Awesome to be next to cool people!



2010





Awesome to be next to cool people!



2010







Awesome to be next to cool people!



2010 - GrTr Conf. on Stat. Mech. and Dyn. Sys. [Turunc - Rhodos]

Anyway, everybody super cool here!























Statistical Mechanics for Complexity The 80th BD of Prof. Constantino Tsallis Rio de Janeiro, Nov 6-10, 2023

Network-Oriented Dynamics and Data Science

Deniz Eroglu

DATA-DRIVEN NETWORK DYNAMICS RECONSTRUCTION AND PREDICTION EMERGENT HIGHER-ORDER INTERACTIONS AND CRITICAL PHENOMENA





time



INVERSE PROBLEM



time

BRAIN

clusters



WHAT WE WANT TO SOLVE

rich-club structures



Scannell, J. W. & Young, M. P. Curr.(1993) Van Den Heuvel, M. P., & Sporns, O. (2011)





WHAT WE WANT TO SOLVE

rich-club structures



time series = local dynamics + coupling



Scannell, J. W. & Young, M. P. Curr. (1993) Van Den Heuvel, M. P., & Sporns, O. (2011)



MICROSCOPIC INVESTIGATION

reduction theorem and sparse regression



Eroglu, D., Tanzi, M., van Strien, S., Pereira, T. *Physical Review X* 10 (2020) Candes, E J., Justin K. R, and Terence T., Comm. Pure Appl. Math 59 (2006)



time series = mean field + fluctuations





ROBUST AGAINST NOISE

better reconstruction, better prediction!





Eroglu, D., Tanzi, M., van Strien, S., Pereira, T. *Physical Review X* 10 (2020) Topal, I and Eroglu, D. Physical Review Letters 130 (2023)

PREDICTION

critical transitions



Eroglu, D., Lamb J., Pereira T. Contemporary Physics 58 207 (2017) Pereira, T., Eroglu, D., Bagci, GB., Tirnakli, U., Jensen, HJ.,. Physical Review Letters 110 (2013) Duan, C., Nishikawa, T., Eroglu, D., Motter., AE. Science Advances 8 (2022)

mean-field measurements



 $\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N} \sum_{n=1}^{N} \sin(\psi_{kn} - \psi_{km})$

mean-field measurements



 $\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N} \sum_{n=1}^{N} \sin(\psi_{kn} - \psi_{km}) + \sum_{\ell=1}^{4} A_{k\ell} \left(\frac{\alpha}{N} \sum_{n=1}^{N} \sin(\psi_{\ell n} - \psi_{km}) \right)$



mean-field measurements



$$\omega_{km} + \frac{\mu}{N} \sum_{n=1}^{N} \sin(\psi_{kn} - \psi_{km}) + \sum_{\ell=1}^{4} A_{k\ell} \left(\frac{\alpha}{N} \sum_{n=1}^{N} \sin(\psi_{\ell n} - \psi_{km})\right)$$

In terms of mean-fields

$$= \omega_{km} + \operatorname{Im}\left(\mu z_{k} + \alpha \sum A_{kl} z_{\ell}\right) e^{-i\psi_{km}}$$





assume infinitely many neurons



Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

undirected and cubic polynomial interaction



Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

where

 $f_k(z_k) = \gamma_k z_k + \beta_k z_k \ z_k^2; \ \gamma_k = (i\Omega_k + \mu - \sigma_k)$ $\beta_k = -\mu; \ h(z_k, z_\ell) = \alpha z_\ell + \alpha \overline{z}_\ell z_k^2$

undirected and cubic polynomial interaction



Resonance satisfying condition: ω

Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

where

$$f_k(z_k) = \gamma_k z_k + \beta_k z_k \ z_k^2; \ \gamma_k = (i\Omega_k + \mu - \sigma_k)$$

$$\beta_k = -\mu; \ h(z_k, z_\ell) = \alpha z_\ell + \alpha \bar{z}_\ell z_k^2$$

$$\omega_2 = \omega_1 + \omega_3 \quad \& \quad \omega_4 = \omega_1 + \omega_3$$

simulation



simulation





defining phases



 $Z_k(t) = r_k(t)e^{i\theta_k(t)}$

defining slow variables





 $Z_k(t) = r_k(t)e^{i\theta_k(t)}$ $\vartheta_k(t) = \theta_k(t) - \Omega_k t$

Then we would like to reconstruct ϑ_i from data

defining slow variables





 $Z_k(t) = r_k(t)e^{i\theta_k(t)}$ $\vartheta_k(t) = \theta_k(t) - \Omega_k t$

COMPRESSED SENSING

model reconstruction from data



Candes, E J., Justin K. R, and Terence T., 2006



COMPRESSED SENSING

model reconstruction from data



Candes, E J., Justin K. R, and Terence T., 2006



reconstruction of slow phases



$$\begin{split} \dot{\theta}_1 &= 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_2 &= 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3) \\ \dot{\theta}_3 &= 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_4 &= 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3) \end{split}$$

