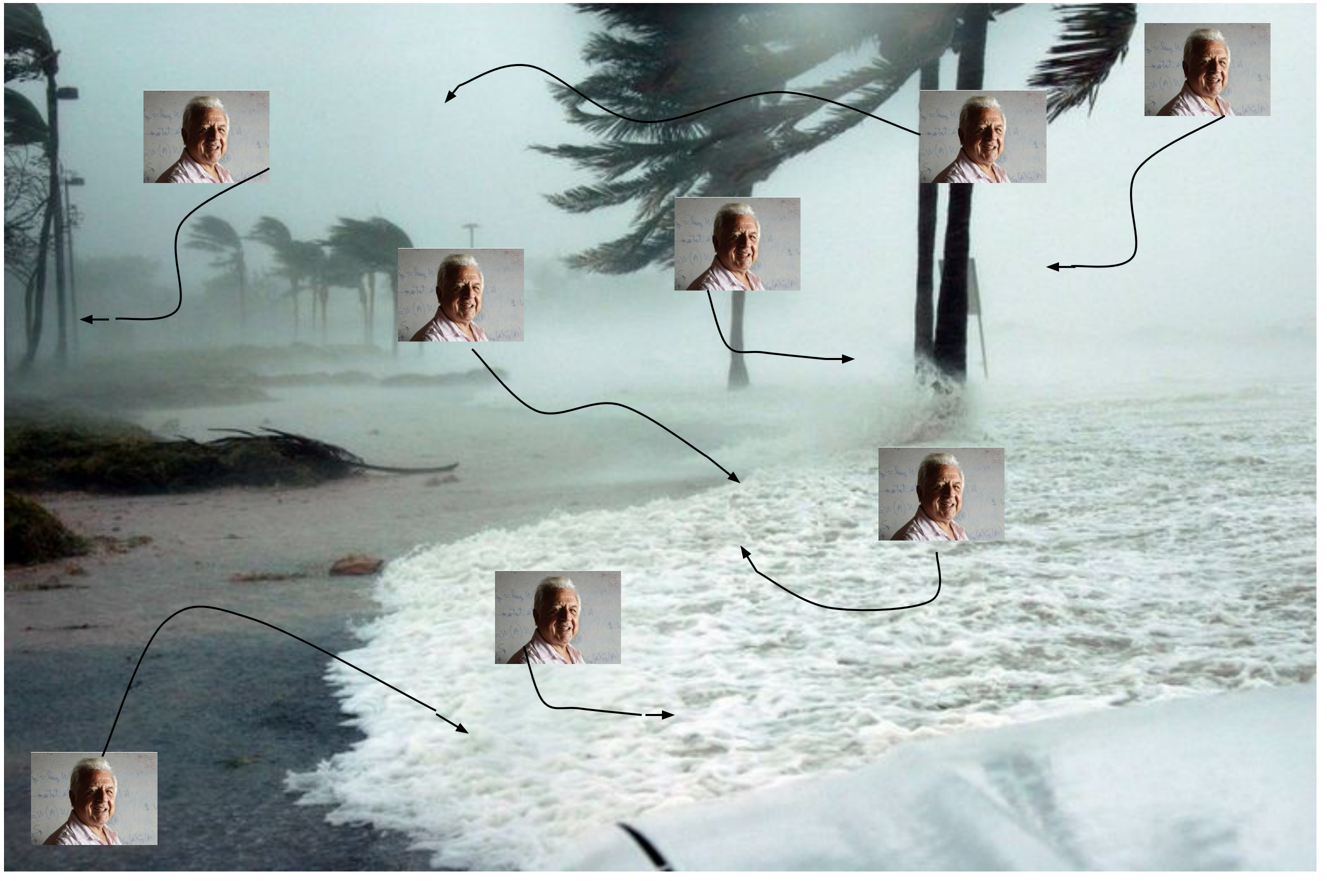
# Tsallis' statistics applications

- Description of structures in the Universe

In summary, we have obtained complementary results that reveal turbulent-like structures that would normally remain hidden in instantaneous density plots simulated from the $\Lambda$CDM model. In addition to the characterization we have shown in this paper, further investigation can apply the GEV statistics to distinguish different versions of the cold dark matter model including possible more realistic $\Lambda$CDM extensions (for example, to allow quintessence rather than a cosmological constant).

# Tsallis' statistics applications

- INPE: School of Emergent Sciences
  (Constantino Tsallis' storm)



# Tsallis' statistics applications

- Fuzzy operators

1999 IEEE International Fuzzy Systems Conference Proceedings
*August 22-25, 1999, Seoul, Korea*

**A New Family of Fuzzy Operators and its Use in the Break-Collapse Method**

Constantino Tsallis*, Camilo Rodrigues Neto†, Sandra Sandri†

# Tsallis' statistics applications

- INPE: School of Emergent Sciences
  (Constantino Tsallis' storm)

# Tsallis' statistics applications

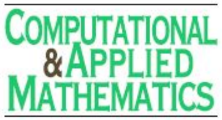
- Application to inverse problems

COMPUTATIONAL & APPLIED MATHEMATICS
Volume 25, N. 2-3, pp. 1–24, 2006
Copyright © 2006 SBMAC
ISSN 0101-8205
www.scielo.br/cam

**A unified regularization theory: the maximum non-extensive entropy principle**

HAROLDO F. DE CAMPOS VELHO[1], ELCIO H. SHIGUEMORI[2]
FERNANDO M. RAMOS[1] and JOÃO C. CARVALHO[3]

# Inverse problem: Regularization method



Volume 25, N. 2-3, pp. 1–24, 2006

ISSN 0101-8205
www.scielo.br/cam

## A unified regularization theory: the maximum non-extensive entropy principle

HAROLDO F. DE CAMPOS VELHO[1], ELCIO H. SHIGUEMORI[2]
FERNANDO M. RAMOS[1] and JOÃO C. CARVALHO[3]

**Theorem.** *For particular values for non-extensive entropy $q = 1$ and $q = 2$ are equivalents to the extensive entropy and Tikhonov regularizations, respectively.*

# Tsallis' statistics applications

- Application to inverse problems

Physica A: Statistical Mechanics and its Applications
Volume 261, Issues 3–4, 15 December 1998, Pages 555-568

## A non-extensive maximum entropy based regularization method for bad conditioned inverse problems

L. Rebollo-Neira [a], J. Fernandez-Rubio [a], A. Plastino [b]

q = 1/2
(only)

# Inverse problem: Regularization method

COMPUTATIONAL & APPLIED MATHEMATICS
Volume 25, N. 2-3, pp. 1–24, 2006

ISSN 0101-8205
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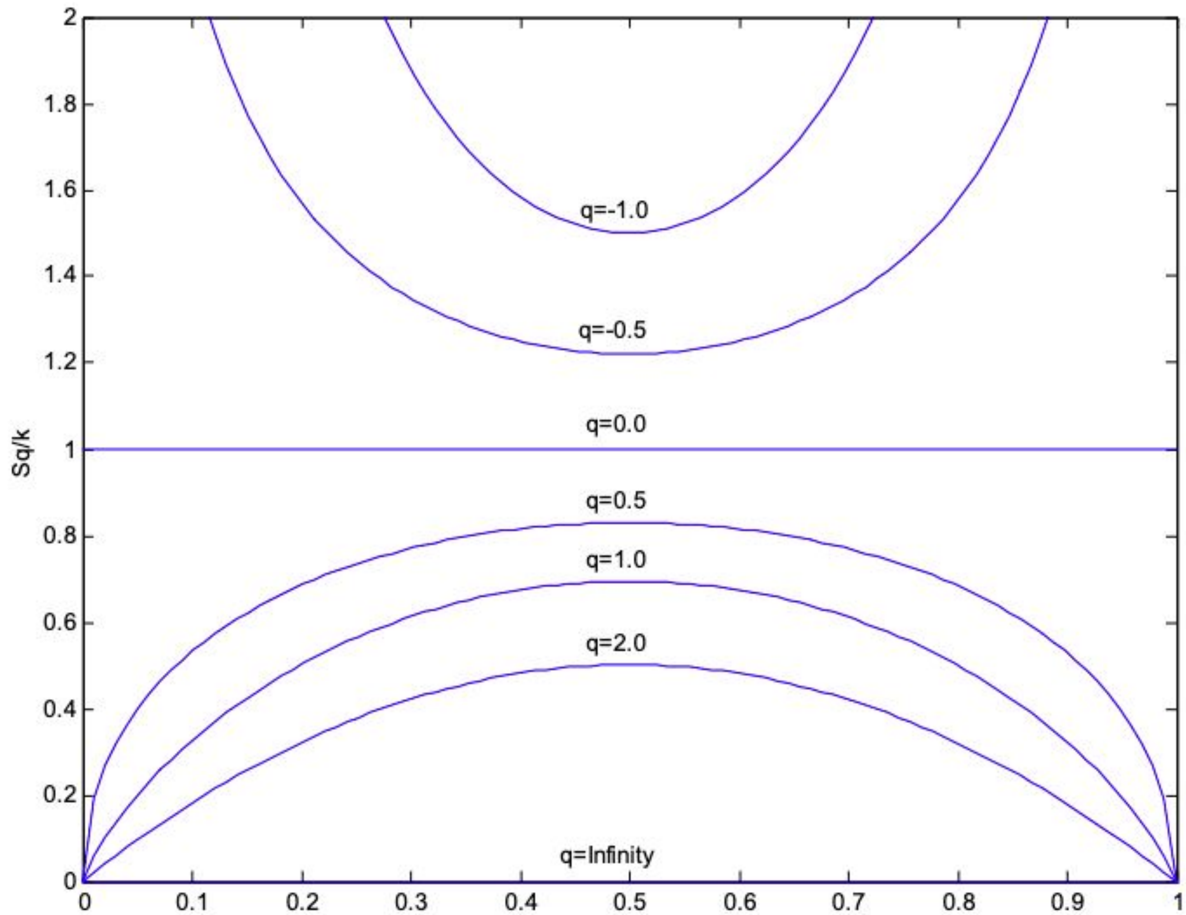

# Inverse problem: Regularization method

COMPUTATIONAL & APPLIED MATHEMATICS
Volume 25, N. 2-3, pp. 1–24, 2006
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ISSN 0101-8205
www.scielo.br/cam

$$I_\lambda(0) = B_\lambda(T_s)\Im_\lambda(p_s) + \int_{p_s}^{0} B_\lambda[T(p)]\frac{\partial \Im_\lambda(p)}{\partial p}dp,$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{[e^{hc/k_B\lambda T} - 1]}$$

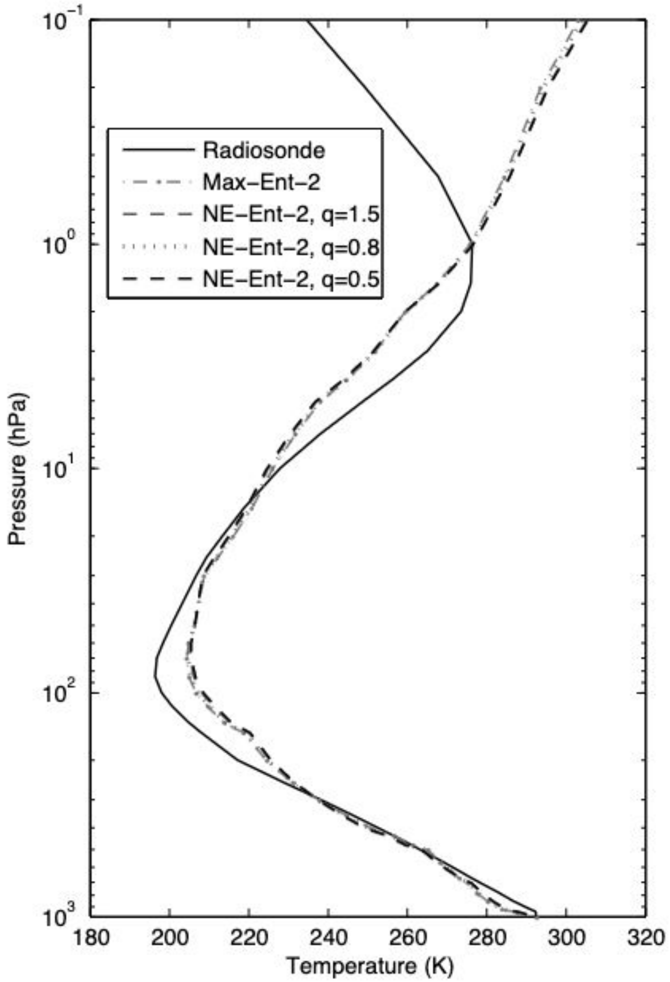


Figure 10 – Reconstructions for temperature profile: $q = 2.0$.

# Inverse problem: neural network

*Int. J. Information and Communication Technology, Vol. x, No. x, xxxx* 1

## Atmospheric temperature retrieval using a Radial Basis Function Neural Network

E.H. Shiguemori

Laboratório Associado de Computação e Matemática Aplicada – LAC,
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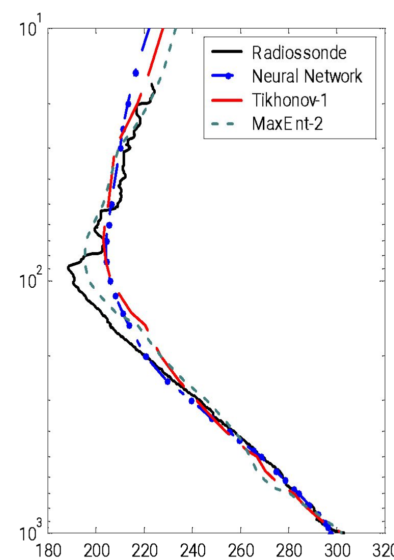
E-mail: joao.carvalho@ana.gov.br

# Aerial drone autonomous navigation

- UAV[a] autonomous navigation by image processing – **Why?**

  [a] Unmanned Aerial Vehicle

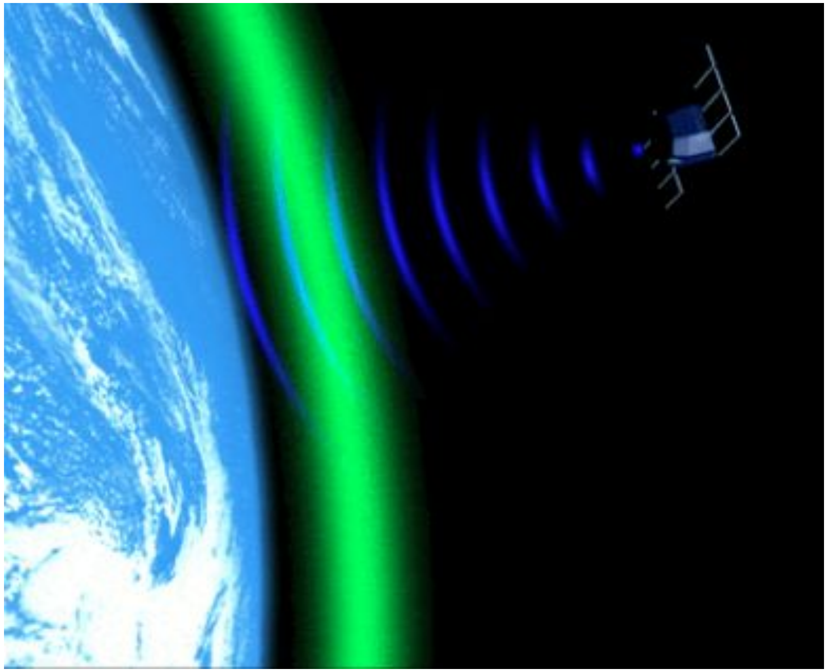
- In general, UAV autonomous navigation is done by using GNSS[b] signal

