Two-coupled Fisher-Kolmogorov equations applied to macroeconomic systems

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1. Introduction

A "complex system" comprises many components that interact with each other and the environment, creating more interactions or information than the sum of its own parts, such as the behavior of bacteria, cells, chemicals, or animals. With the power of complex systems we can study the economy and understand reality in which any incident in any part of the world, however minuscule and innocuous it may seem, has a direct or indirect impact on the world economy, evoking a *diffusive process* that is a well-known behavior population physics. This extension is applied to macroeconomic systems, using macroeconomic indicators such as Gross Domestic Product,the Consumer Price Index, and the Debt Ratio as input data; these are used to write two coupled Fisher-Kolmogorov-like equations. The solutions of this system represent the distributions of per capita income and worker income in a country, allowing for the identification and study of inequality.

2. Theoretical background

This analysis comes from population physics, where two Fisher-Kolmogorov-type equations are coupled in the following form:

$$\frac{\partial W}{\partial t} = k\frac{\partial^2}{\partial x^2}\log(W) + rW - \mu(t)c(x)W^2 \quad (1)$$

$$\frac{\partial c(x,t)}{\partial t} = \frac{k_2}{2}\frac{\partial^2}{\partial x^2}c^2(x,t) + \alpha c(x,t) - \mu\beta(x,t)W(x,t) \quad (2)$$

The parameters correspond to the following macro-economic indicators:

- r, the rate of existent resources and population.
- $\mu(t)$, the variation of available facilities.
- $c(x,t)$, the distribution of local resources necessary for population growth.
- $\beta(x,t)$, the increasing modulation rate of facilities for the population growth.
- α, a rate of the population without debt.

Equation (1) will be assessed in a *super-diffusive* regime as the distribution $W(x,t)$ adheres to a Lorentzian fit (2) and is acknowledged as the equation's solution.
The modulating term obeys the following rule of normalization: $\int_0^{50}\beta(x,t)dx = 50R$, where R represents the increasing income factor for the population.

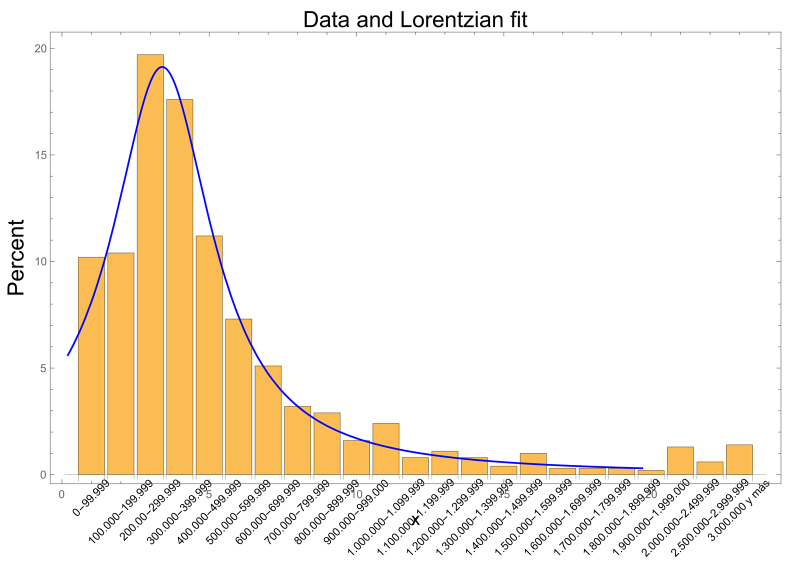


Figure 1: Income distribution fits a Lorentz-type curve as function of time.

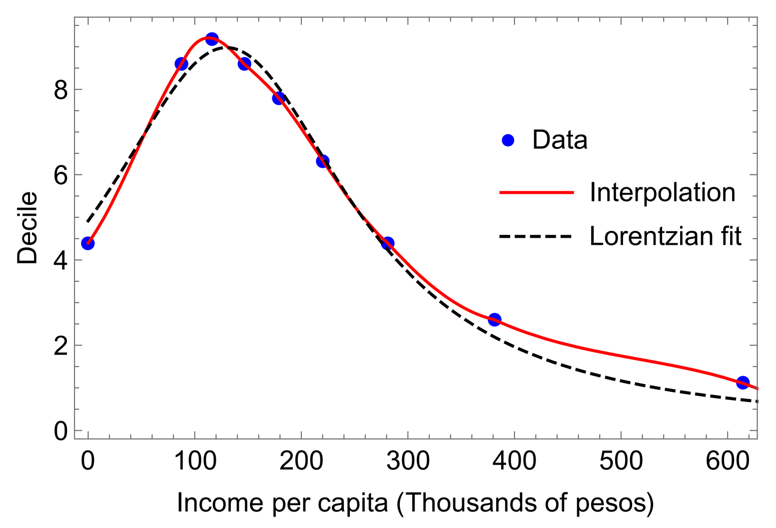


Figure 2: A Lorentzian fitting of the per capita income as a function of time.

3. Procedure and some Results

The form of $\beta(x,t)$ in (2) to solve the system of differential equations takes the following form:

$$\beta(x,t) = \frac{1 - a(x - x_p)}{W(x,t)}. \quad (3)$$

It is possible to solve the coupled differential equations with two modified Plastino-Plastino ansatz.