$0.001\cos(\theta_1 - \theta_4 + \theta_3)$

 $0.001\cos(\theta_1 - \theta_4 + \theta_3)$

emergent hypergraphs



$$\begin{split} \dot{\theta}_1 &= 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_2 &= 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3) \\ \dot{\theta}_3 &= 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_4 &= 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3) \end{split}$$

original network

 $0.001\cos(\theta_1 - \theta_4 + \theta_3)$

 $0.001\cos(\theta_1 - \theta_4 + \theta_3)$

emergent hypergraphs

$$\begin{split} \dot{\theta}_1 &= 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_2 &= 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3) \\ \dot{\theta}_3 &= 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - \\ \dot{\theta}_4 &= 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3) \end{split}$$

original network

reconstructed hypernetwork

 $0.001\cos(\theta_1 - \theta_4 + \theta_3)$

 $0.001\cos(\theta_1 - \theta_4 + \theta_3)$

looking for the sparsest solution

N $\dot{z}_k = f_k(z_k) + \alpha \sum A_{k\ell} h_k(z_k, z_\ell)$ $\ell = 1$

looking for the sparsest solution

a coordinate transformation of the form $w_k = z_k - \alpha P_k(z)$

for some polynomials $P_k(z) = \sum A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$

 $\dot{z}_k = f_k(z_k) + \alpha \sum A_{k\ell} h_k(z_k, z_\ell)$ $\ell = 1$

 $\ell = 1$

looking for the sparsest solution

a coordinate transformation of the form $w_k = z_k - \alpha P_k(z)$

for some polynomials $P_k(z) = \sum A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$

 $\ell = 1$

looking for the sparsest solution

$\ell = 1$

transformation generates additional undesired terms from the isolated dynamics

looking for the sparsest solution

second coordinate transformation of the form $u_k = w_k - \alpha Q_k(w)$

looking for the sparsest solution

second coordinate transformation of the form $u_k = w_k - \alpha Q_k(w)$

special bracket [.]] on the space of polynomials.

nontrivial combinatorial problem tackled by introducing a

rules for resonant and nonresonant terms

rules for resonant and nonresonant terms

rules for resonant and nonresonant terms

rules for resonant and nonresonant terms

rules for resonant and nonresonant terms

EMERGENT HYPERNETWORK

mean-field pops nonlinearity

EMERGENT HYPERNETWORK

mean-field pops nonlinearity

 $\dot{\theta}_1 = 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_2 + \theta_3)$ $\dot{\theta}_2 = 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3)$ $\dot{\theta}_3 = 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_3)$ $\dot{\theta}_4 = 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3)$

$$\theta_1 - \theta_4 + \theta_3$$

$$\theta_1 - \theta_4 + \theta_3$$

 $\varphi_1 = \theta_1 - \theta_2 + \theta_3$ $\varphi_2 = \theta_2 - \theta_4 + \theta_4$

EMERGENT HYPERNETWORK

mean-field pops nonlinearity

SURPRISING PREDICTIONS!

ring topology with quadratic coupling

$h(z,w) = z\bar{w}$

pairwise coupling function

Jacobian vanishes at the origin

NOT a diffusive coupling!

SURPRISING PREDICTIONS!

ring topology to driven system

original network

reconstructed hypernetwork

ANOMALOUS SYNC

synchronization tongue

Nijholt, E., Ocampo-Espindola, JL., Eroglu, D., Kiss, IZ., Pereira, T. Nature Communications, 2022; 13:4849

I AM NOT ONLY RECONSTRUCTING BRAIN!

TSALLIS NETWORK

Bukman D.J.

Gomes Jr. S.R.