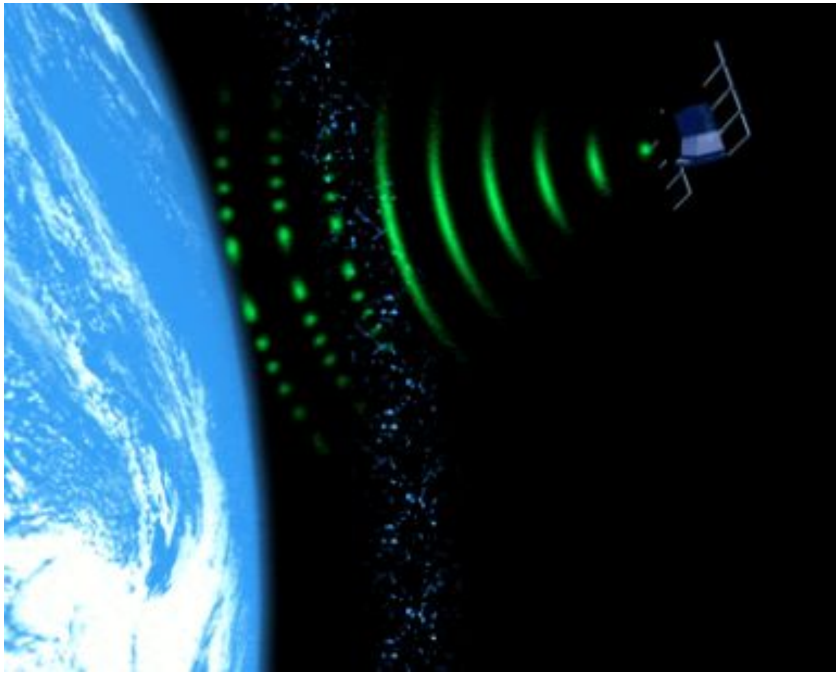
  [b] GNSS: Global Navigation Satellite System

- **However**, the GNSS signal can fail, due to:
  - Malicius attack
  - Natural phenomena: scintillation

# Motivation

- Ionospheric scintillation (GNSS signal propagation)



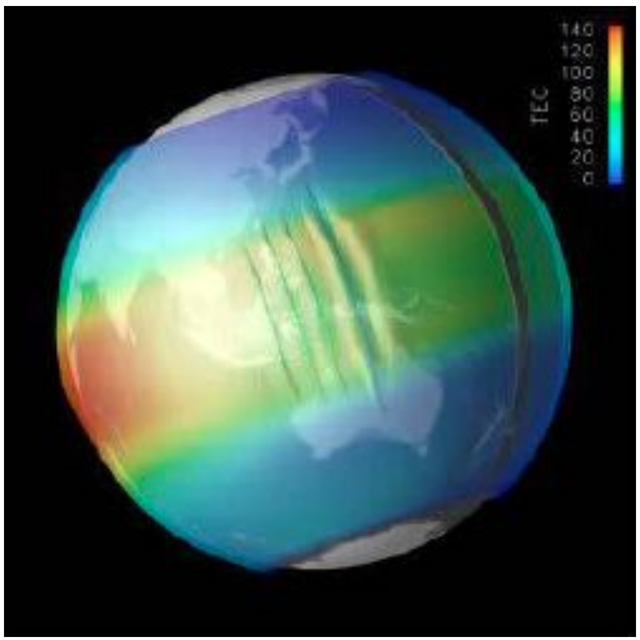


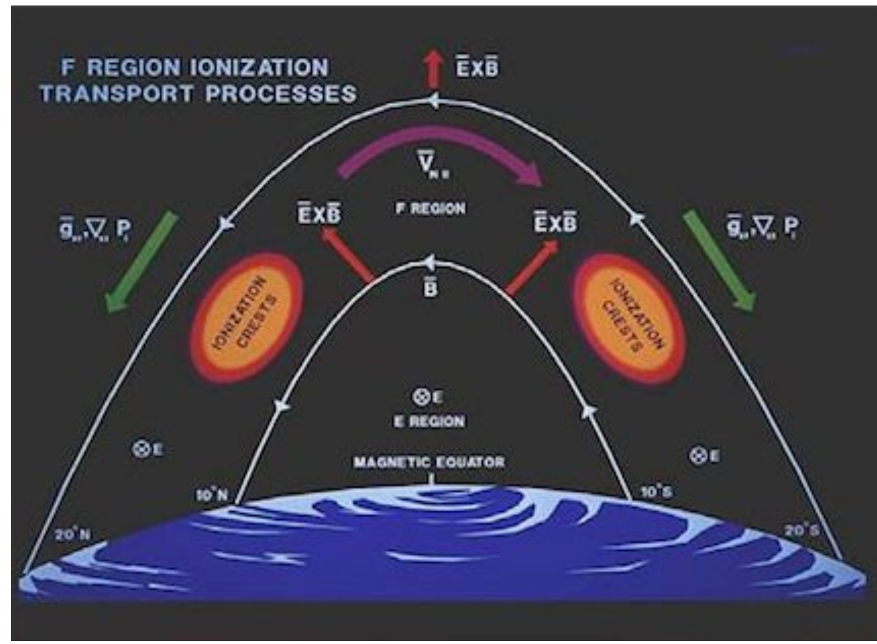
Signal delay: proportional to TEC (Total Electronic Content)

Scintillation

# Motivation

- Ionospheric scintillation (GNSS signal propagation)



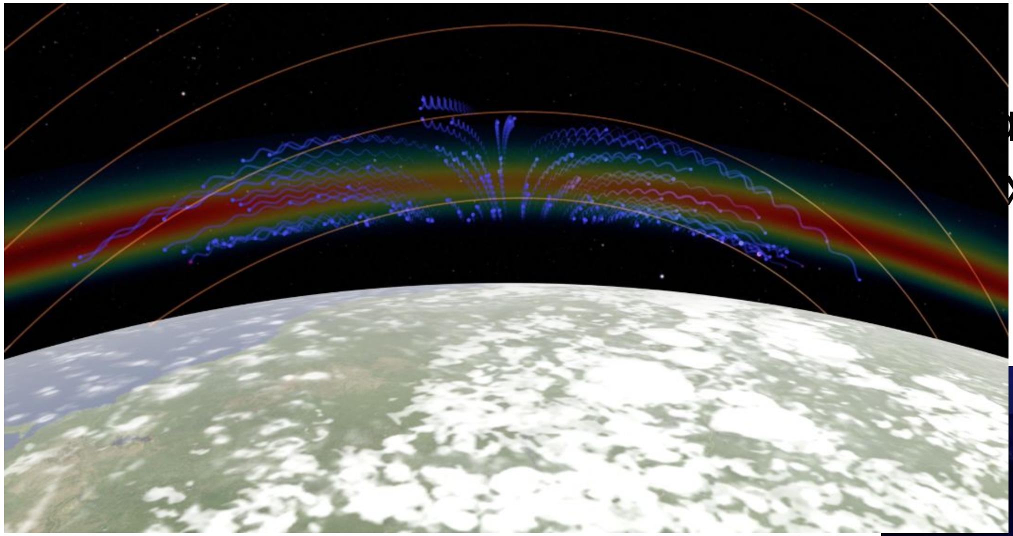




Ionospheric bubbles

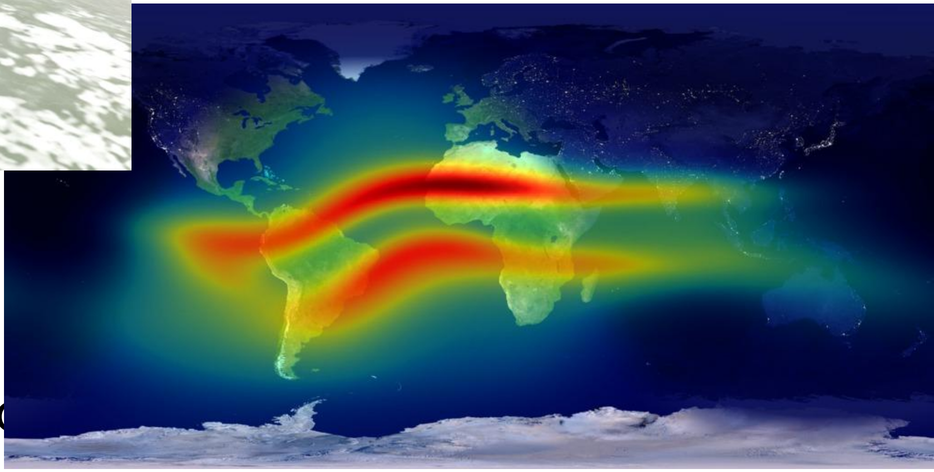
EIA: Equatorial Ionospheric Anomaly

# Motivation

- EIA: Equatorial Ionospheric Anomaly



ıy period for $O^+$
xigen ion concentration)



Ions traveling to North and South
from the Earth's magnetic equato

# SUPIM: Space weather prediction

Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 54 (2014) 22–36

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

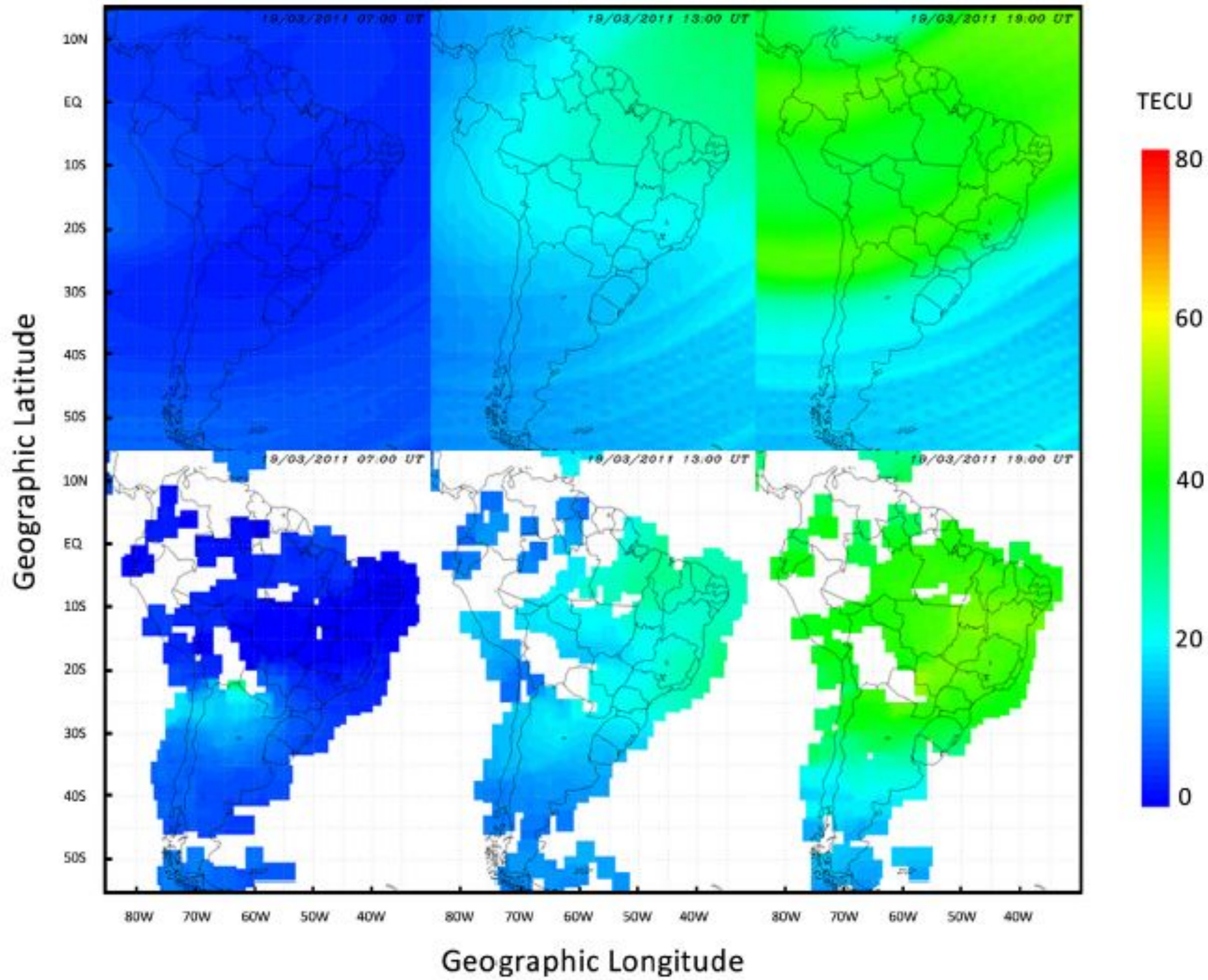
www.elsevier.com/locate/asr

First results of operational ionospheric dynamics prediction for the Brazilian Space Weather program

Adriano Petry [a,*], Jonas Rodrigues de Souza [b,1], Haroldo Fraga de Campos Velho [c,2], André Grahl Pereira [d,3], Graham John Bailey [e]