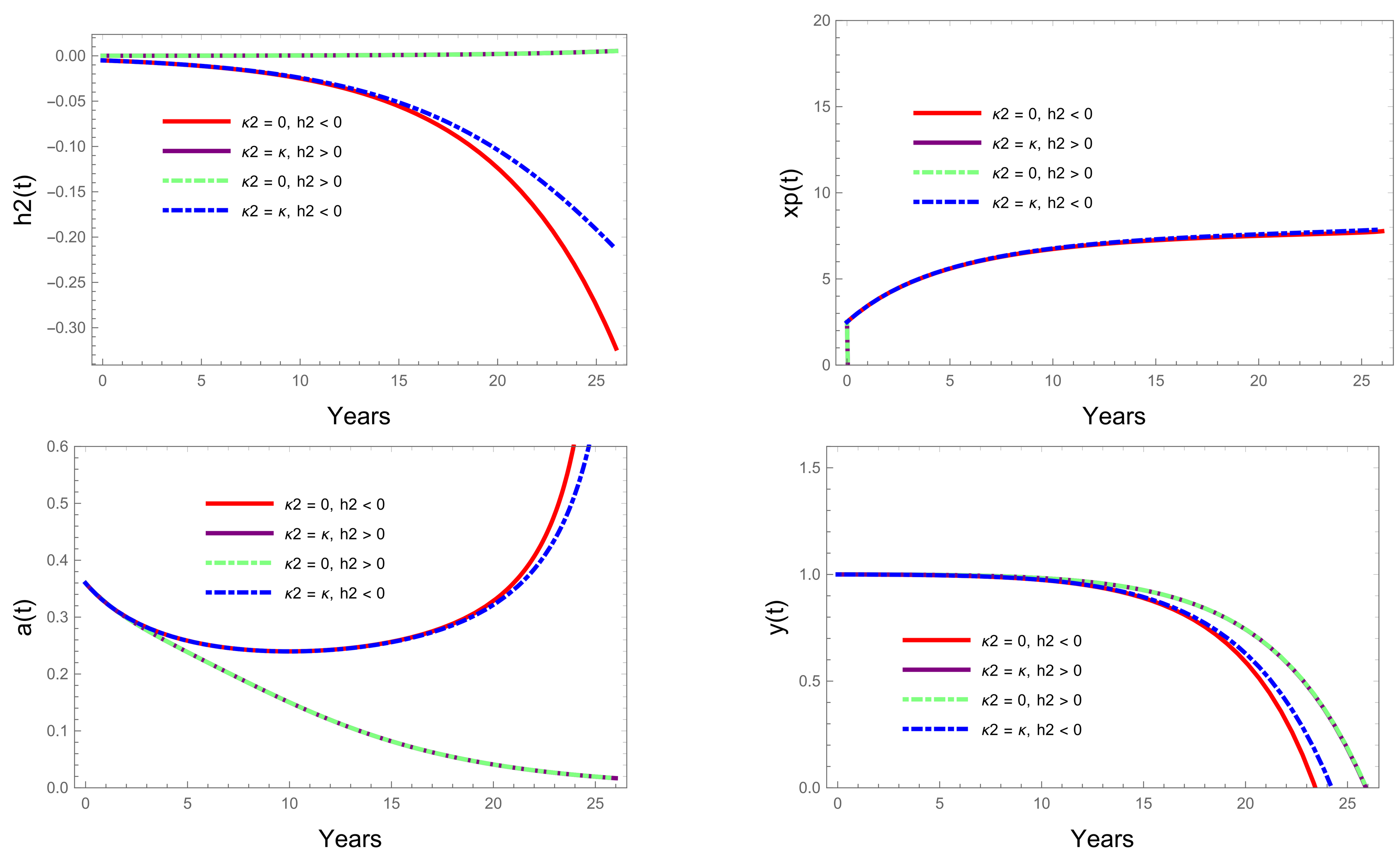
$$W(x,t) = \left(A(t) + \frac{(x - y(t))^2}{4S(t)}\right)^{-1}. \quad (4)$$

$$c(x,t) = h_1(t) + h_2(t)(x - x_p(t))^2. \quad (5)$$

According to distribution properties, $A(t)$ represents the inverse of the amplitude of the distribution. The function $y(t)$ represents the position of the mode.
Replacing the ansatz (4) and (5) into, the system of equations (1) and (2), we obtain the following set of six equations; Which can be analitically solved

$$\begin{aligned} h_1'(t) &= 2k_2h_1(t)h_2(t) + \alpha h_1(t) - \mu \\ h_2'(t) &= 6k_2h_2(t)^2 + h_2(t) \\ x_p'(t) &= -\frac{\mu a}{2h_2(t)} \\ s'(t) &= \frac{k}{2} + rs(t) - 4\mu h_2(t)s(t)^2 \\ y'(t) &= -4\mu h_2(t)(y(t) - x_p(t))s(t) \\ A'(t) &= \frac{k}{2}\frac{A(t)}{s(t)} - rA(t) + \mu h_1(t) + \mu h_2(t)(y(t) - x_p(t))^2 \end{aligned}$$

The parameters, $h_2(t)$, $x_p(t)$, $a(t)$, and $y(t)$ are the most interesting indicators, as they are strongly related to the configuration of our distributions $c(x,t)$ and $W(x,t)$, respectively.



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5. References

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