Loh Mite D.R. Bologpa M. Ferreiga Bui A. Andrad Villela T. Plastino A.R. Farmer D. Bagchi D. De Albuquerque M.P. Leo M. Da Silva Echigolini P. de Souza M.d.S. Ludescher J. Jauregui M. Monteiro MarizWilk G. Portesi M. De Oliveira M.J. Sá Barreto F.C. Assis V.R.V. Vieira C.M. Castro De Oliveira P.M. Cedeño C.E. Tempesta P. Viry Beer R. Thistleton W.J. Elias VBunde A. White S. Dos Santos R.J.B. Lamberti P.W. West B. Bouillot J. Sicuro G. Caruso F. Eroglu D. Verona De Resendesdusa VeiMagathes A.C.N. Moreira A.A.Ribeiro M.S. Stanley H.E. Nelson K.P. de Pinho da Silva E. Abramov D.M. Jensen H Kejžar N Zamora D.J. Leo R.A. / Bountis T. Kim S. Bettini C. Carmona H.A. Li N. Wong C.-Y. Cetin K. -Li B'. Cabral B.J.C. Damião Soares I. Levy S.V.F. Cirto L.J.L Lima R.A.T. Brigatti E. Curado E.M.F. Sato Y. Lazo E. Araújo J.M. Scuta M.S. Christodoulidi H. Rodríguez A. Casati G. Machet R. Tavares D.M. Arenas Z.G. Essam J.W. Rajagopal A.K. Rapčan P. Marsh J.A. Brito S. Vinciguerra S.C. Martins M.L. de Lima R.A.T. Pluchino A. De Souza J. Prato D. Barci D.G. Bezerra C. Marques M. Coutinho Dos Santos B. Drossos L. Livadiotis G. Fichera G. Curilef S. Soares D.J.B. Wedemann R.S. Maynard R. Nobre F.D. Tirnakli U. Rohlf T. Ruiz G. Lucena L.S. Jund Nunes T.C. **Baldovin F.** Olivei**r**a P.M. Kyriakopoulos F. Van Wijland F. Zheng W.-M. Hauser P.R. Ay N. Gell-Mann M. **Tsallis C.** Greco A. Zebende fveira R.M. Fuentes M.A. Souza A.M.C. Añaños G.F. Rapisarda A. Honmura R. Fulco P.Oliveira R. Megías E. Penna T.J.P.Da Silva L.R. Plastino A.R. Moyano L.G. Albuquerque E.L. Queirós S.M.D. Benedetoatrafatto L. Schwachheim G. Zorzenon Dos Santos R.M. Dos Santos R M Z Alvarez-Estrada R.F. Anteneodo C. Vallejos R.O. Lyra M.L. Herrmann H.J. 🗡 Machado S.F. Mariz A.M. Rego H.H.A. Haubold H.J. Deppman Schulze B. Cinardi N. Borges E.P. Umarov S. Souletie J. Carbone A. Rego-Monteiro M.A. Kaniadakis G. Thumer S. Beck C. Ito N. Majtey A.P. Fittipaldi I.P. Isekouras G.A. Coniglio A. Redner S. Moroni D. Robledo A. Kodama T. Borland L. De Magalhães A.C.N. Steinberg S. Sarmento E.F. Stinchcombe R.B. Zanette S.I. Pasechnik R. Tamarit F.A. Hilhorst H.J. Baranger M. Costa U.M.S. Da Silva L.R. Mundim K.C. Dos Santos R.R. Latora V. de OliveMarandaCJ.G.V. Roux S. Mendes R.S. Stella A. Martinez MacKay R.S. Toscano F. Osorio R. Malarz K. da Cruz H.R. Bemski G. Caride A.O. Scarfone A.M. Quarati P. Papa A.R.R. Buck B. Anjos J.C.Andrade R.F.S.anelagy T.F. Provata A. Lavagno A. Gazeau J.P. Stariolo D.A. Chame A. D.A. Cavalcanti S.B. Mahanti S.D. Cannas S.A. Mahanti S.D. Stefanovska A. Giansanti A. Drazer G. Dos Santos R.J.V. Sandrissindisa S.R. Weinstrijearaz Wo H.S. Sandrigues

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THANK

LoheWhite D.R. Ferreiga Ruj A lastino A.R. Farmer D. Leo M. Da Silva Echigolini P. Monteiro MarizWilk G. Assis V.R.V. Castro De Oliveira P.M. Tempesta P. 🛛 👔 🏹 Thistleton W.J. Dos Santos R.J.B. Lamberti P.W. West B. Bouillot J. Eroglu D Stanley H.E. Nelson K.P. de Pinho da Silva E. Jensen H.kejžar N. Leo R.A. Bountis T. Li N. Wong C.-Y. -Li B⁄. Levy S.V.F. Cirto L.J.V Lima R.A.T. Brigatti E. Sato Y. Christodoulidi H. Rodríguez A. Casati G. Machet R. Arenas Z.G. Marsh J.A. Vinciguerra S.C. de Lima R.A.T. De Souza J. Pluchino A. Prato D. Barci D.G. Livadiotis G. Fichera G. Maynard R. Tirnakli U. Rohlf T. Ruiz G. Baldovin F. Olivei**r**a P.M. Kyriakopoulos F. Gell-Mann M. Greco A. Fuentes M.A. Souza A.M.C. Añaños G.F.I. Rapisarda A. Megías E. Moyano L.G. Queirós S.M.D. Benedetoatrafatto L. Schwachheim G. Anteneodo C. Herrmann H.J. Valleios R.O. Lvra M.L. Deppman Schulze B.Cinardi N. Umarov S. Borges E.P. Thumer S. Beck C. lto N. Majtey A.P. Robledo A. Kodama T. Borland L. Steinberg S. Pasechnik R. Tama**r**it F.A. Baranger M. Da Silva L.R. Latora V. de OliveMarandaCI.G.V. Roux S. Martinez MacKay R.S. Toscano F. Osorio R. Papa A.R.R Buck B. Anjos J.C.Andrade R.F.S. Helman J.S. Cavalcanti S.B. Martin H.O. da Silva D.M.H. Maha**n**ti S.D. Stefanovska A. Lloyd S. Lenzi E.K.

Network-Oriented Dynamics and Data Science

twitter: nodds_lab

Science Academy, TUBITAK ERC, EU MSCA CNPq, FAPESP

Deniz Eroglu

deniz.eroglu@khas.edu.tr