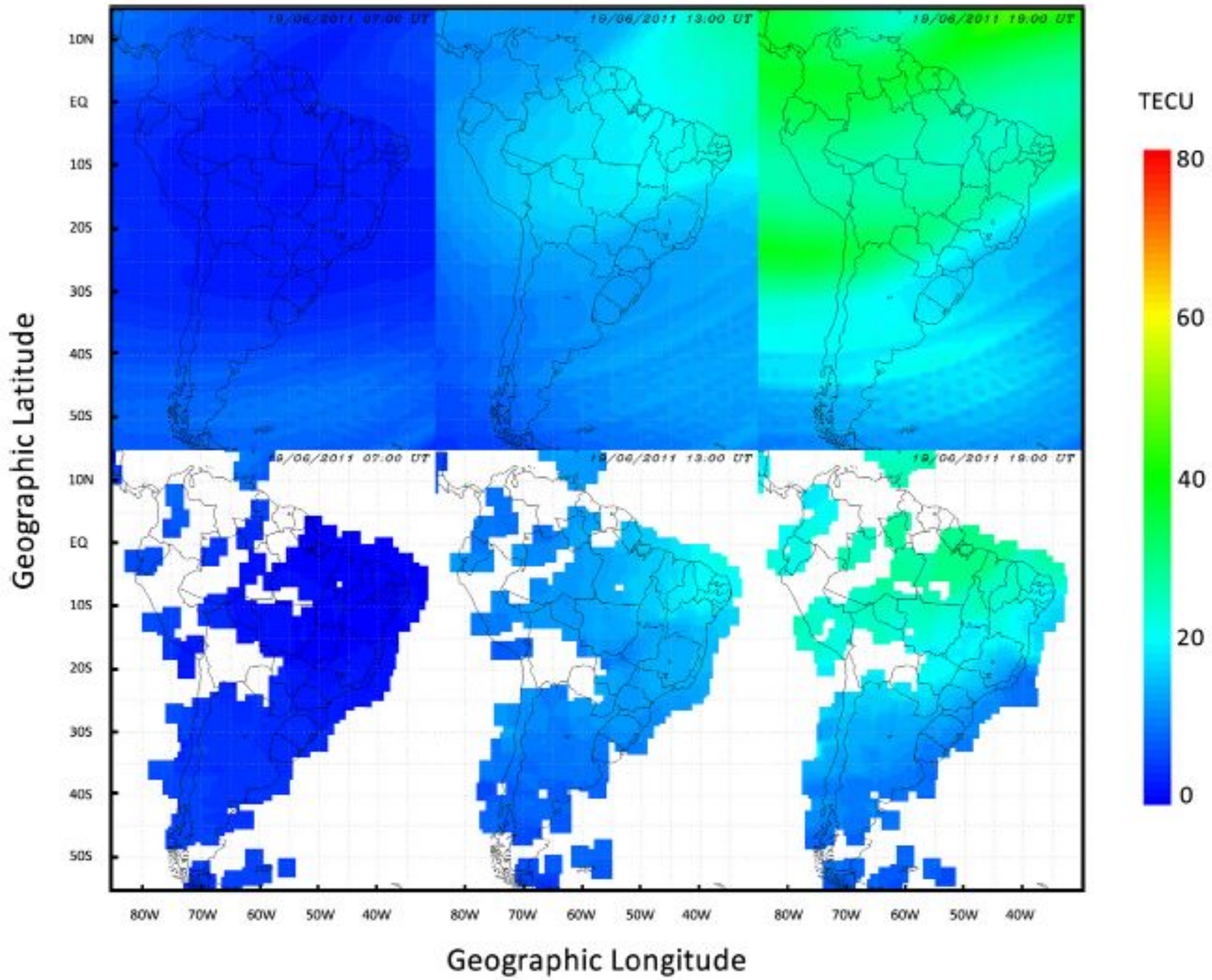
# SUPIM: Space weather prediction

- 7, 13, 19 UT: March 19th, 2011



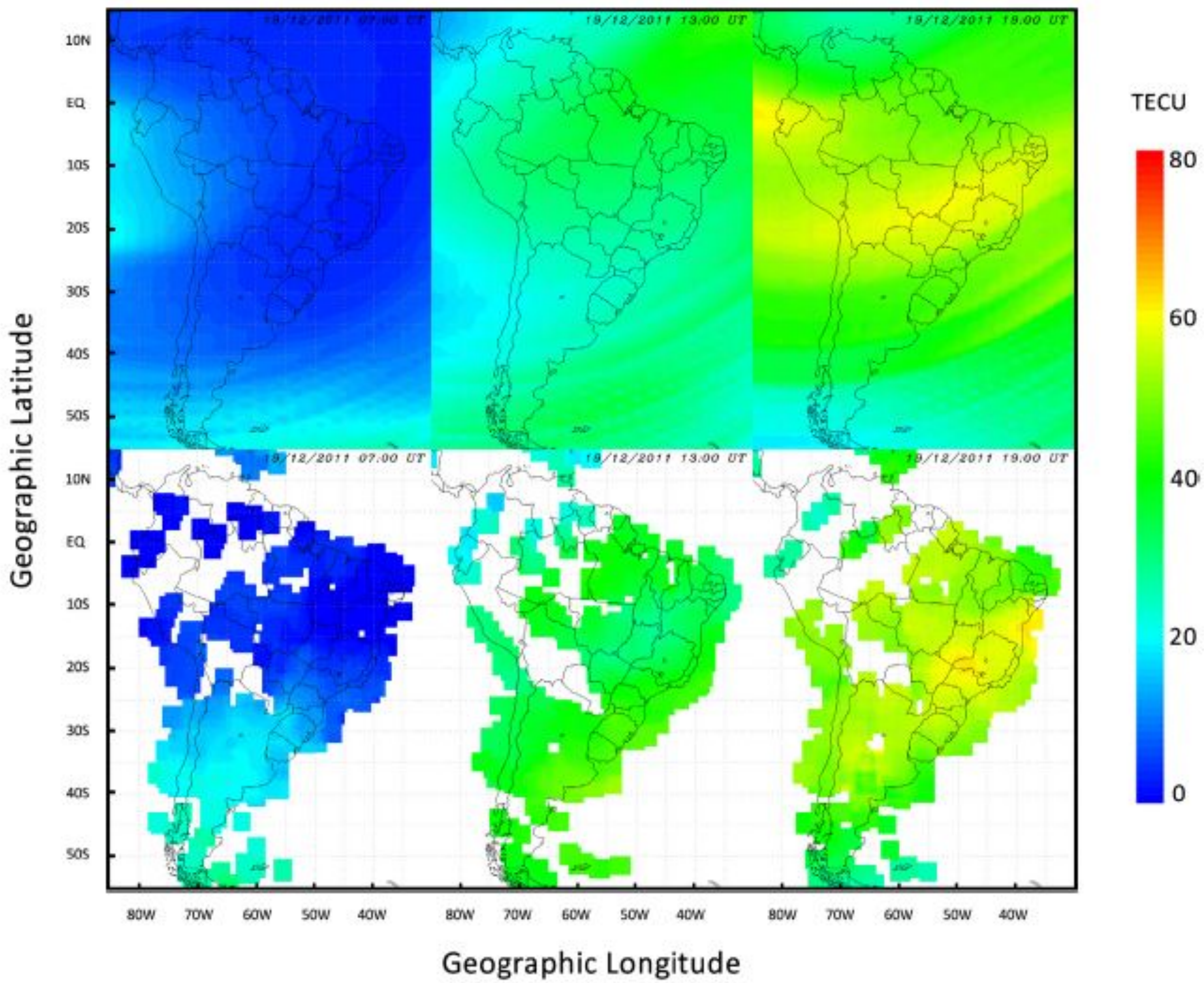
# SUPIM: Space weather prediction

- 7, 13, 19 UT: June 19$^{th}$, 2011



# SUPIM: Space weather prediction

- 7, 13, 19 UT: December 19th, 2011

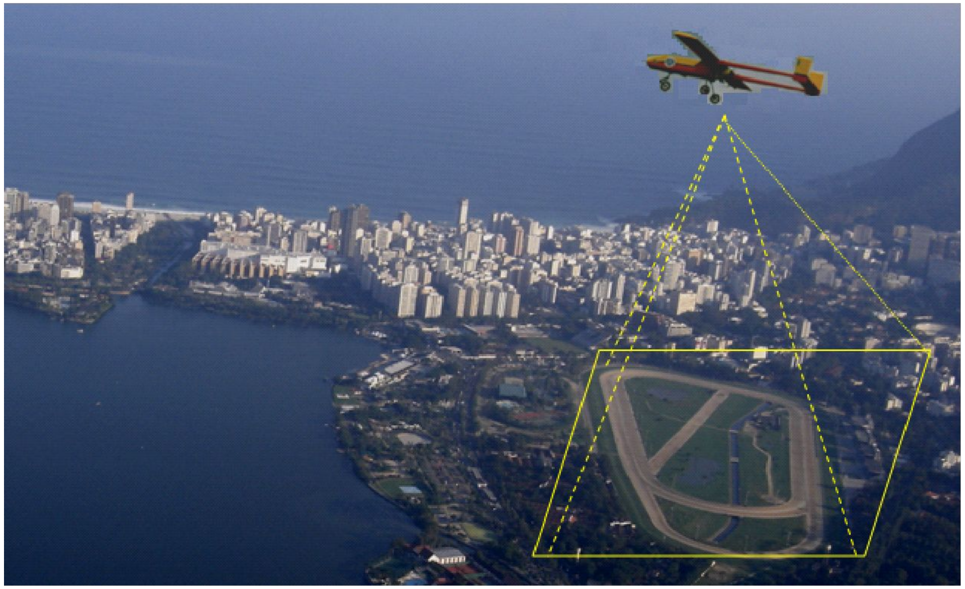


# Drone navigation without GNSS signal

A standard configuration for an UAV navigation system is composed of GNSS and inertial sensors (INS).

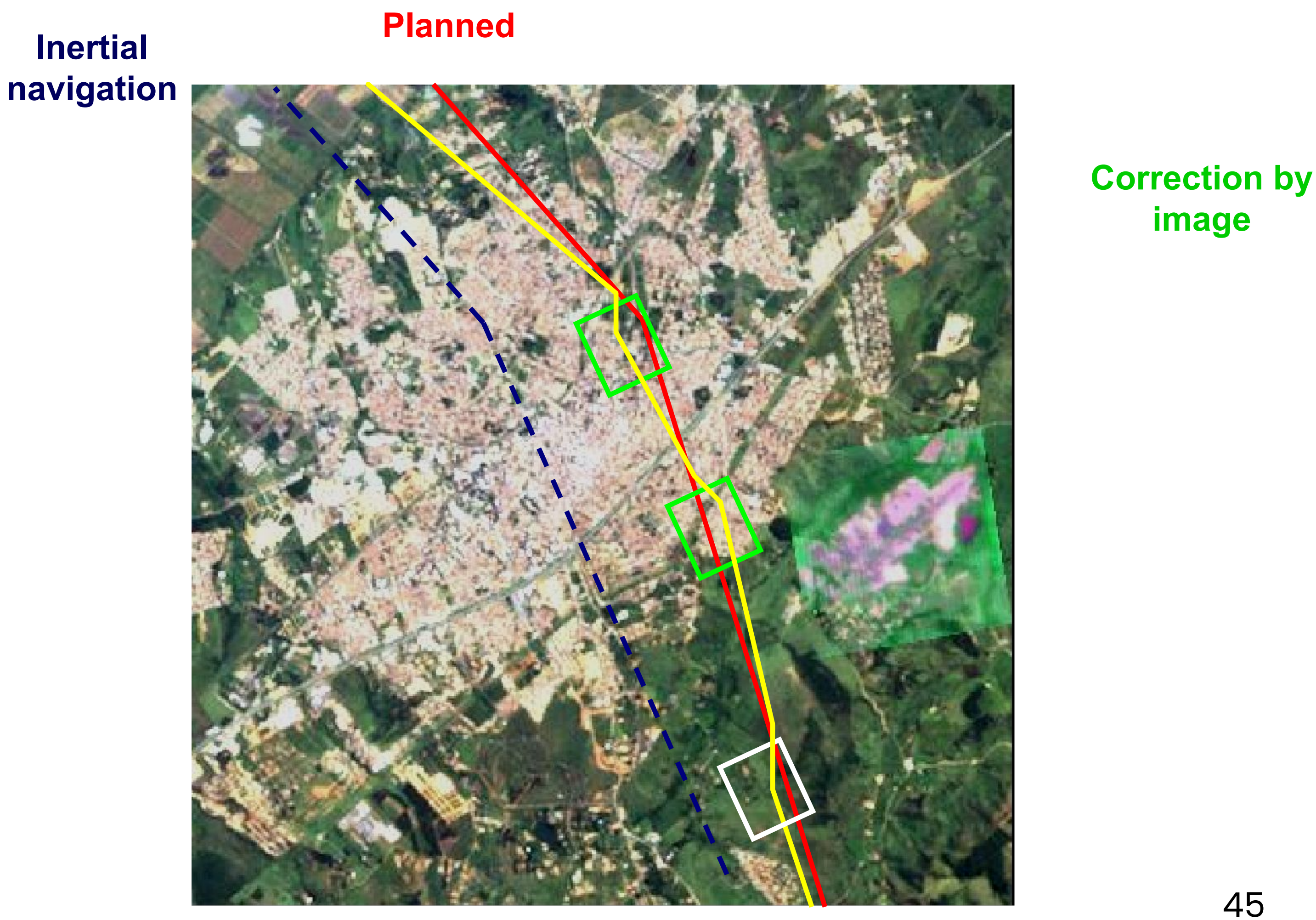
However, there are some **critical** applications where an alternative strategy to the GNSS signal is needed.

# Drone navigation without GNSS signal



One alternative without the use of a GNSS signal is an UAV navigation system based on vision system.

# Drone trajectory correction based on aerial images information



# Drone navigation without GNSS signal

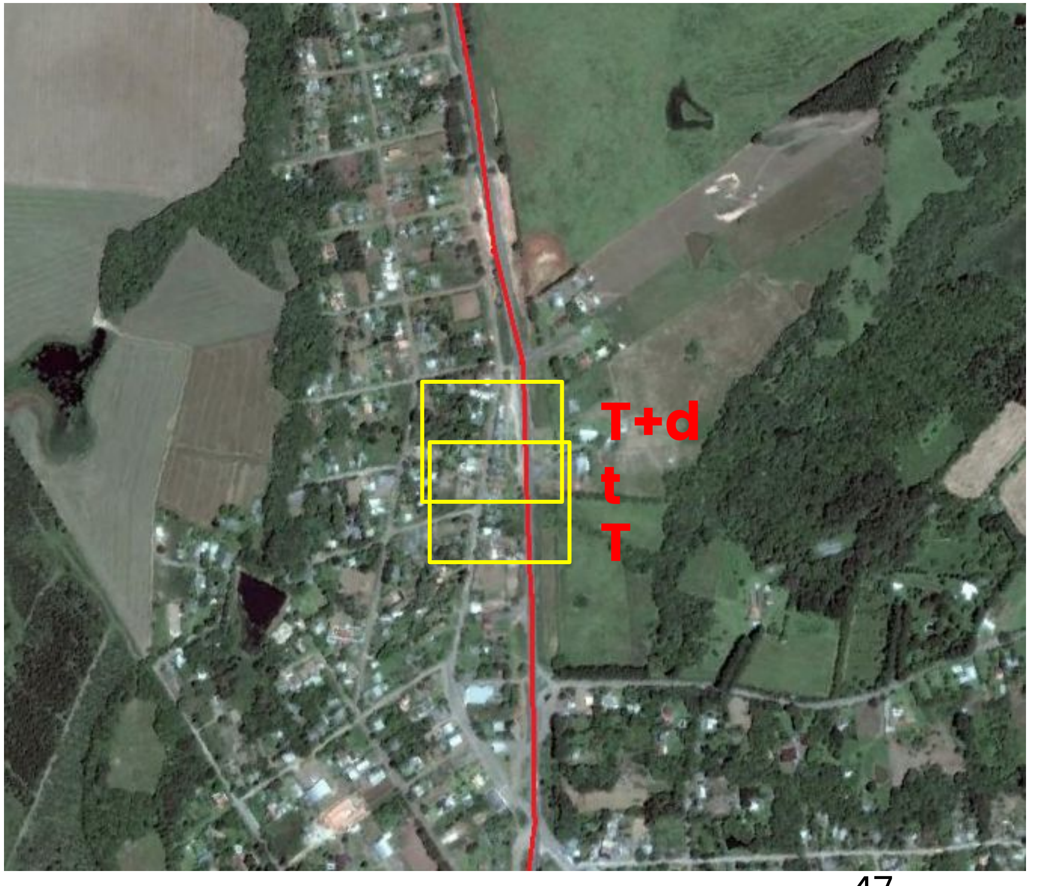
## Image processing with NN

- UAV used in former experiments

G. A. M. Goltz, J. D. S. Silva, H. F. Campos Velho, E.H. ; Shiguemori (2009): "Edge detection with aerial and satellite using artificial neural networks". In: National Congress on Computational and Applied Mathematics. Proceedings of XXXII CNMAC. São Carlos (SP), Brazil, pp. 1044-1045 - In Portuguese.

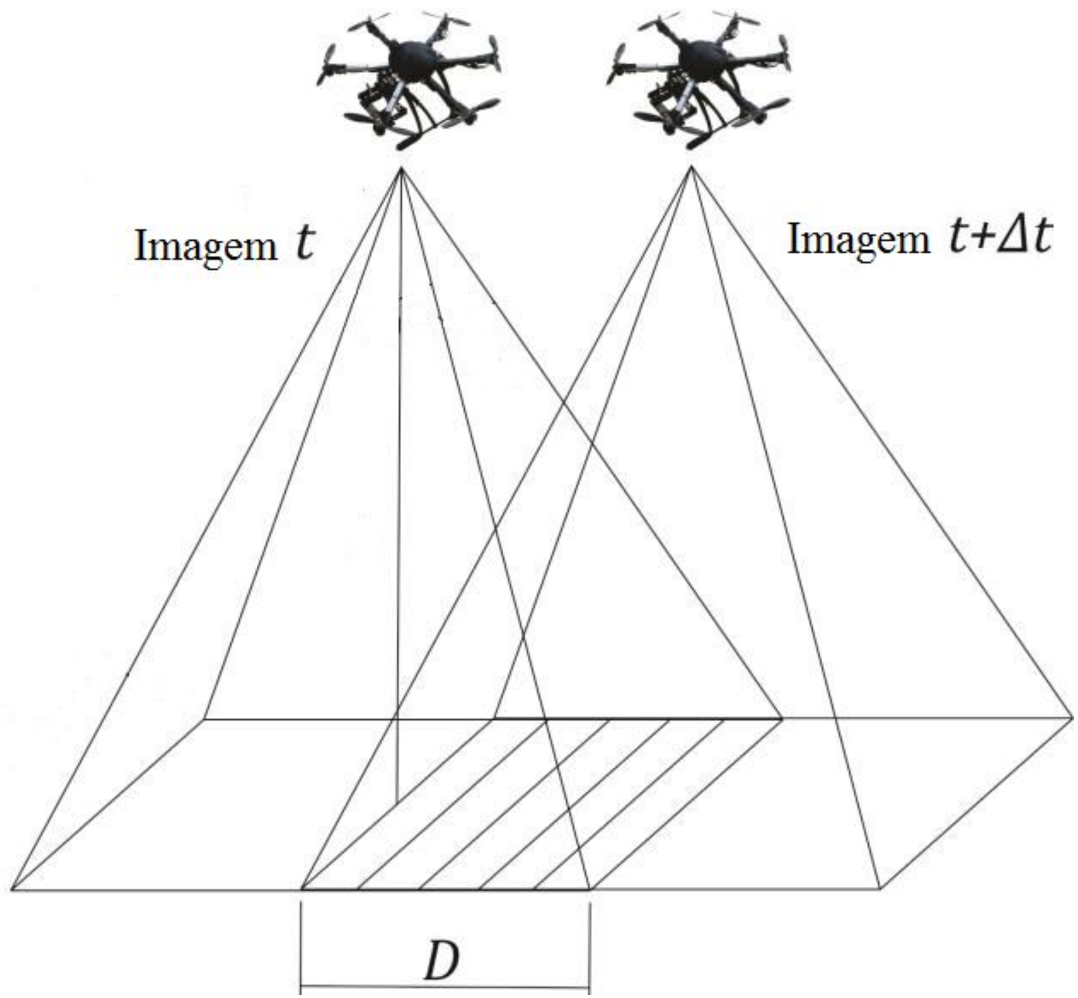
# Drone navigation without GNSS signal

- **Visual odometry**



# Drone navigation without GNSS signal

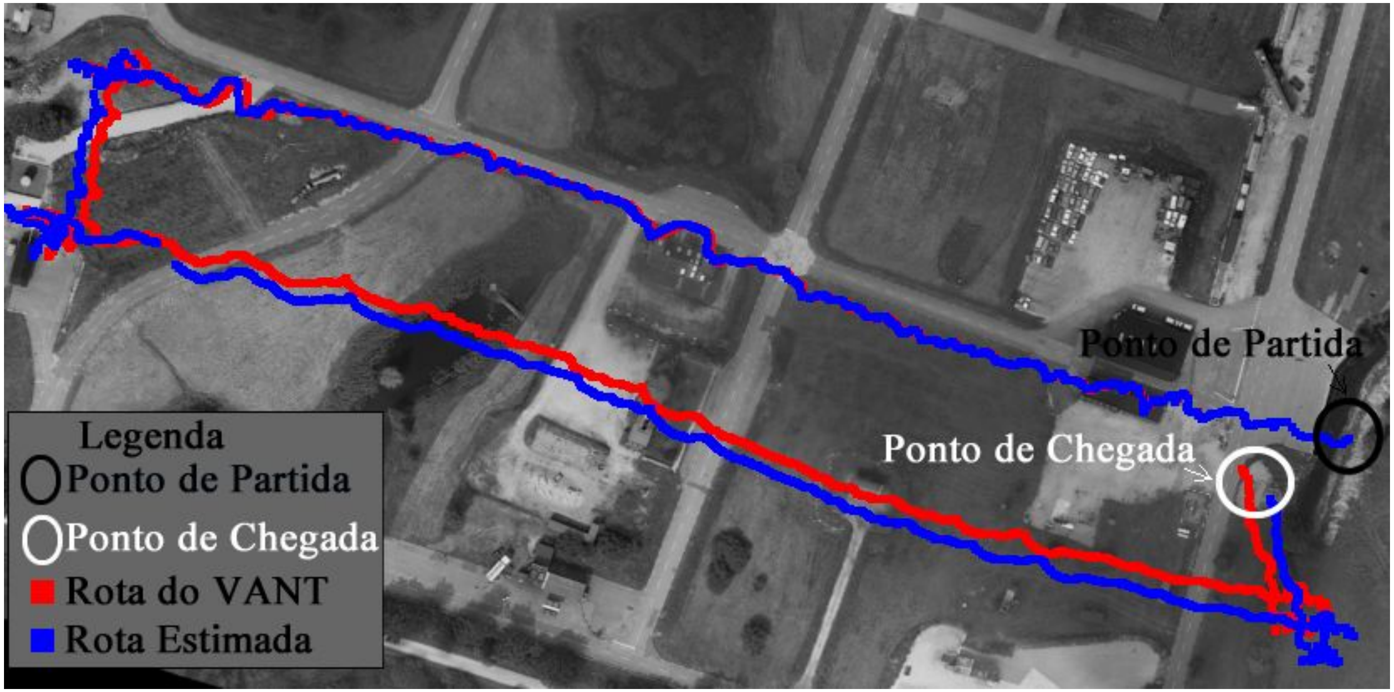
- **Visual odometry**



# UAV positioning algorithm: embedded system

1. **Drone trajectory correction**

   Without GNSS signal: visual odometry



# Drone navigation without GNSS signal

## Computer vision by image segmentation

- Examples


Original image


True


Canny

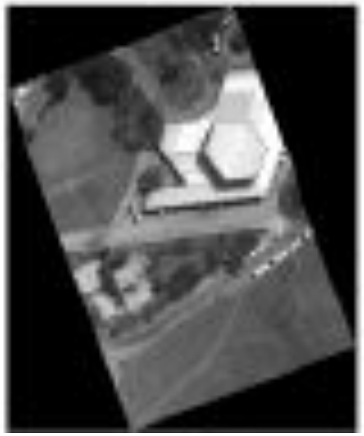
# Previous Works

## Image acquisition

Acquisition

Rotation / Scale correction

**UAV**



**Satellite**

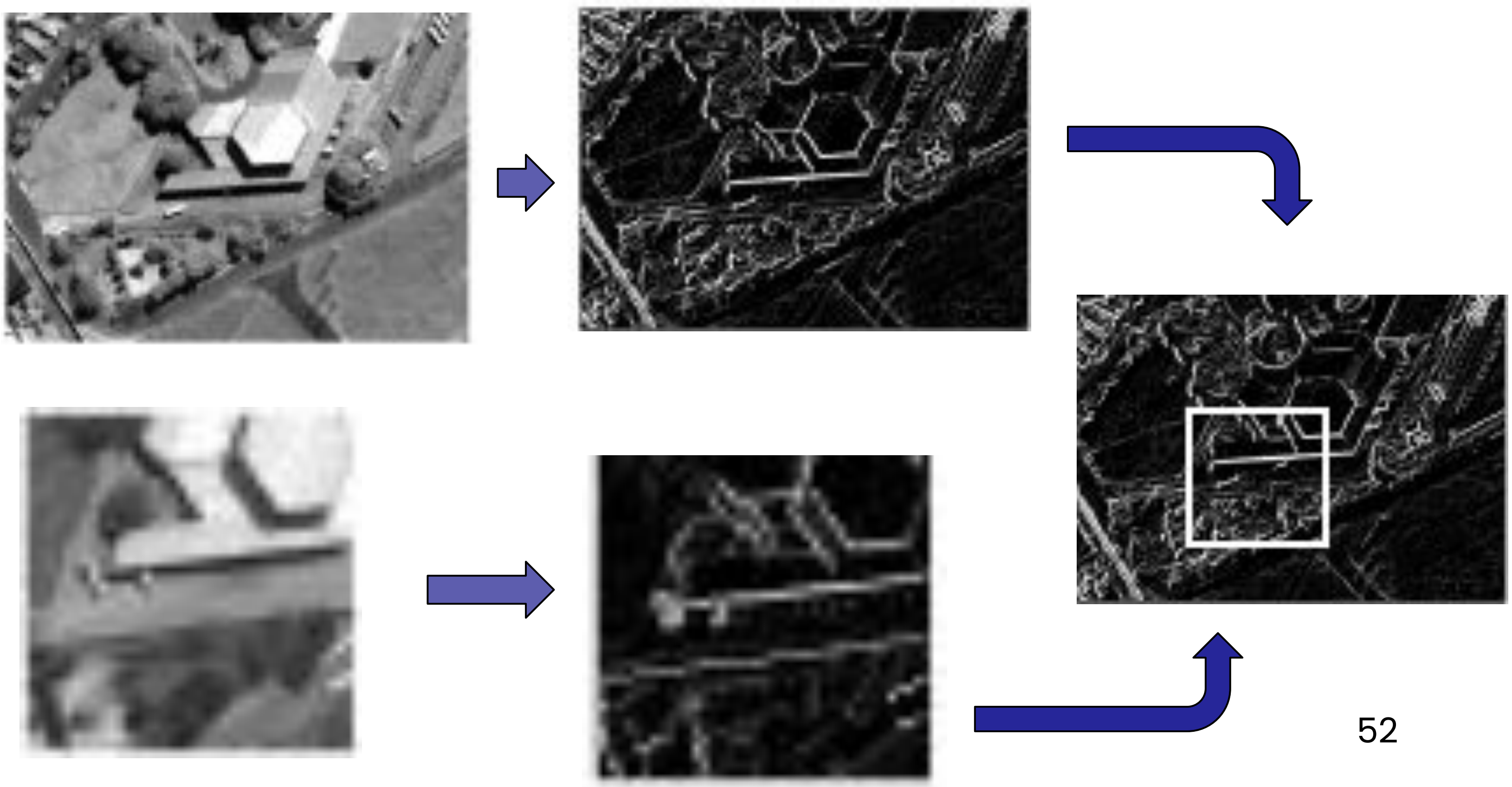
# Previous Works

## Image segmentation and correlation

**Segmentation** **Correlation**

Satellite

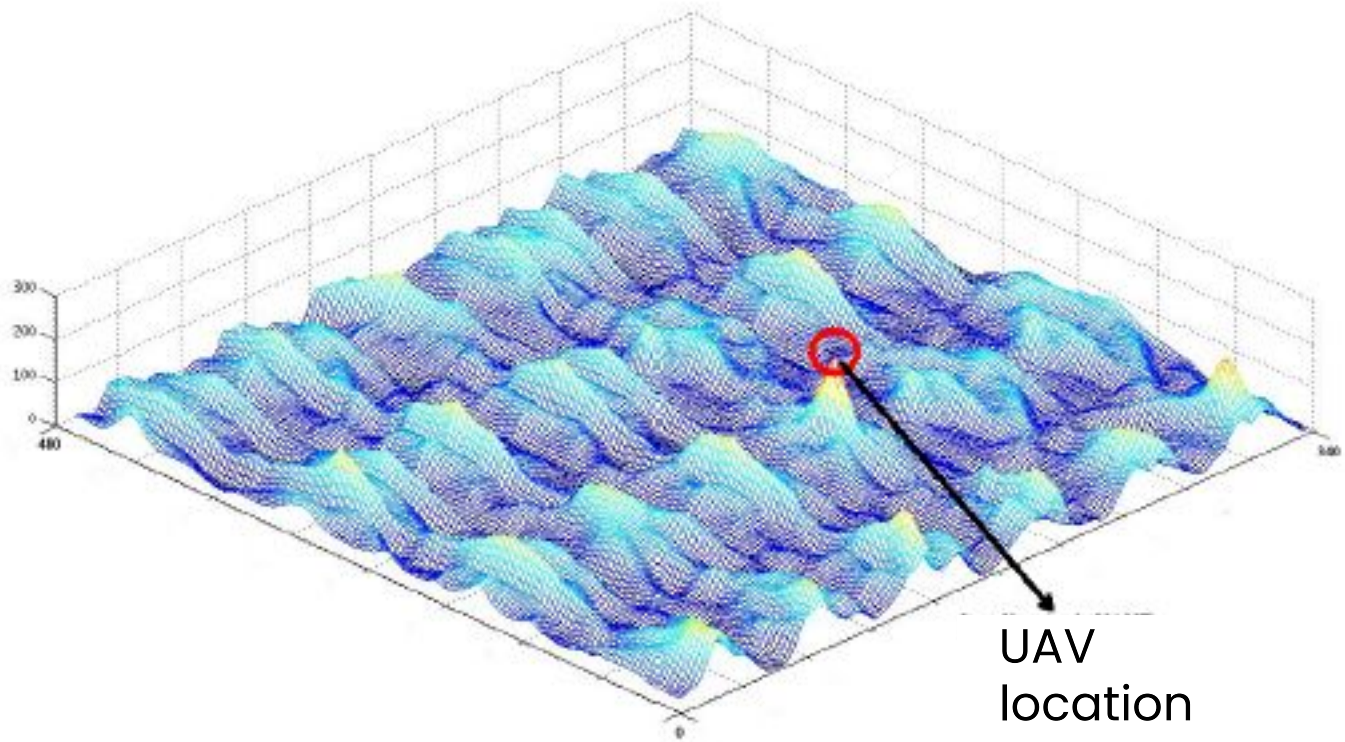
UAV



# Matching different images

Images correlations

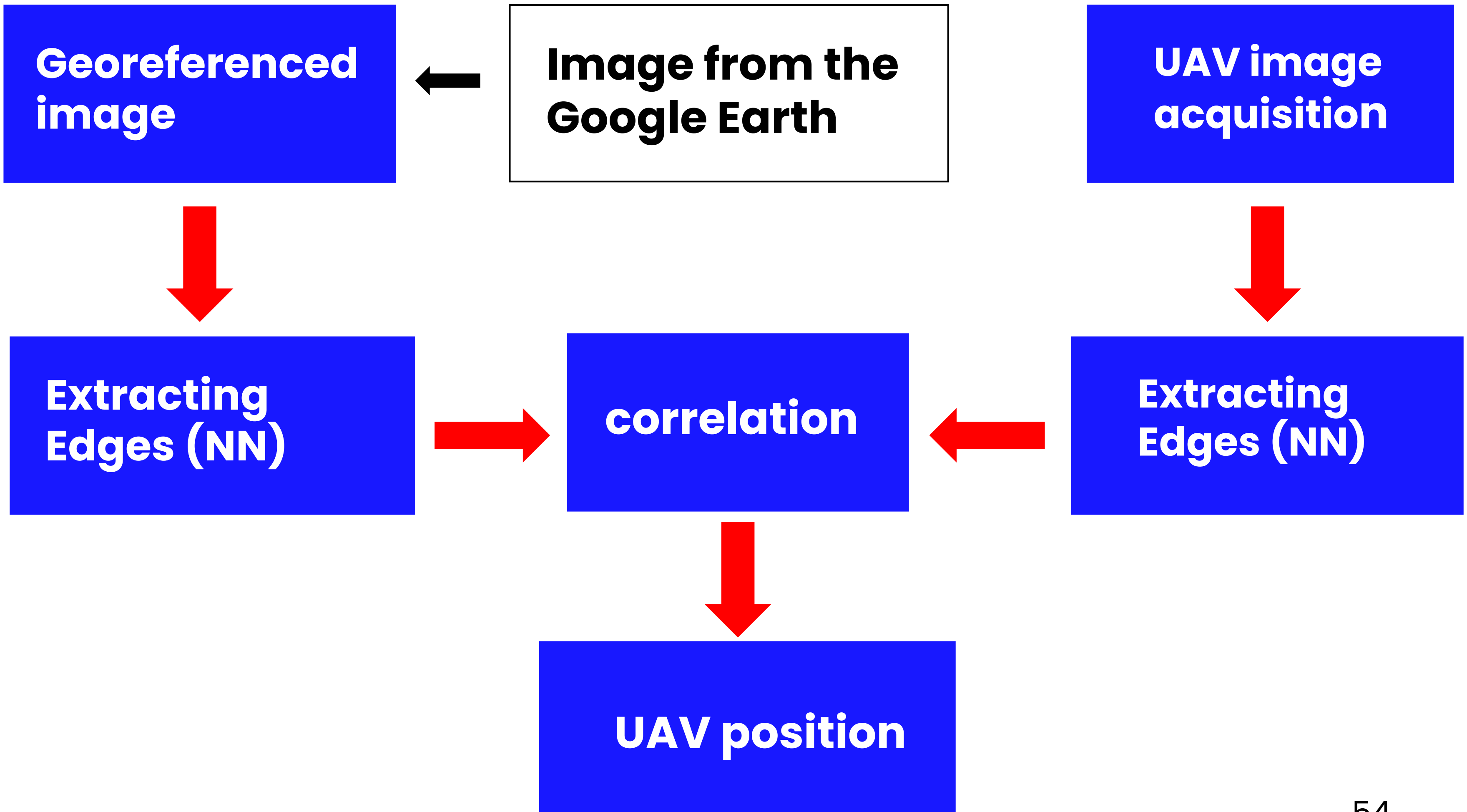
$$c(s,t) = \sum\sum f(x,y) \times w(x-s, y-t)$$



UAV location

# Drone positioning algorithm

## Image segmentation and correlation



# PART 2 – Optimal neural network

- Design of supervised neural network:

  Optimization problem – see cost function:

$$E_{trein} = \frac{1}{N}\sum_{k=1}^{N}(d_k - s_k)^2$$

$$E_{gen} = \frac{1}{(M-N+1)}\sum_{k=N+1}^{M}(d_k - s_k)^2$$

$$F_{obj} = penalty * \frac{(\rho_1 * E_{trein} + \rho_2 * E_{gen})}{\rho_1 + \rho_2}$$

$$penality = \underbrace{\left(c_1 * \left(e^{\#neuron}\right)^2\right)}_{\text{complexity factor-1}} \times \underbrace{\left(c_2 * (\#epoch)\right)}_{\text{complexity factor-2}} + 1$$

# MPCA: Multi-Particle Collision Algorithm

Available for download:
www.epacis.net/jcis/PDF_JCIS/JCIS11-art.01.pdf

PAN-AMERICAN ASSOCIATION OF COMPUTATIONAL INTERDISCIPLINARY SCIENCES

Journal of Computational Interdisciplinary Sciences (2008) 1(1): 3-10

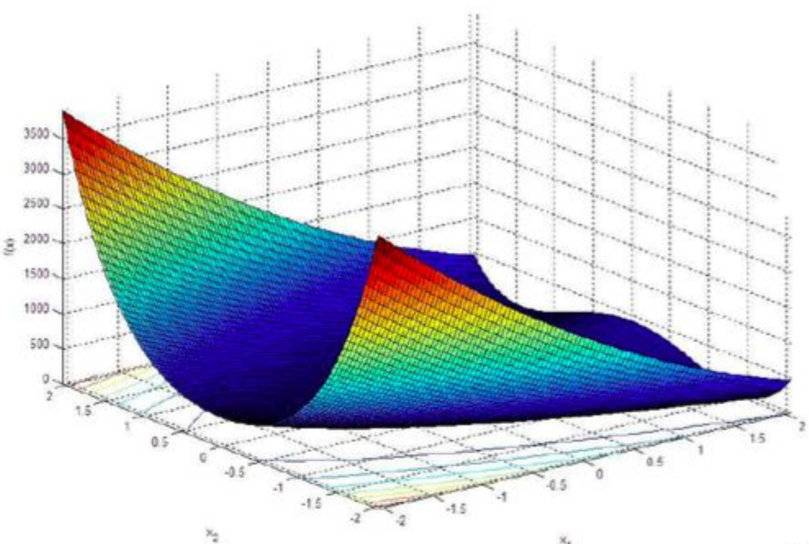
ISSN 1983-8409
http://epacis.org

## A new multi-particle collision algorithm for optimization in a high performance environment

Eduardo Fávero Pacheco da Luz, José Carlos Becceneri and Haroldo Fraga de Campos Velho

Manuscript received on July 31, 2008 / accepted on October 5, 2008

# PCA vs MPCA



**Rosenbrok function:**

$$f(x_1,x_2)=100(x_2-x_1^2)^2+(1-x_1)^2$$

$$\left\|(x_1,x_2)\right\|^2 \leq 2048$$

$$\min:(1,\ 1),\quad f(\pi,\pi)=0$$

| **PCA** | **MPCA** |
|---|---|
| **(1,1)** | **(1,1)** |
| $f(x_1,x_2)=5\times 10^{-5}$ | $f(x_1,x_2)=9\times 10^{-9}$ |

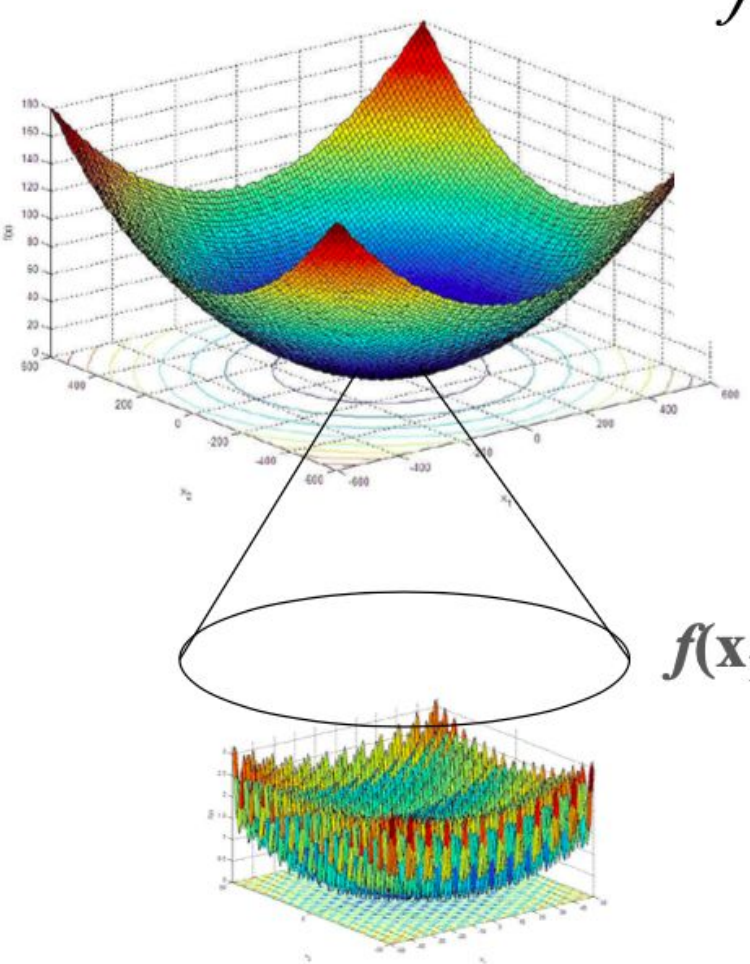
# PCA vs MPCA

**Griewank function**

$$f(x_1,\ldots,x_n) = 1 + \sum_{j=1}^{n} \frac{x_1^2}{4000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right)$$

$$\left\|(x_1,\ldots,x_2)\right\|_2^2 \le 600$$

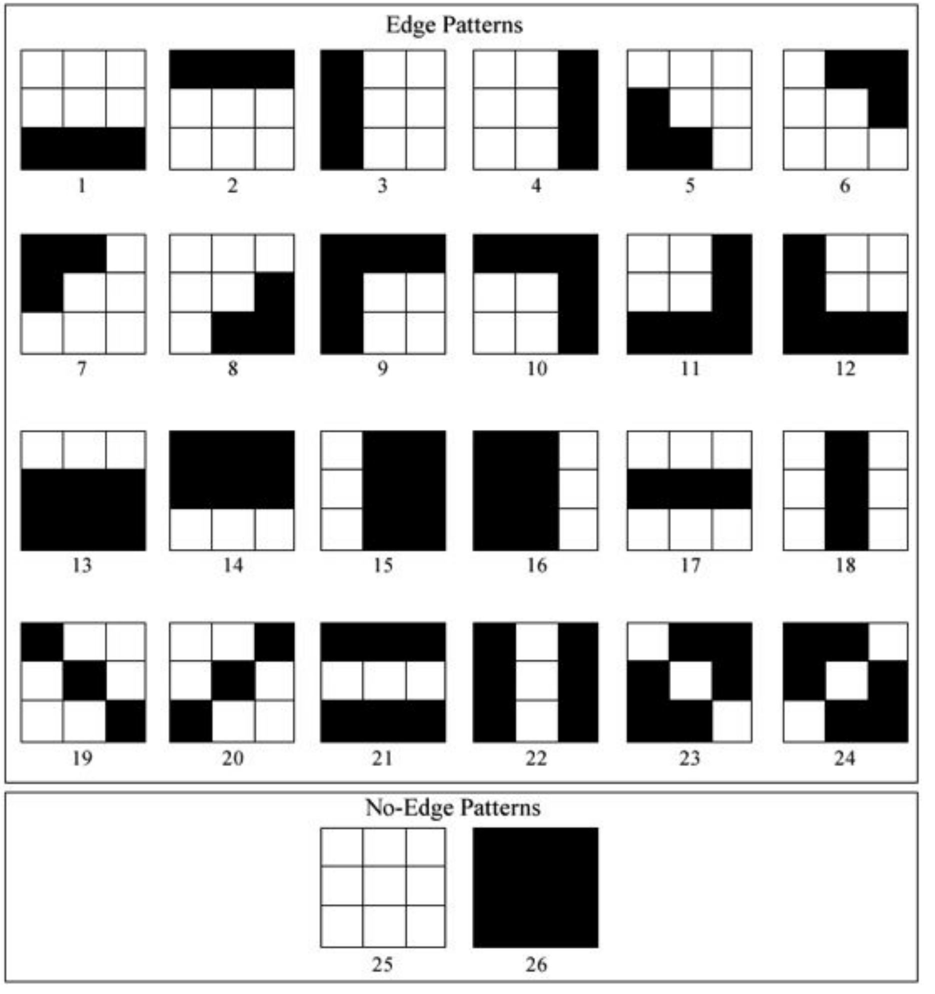
$$\min : (0,\cdots,0), \quad f(0,) = 0$$



| PCA | MPCA |
|---|---|
| (-3.14, 4.43) | $(-1.8\text{x}10^{-8}, -3.3\text{x}10^{-8})$ |
| $f(x_1,x_2) = 7.4 \text{ x } 10^{-3}$ | $f(x_1,x_2) = 3.3 \text{ x } 10^{-16}$ |

# UAV positioning algorithm: embedded system

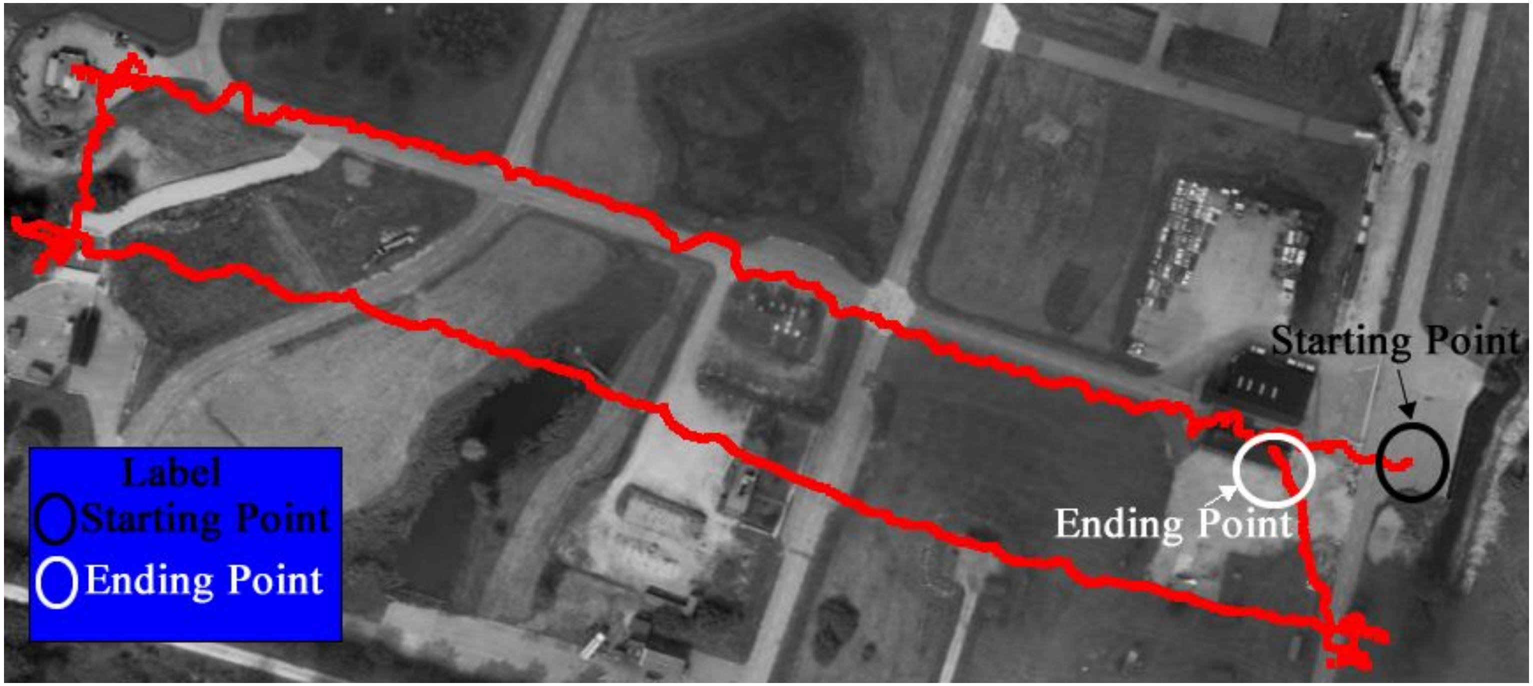
1. **Edge patterns**
   a) 24 are selected
   b) MLP-NN classifies
   the other patterns
   c) Classification table



# UAV positioning algorithm: embedded system

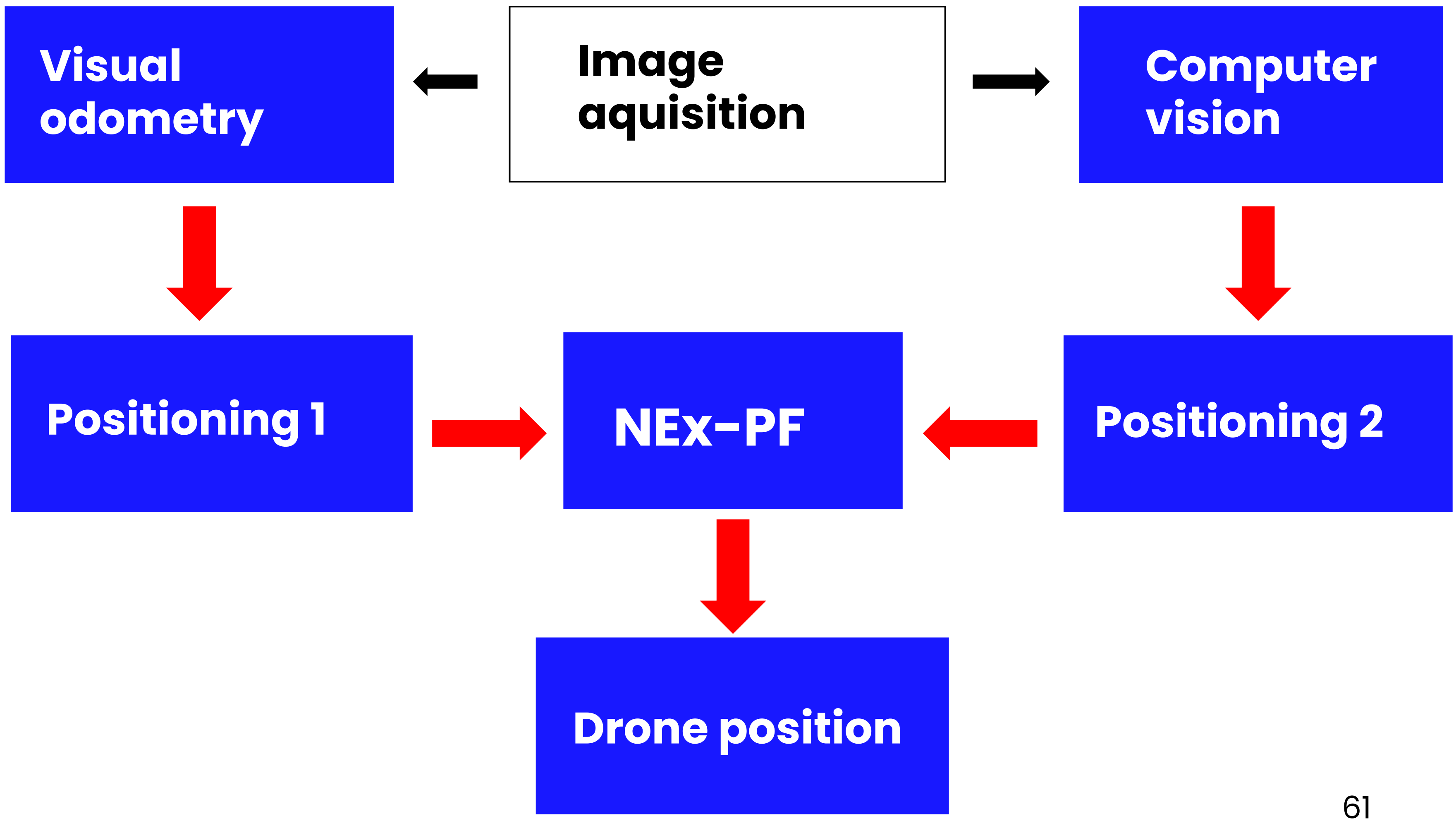
1. **Drone trajectory correction**

   Without GNSS signal: edge extraction



# Drone positioning algorithm

**Data fusion: Visual odometry + Computer vision**



# State estimation: several goals

- **Prediction – Filtering – Fixed-lag Smoothing**
  - $\pi(x,z)$: conditional probability distribution
  - $x$ : state variable
  - $z$ : observations
  - **Prediction:** determination of $\pi(x_k, z_{1:k-1})$
  - **Filtering:** determination of $\pi(x_k, z_{1:k})$
  - **Fixed-lag Smoothing:** determination of $\pi(x_k, z_{1:k+p})$
    - where $p \geq 1$ is the fixed lag.

# Bayesian filters

- **Bayesian strategy – filtering**

1. We know: $\pi(x_0, z_0) = \pi(x_0)$
1. Compute: $\pi(x_k, z_{1:k})$
    a) Based on Bayes's theorem

$$P(A \mid B) = \frac{P(A \cap B)P(A)}{P(B)}$$

    a) Under Markovian process

$$\pi(x_k \mid x_{k-1}, \dots , x_1) = \pi(x_k \mid x_{k-1})$$

2. Bayesian filters: **prediction** and **update**

# Bayesian filters

- **Particle filter: beyond Kalman filter**

1. Compute the initial ensemble: $\left\{x_{0|n-1}^{(i)}\right\}_{i=0}^{M} \sim \pi_{w_0}(x_0)$ (initial PDF: $N(0,1)$)
1. Compute the weights: $q_n^{(i)} = \pi(z_n \mid x_{n|n-1}) = \pi_{\text{et}}[z_n - h(x_n, t_n)]$

$$p_{\text{et}}(z) = \exp(-w^2 / 4\pi)$$

1. Normalize: $\hat{q}_n^{(i)} = q_n^{(i)} \Big/ \left(\sum_{i=1}^{M} q_n^{(i)}\right)$
1. Re-sampling (select particles): $\pi(x_n \mid z_s) \approx \sum_{i=1}^{M} \hat{q}_n^{(i)} \delta(x_n - x_{n|s}^{(i)}) \quad \sum_{i=1}^{M} \hat{q}_n^{(i)} = 1$
1. Compute new particles: $x_{n+1|n}^{(i)} = f\left(w_{n|n}^{(i)}, t_n\right) + \mu_n$

# Bayesian filters

- **Bayesian strategy – filtering**

  1. **Kalman filter**

     a) Linear stochastic process

     b) Gaussian statistics

  1. **Particle filter**

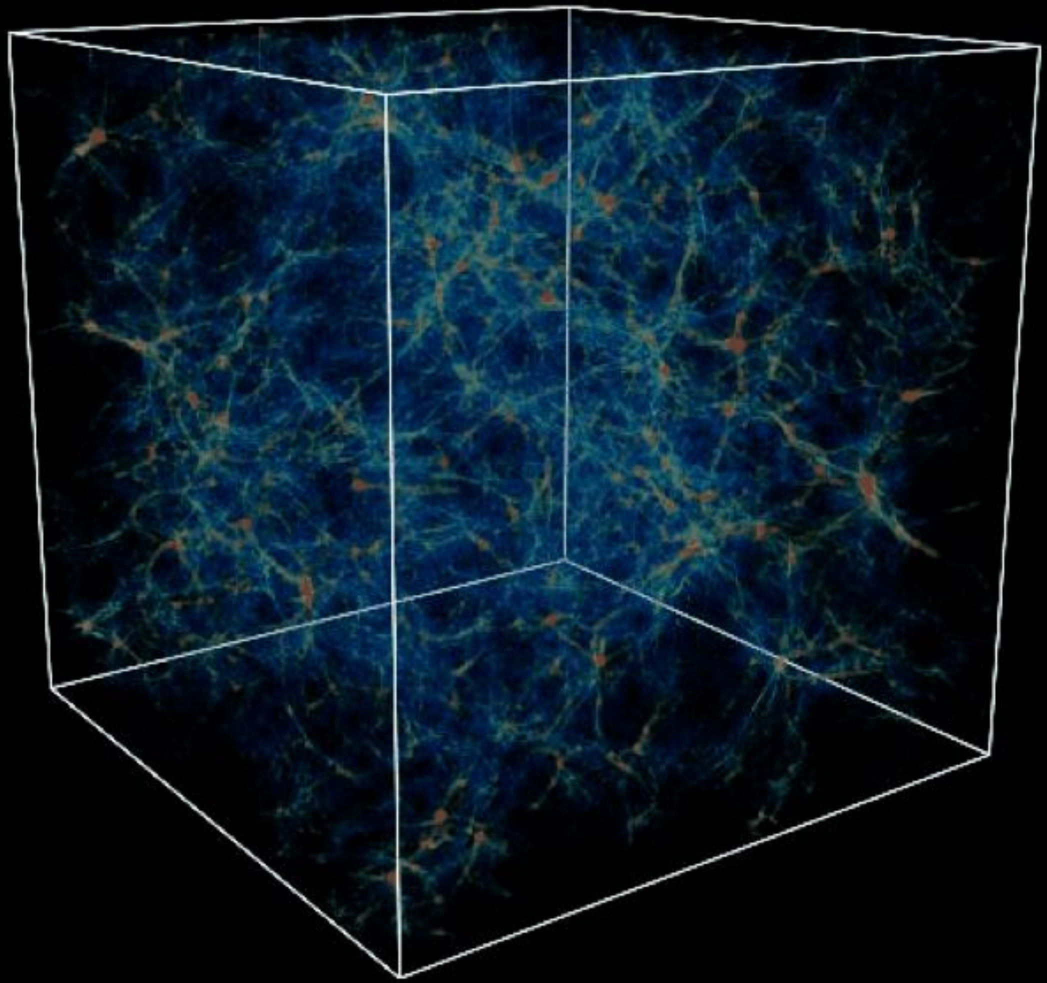
     b) Applied to non-linear process

     c) Applied for non-Gaussian statistical models

     d) Kernel of Particle Filter:

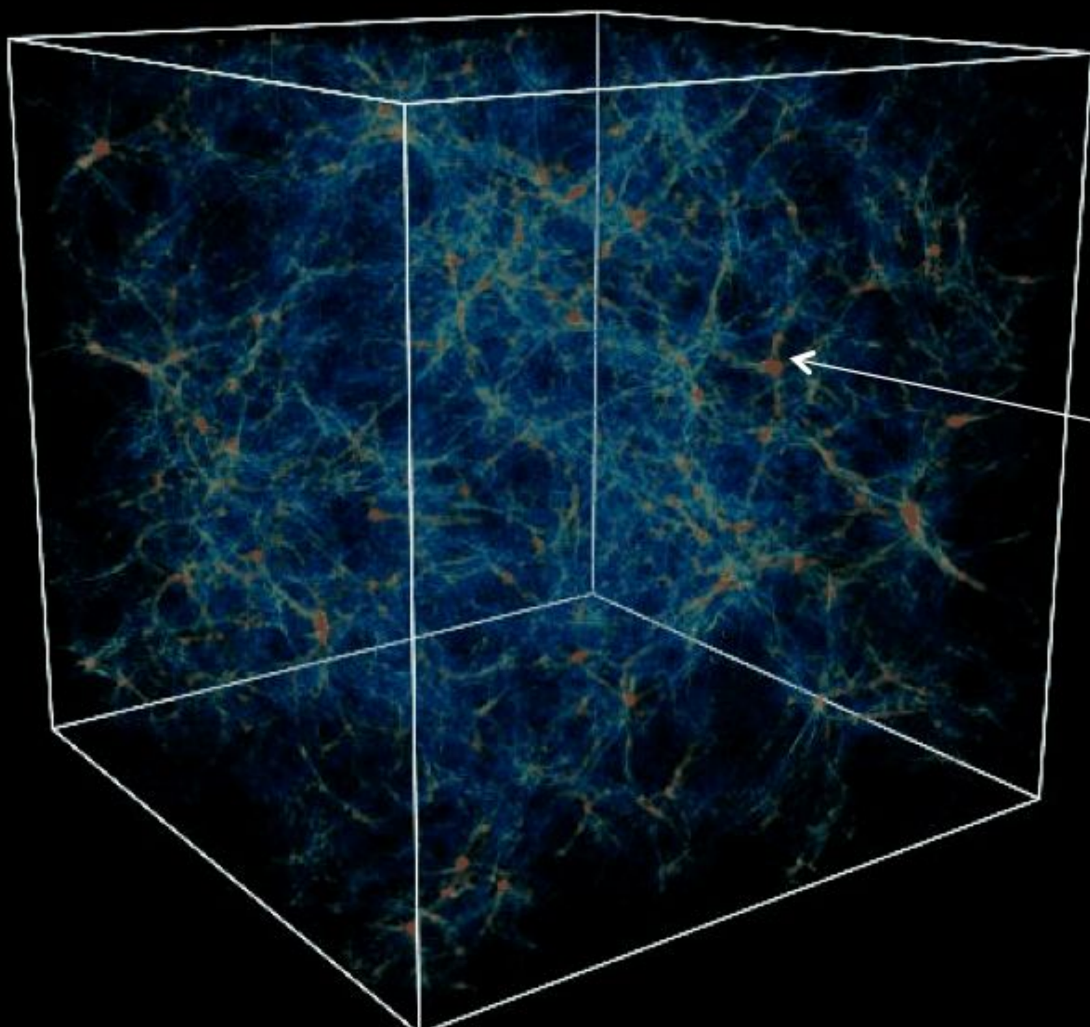
$$\underbrace{p(w_n \mid Y_{n-1})}_{\text{a posteriori}_{(w_n)}} \propto \underbrace{p(y_n \mid w_n)}_{\text{likelihood}_{(w_n)}} \underbrace{p(w_n \mid Y_{n-1})}_{\text{a priori}_{(w_n)}}$$

# Everything is perfect for particle filter?



Where are the attractors?

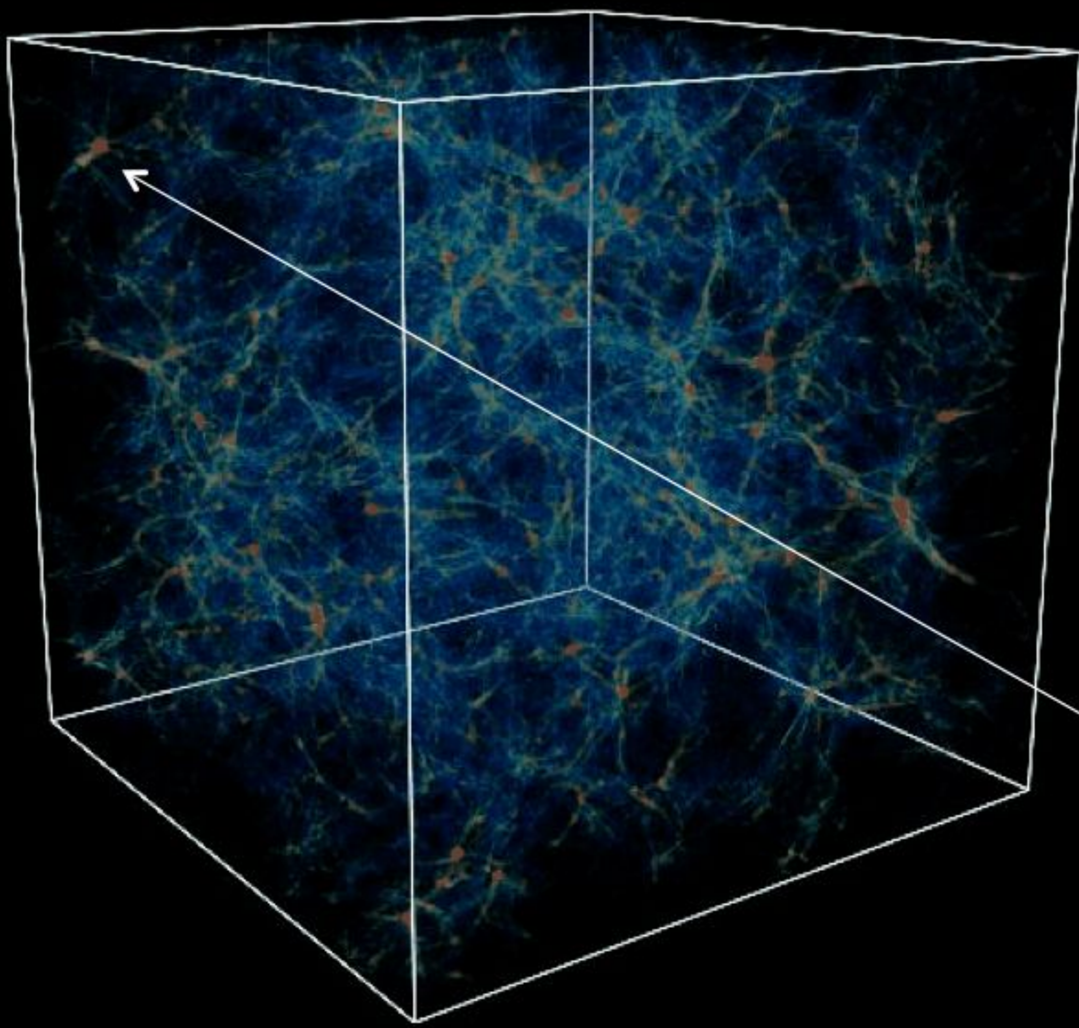
# Everything is perfect for particle filter?



Where are the attractors?

Gaussian?

# Everything is perfect for particle filter?



Where are the attractors?

Lévy?

# Bayesian filters

- **Everything is perfect with particle filter?**

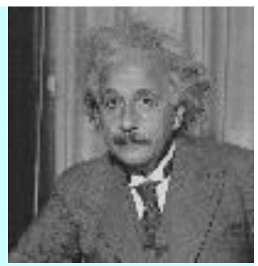
  **Almost!**

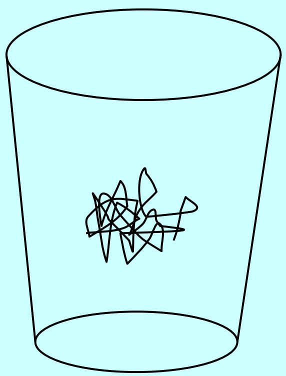
  - In the distribution space there are (at least) two attractors for stable distributions: Gaussian, and Lévy $\alpha$-stable
  - Gaussian (Normal): Central Limit Theorem
  - Lévy $\alpha$-stable: Lévy-Gnedenko Central Limit Theorem.

# Bayesian filters

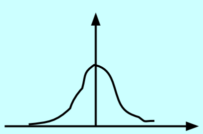
- **Non-extensive particle filter:**
- Non-extensive thermostatistics: motivation

- Albert Einstein
  1905



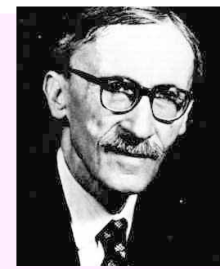


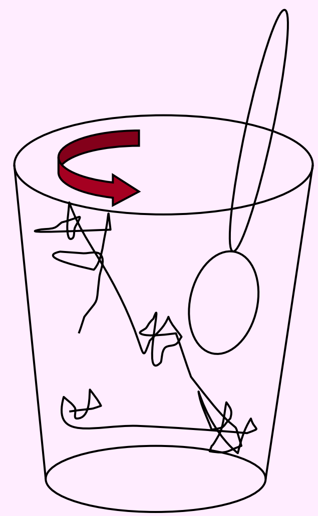
***Brownian motion***



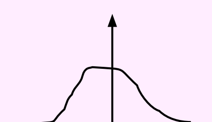
$$e^{-x^2/\sigma^2}$$

- Paul Levy
  1937





***Levy distributions***



$$\frac{1}{\sigma^2 - x^2}$$

# Bayesian filters

- **Non-extensive particle filter:**

  **Beyond particle filter**

- Tsallis' distribution – from non-extensive thermostatistics

- ***q* < 1**

$$\rho(x) = \frac{1}{\sigma}\left[\frac{q-1}{\pi(3-q)}\right]^{1/2} \frac{\Gamma\left(1/(q-1)\right)}{\Gamma\left((3-q)/[2(q-1)]\right)} \frac{1}{\left[1+[(q-1)/(3-q)](x/\sigma)^2\right]^{1/(q-1)}}$$

- ***q* = 1**

$$\rho(x) = \frac{1}{\sigma}\left[\frac{1}{2\pi}\right]^{1/2} e^{-(x/\sigma)^2/2}$$

- ***q* > 1**

$$\rho(x) = \frac{1}{\sigma}\left[\frac{1-q}{\pi(3-q)}\right]^{1/2} \frac{\Gamma\left((5-3q)/[2(1-q)]\right)}{\Gamma\left((2-q)/(1-q)\right)} \left[1-[(1-q)/(3-q)](x/\sigma)^2\right]^{1/(1-q)}$$

# Bayesian filters

- **Non-extensive particle filter:**
- Tsallis's non-extensive distribution
- Choice for using Gaussian distribution can be justified by the Central Limit theorem.
- However, there is another attractor on the distribution space.
- This is the Levy-Gnedenko's central limit theorem.
- For the case of Tsallis' distributions:

$$\rho_q(x) = \begin{cases} G(x) & q < 5/3 \\ L_\gamma(x) & q > 5/3 \end{cases}$$

# Adaptive Non-extensive Particle Filter

- Likehood function: Tsallis' distribution
  (non-extensive formalism of thermodynamics)

$$f_q(x) = \begin{cases} \alpha_q{}^{-}.\left[1 - \frac{1-q}{3-q}\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}, & -\infty < q < 1 \\ \alpha_q{}^{+}.\left[1 - \frac{1-q}{3-q}\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{1-q}}, & 1 < q < 3 \end{cases}$$

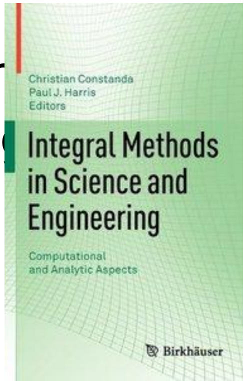
$$f_q \xrightarrow{q \to 1} Gaussian \qquad f_q \xrightarrow{q \to 5/3} Lévy$$

- where "$q$" is the non-extensive parameter:
  to be estimated by a secondary process

# Adaptive Non-extensive Particle Filter

- Result presented during **IMSE-2010**
  (University of Brighton (UK), 12-14/July/2010)

- H. F. Campos Velho, H.C.M. Furtado (2011):
  Adaptive particle filter for stable distributio
  Integral Methods in Science and Engineerin
  p. 47-57, Birkhäuser, New York.



# Drone navigation without GNSS signal

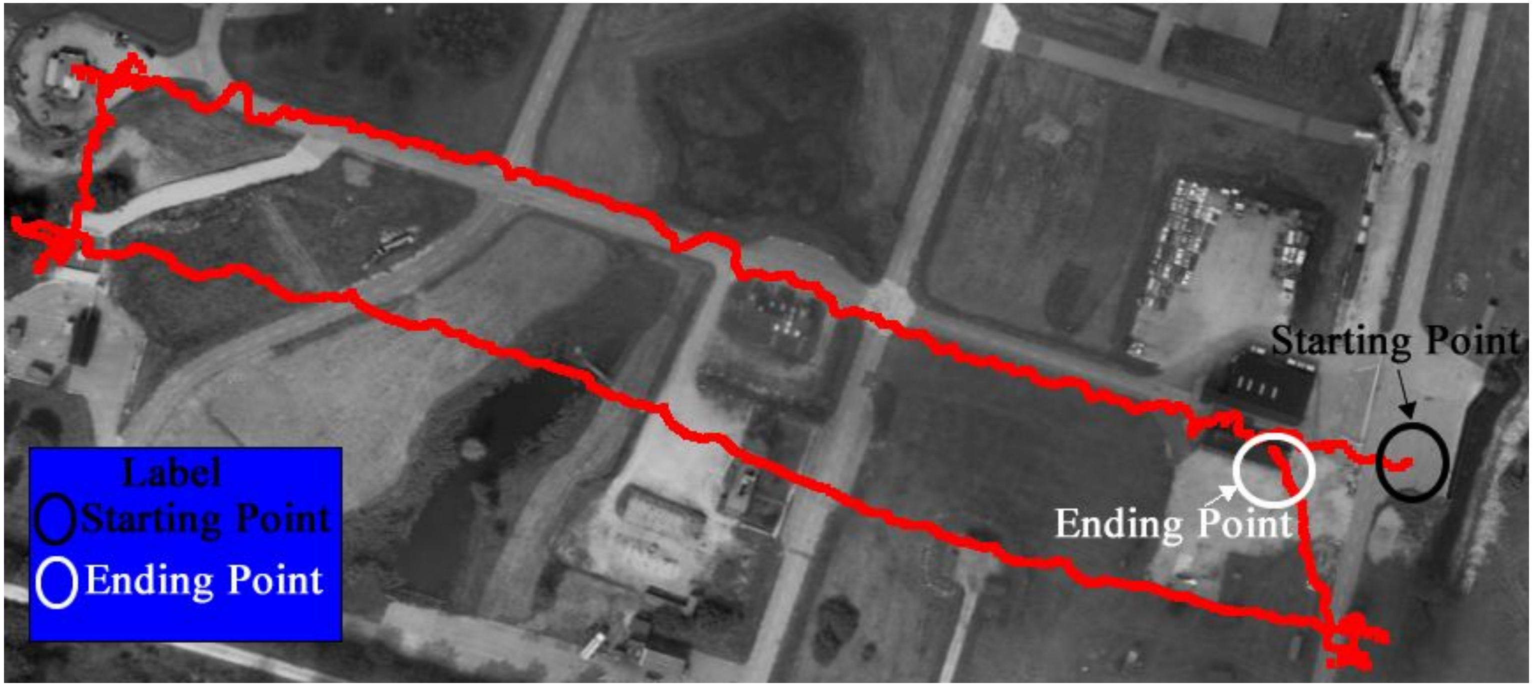
## Image processing with NN

- UAV used in the out door experiments

# UAV positioning algorithm: embedded system

1. **Drone trajectory correction**

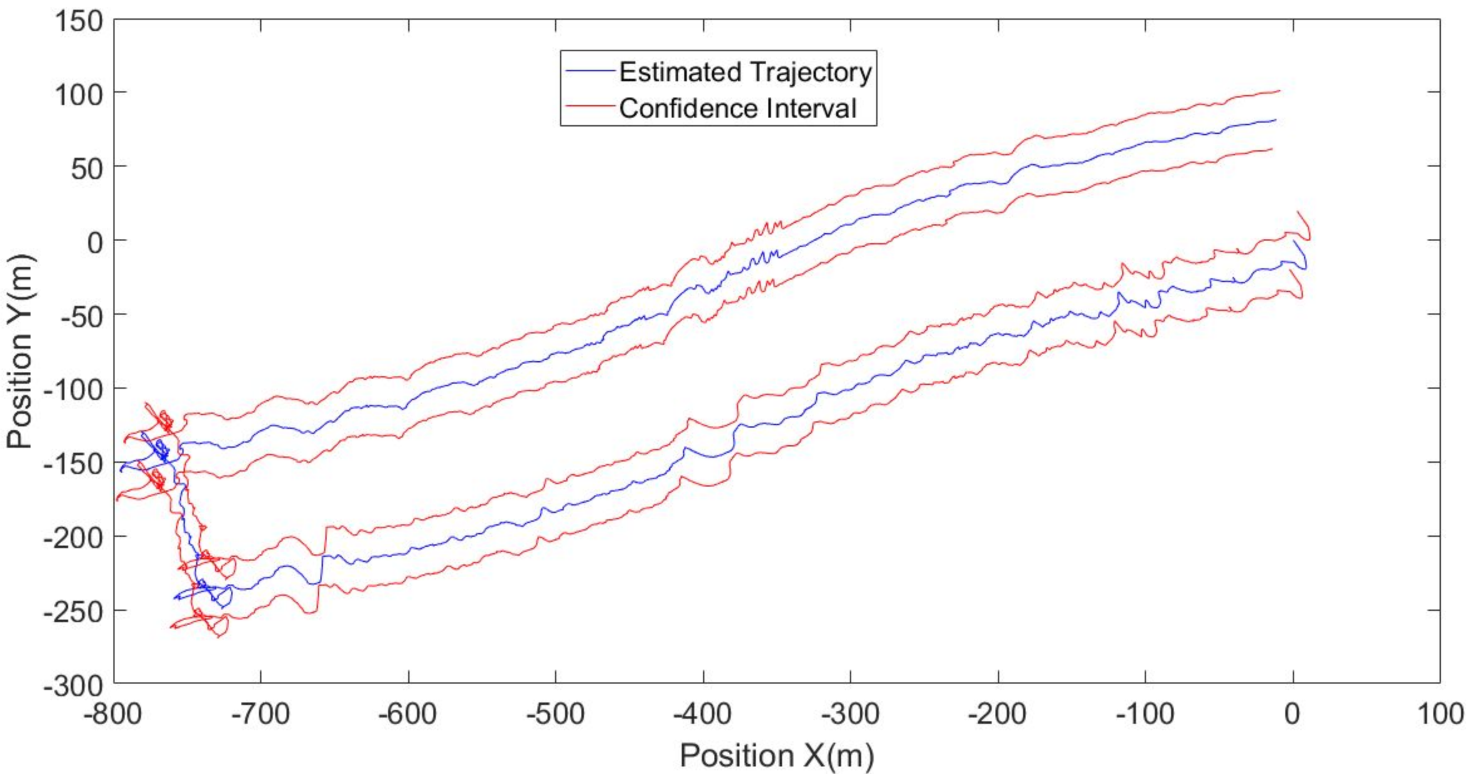
(without GNSS signal)



# UAV positioning algorithm: embedded system

1. **Drone trajectory correction**

Data fusion by **N**on-**Ex**tensive **P**article **F**ilter
Uncertainty quantification



# UAV positioning algorithm: embedded system

1. **Drone trajectory correction**

Data fusion by **N**on-**Ex**tensive **P**article **F**ilter

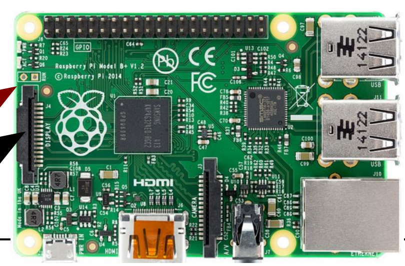
| Method | Average error |
|---|---|
| VO | 2.7572 m |
| CVS | 1.8315 m |
| NExt-PF: $q = 1.00$ | 1.6885 m |
| NExt-PF: $q = 2.57$ | 0.9857 m |

# UAV positioning algorithm: embedded system

## Cooperation with Linköping University

Pipelene processing:

Image acquisition → pre-processing → edge extraction → correlation (position)



(CPU: Raspberry)

(FPGA: Spartan)

# UAV positioning algorithm: embedded system

## 1. Trajectory correction (MLP-NN on FPGA)

| Edge extraction (methods) | # corrected points with error < 10 m | CPU-time (seconds) |
|---|---|---|
| Canny | 197 | 0.25 |
| Sobel | 296 | 0.29 |
| MLP-NN | 414 | 5.00 (20 x slower) |

# UAV positioning algorithm: embedded system

1. **Trajectory correction (MLP-NN with LUT on FPGA)**

| Edge extraction (methods) | # corrected points with error < 10 m | CPU-time (seconds) |
|---|---|---|
| Canny | 197 | 0.25 |
| Sobel | 296 | 0.29 |
| MLP-NN | 414 | 0.13 (2 x faster) |

# Final Remarks

1. Edge extraction by optimal NN was effective, obtaining **better** results than Canny approach.
2. Visual odometry can be used to estimate the drone trajectory.
3. Data fusion (computer vision + visual odometry) by Non-extensive particle filter **improved** the results.
4. A parametric study was applied to determine the **best** non-extensive parameter (q).
5. Our methodology is using a parallel multi-processor CPU and multi-FPGA hybrid computing.

# Applications for NEx-PF

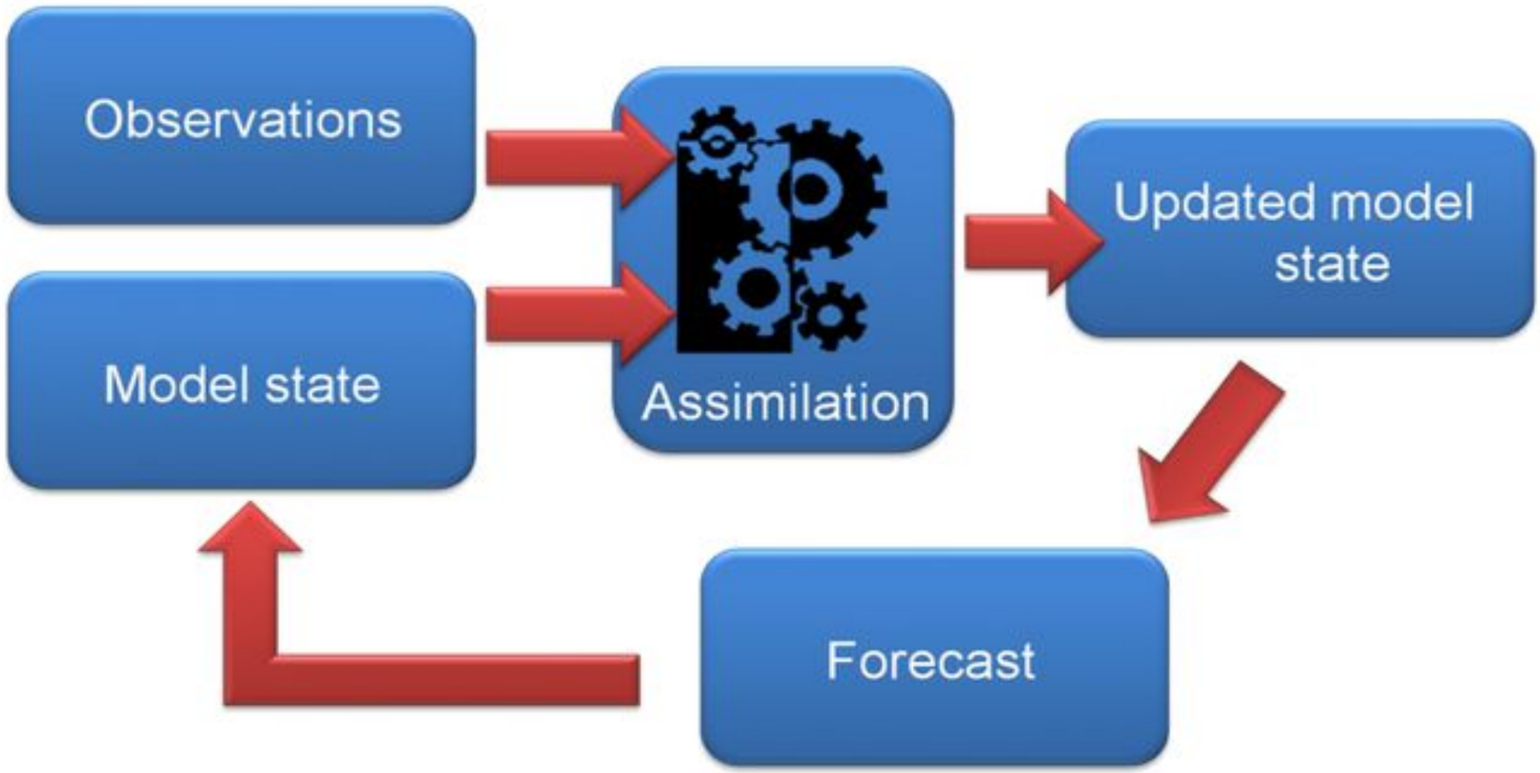
Journal of Computational Biology > Vol. 22, No. 7 > Research Articles

## Estimation of Tumor Size Evolution Using Particle Filters

Jose M.J. Costa, Helcio R.B. Orlande, Haroldo F. Campos Velho, Suani T.R. de Pinho, George S. Dulikravich, Renato M. Cotta, and Silvio H. da Cunha Neto

**Published Online:** 10 Jul 2015 | https://doi.org/10.1089/cmb.2014.0003

# Applications for NEx-PF

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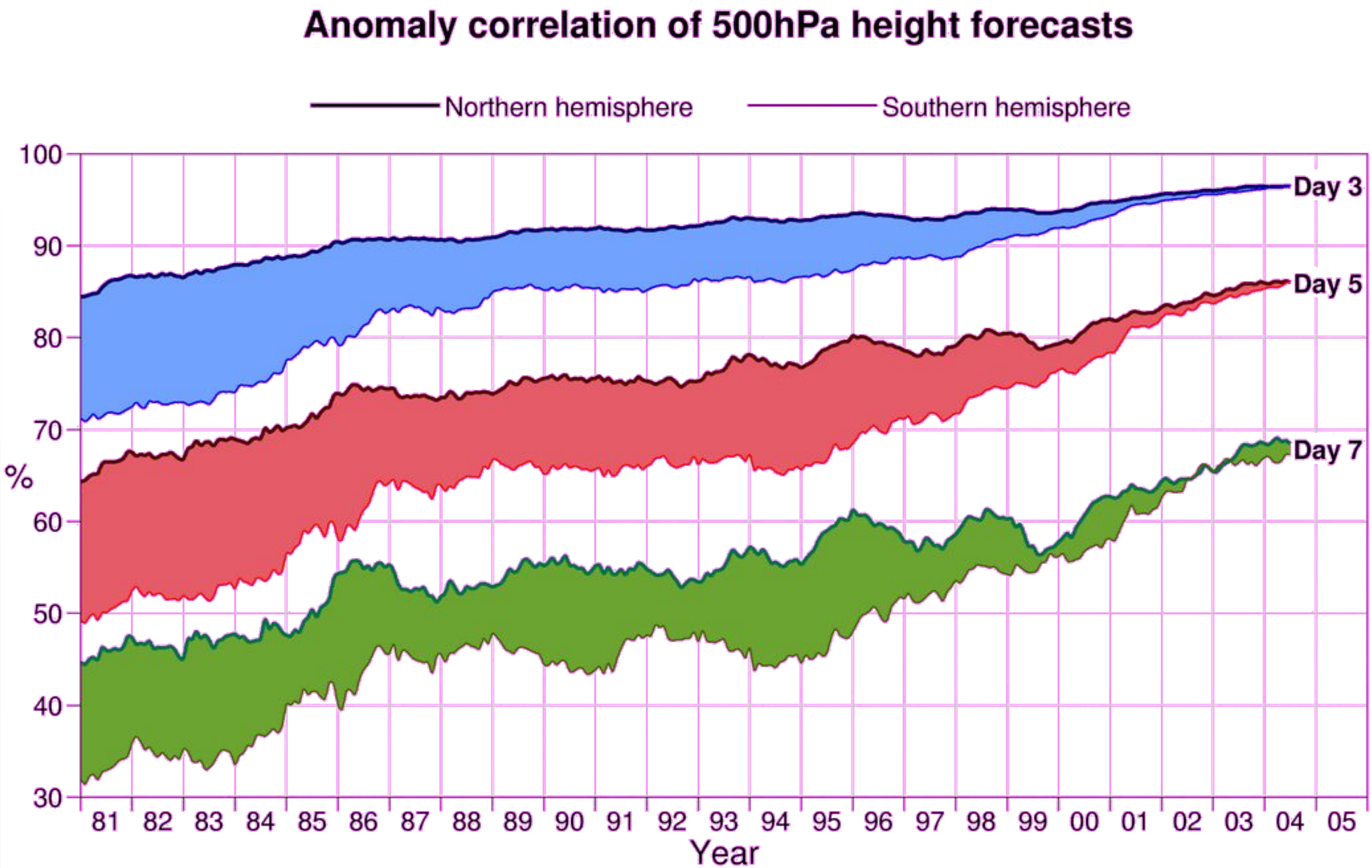
# Data assimilation: what's that?

# Forecasts Scores

**ECMWF**




Adrian Simmons

# Forecasts Scores

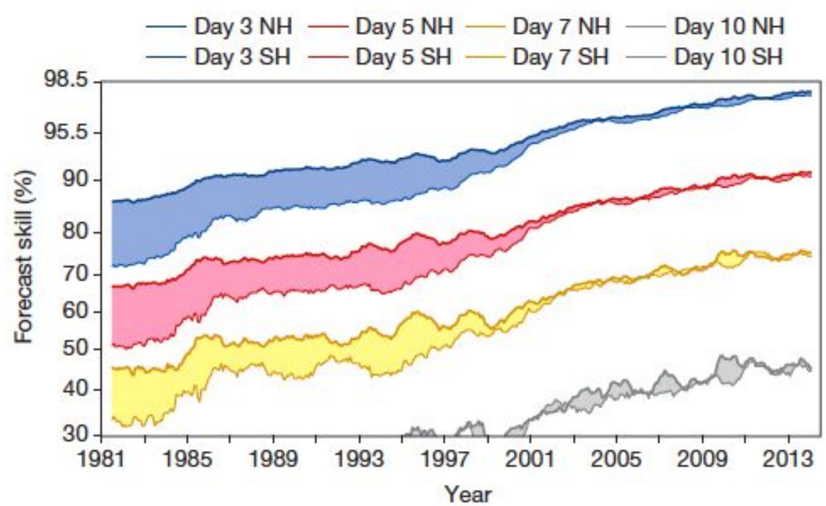
**ECMWF**

REVIEW

doi:10.1038/nature14956

## The quiet revolution of numerical weather prediction

Peter Bauer[1], Alan Thorpe[1] & Gilbert Brunet[2]



MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

# From Least Squares to Kalman Filter, Particle Filter, and Beyond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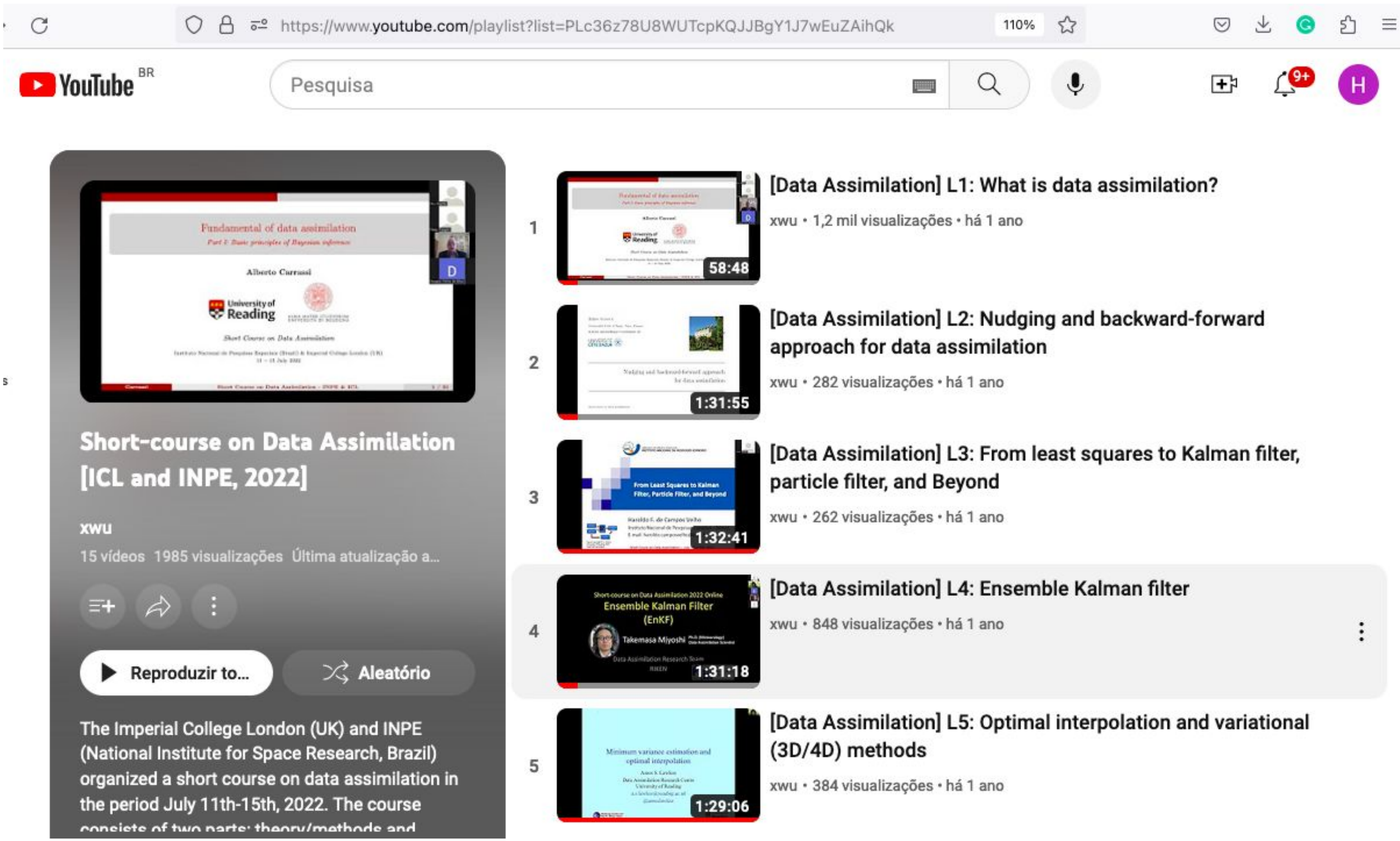
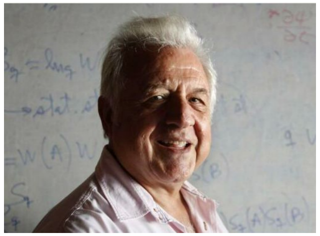
Short Coure on Data Assimilation – July 11th-15th, 2022

# Applications for NEx-PF





# Constantino Tsallis

# Happy birthday!!

# Thank you!