

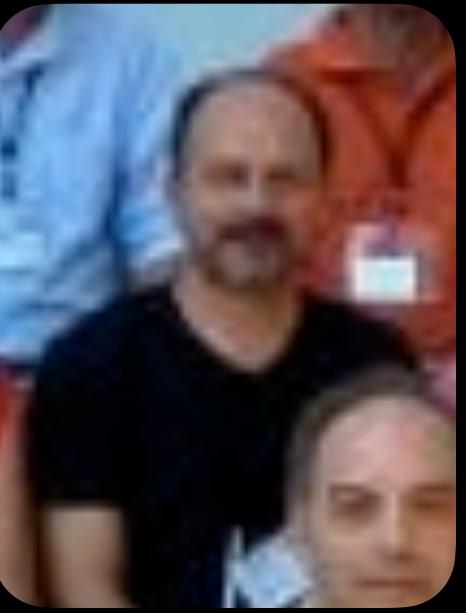
**Awesome to be next to cool people!**

2010



Awesome to be next to cool people!

2010



Awesome to be next to cool people!

# 2010 - GrTr Conf. on Stat. Mech. and Dyn. Sys. [Turunc - Rhodos]



Anyway, everybody super cool here!













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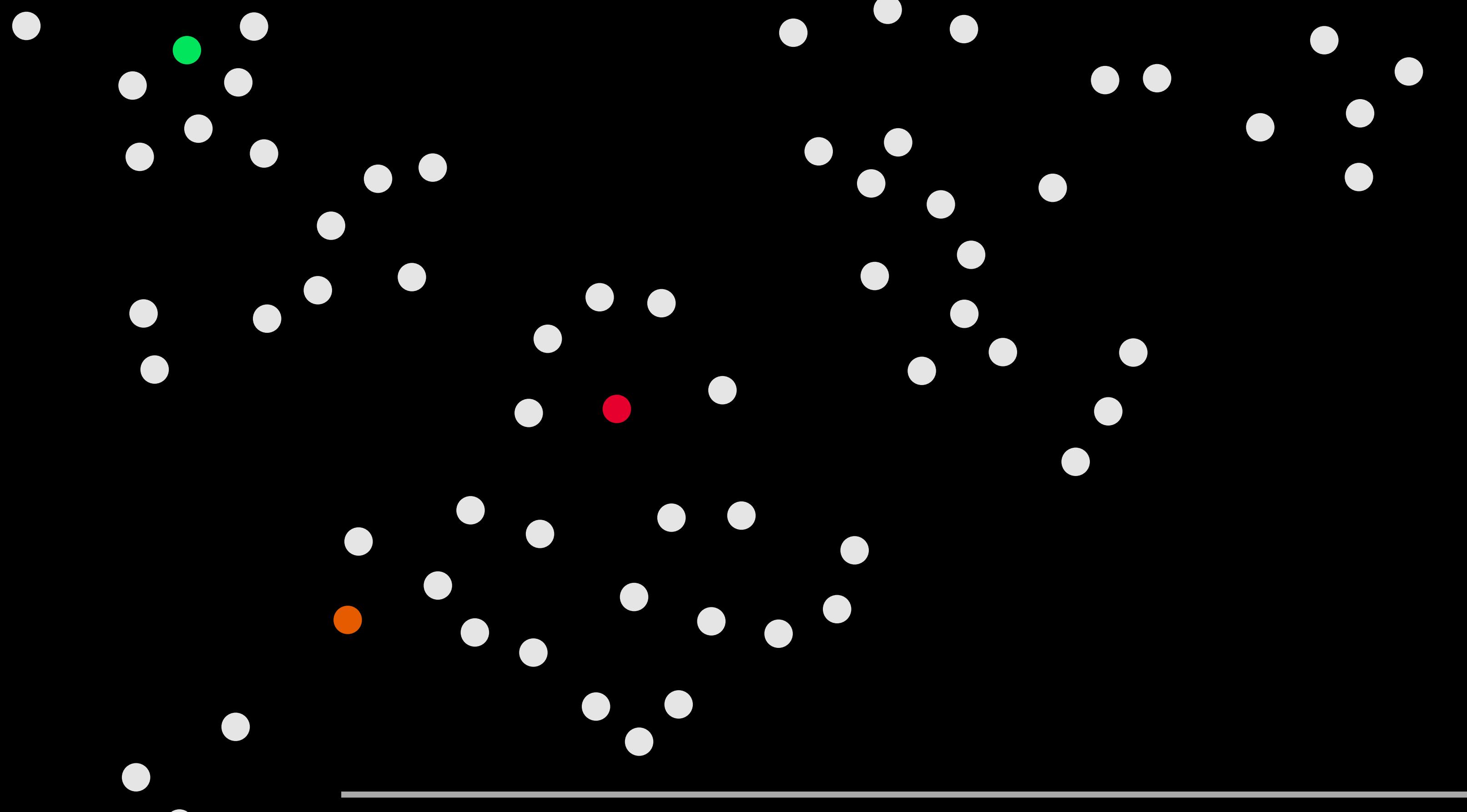




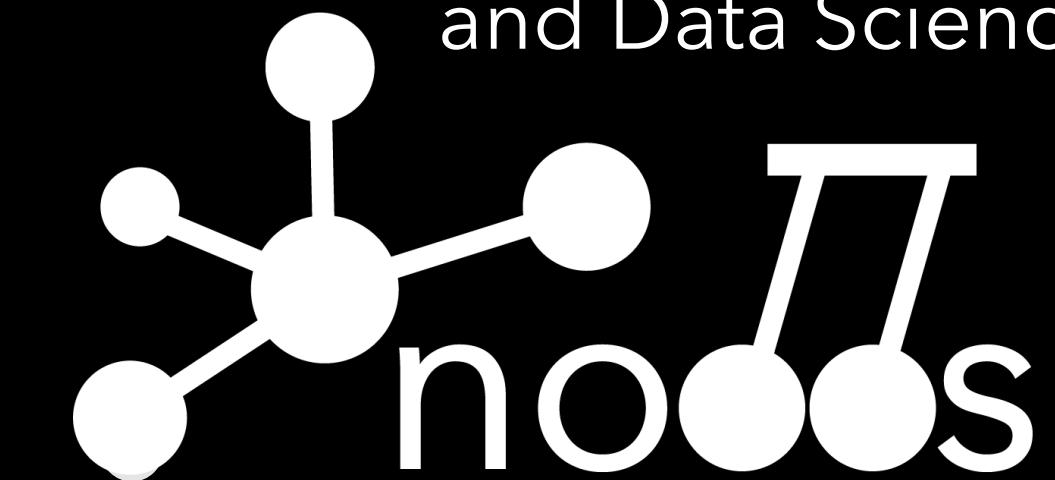
Statistical Mechanics for Complexity

# The 80th BD of Prof. Constantino Tsallis

Rio de Janeiro, Nov 6-10, 2023



Network-Oriented Dynamics  
and Data Science

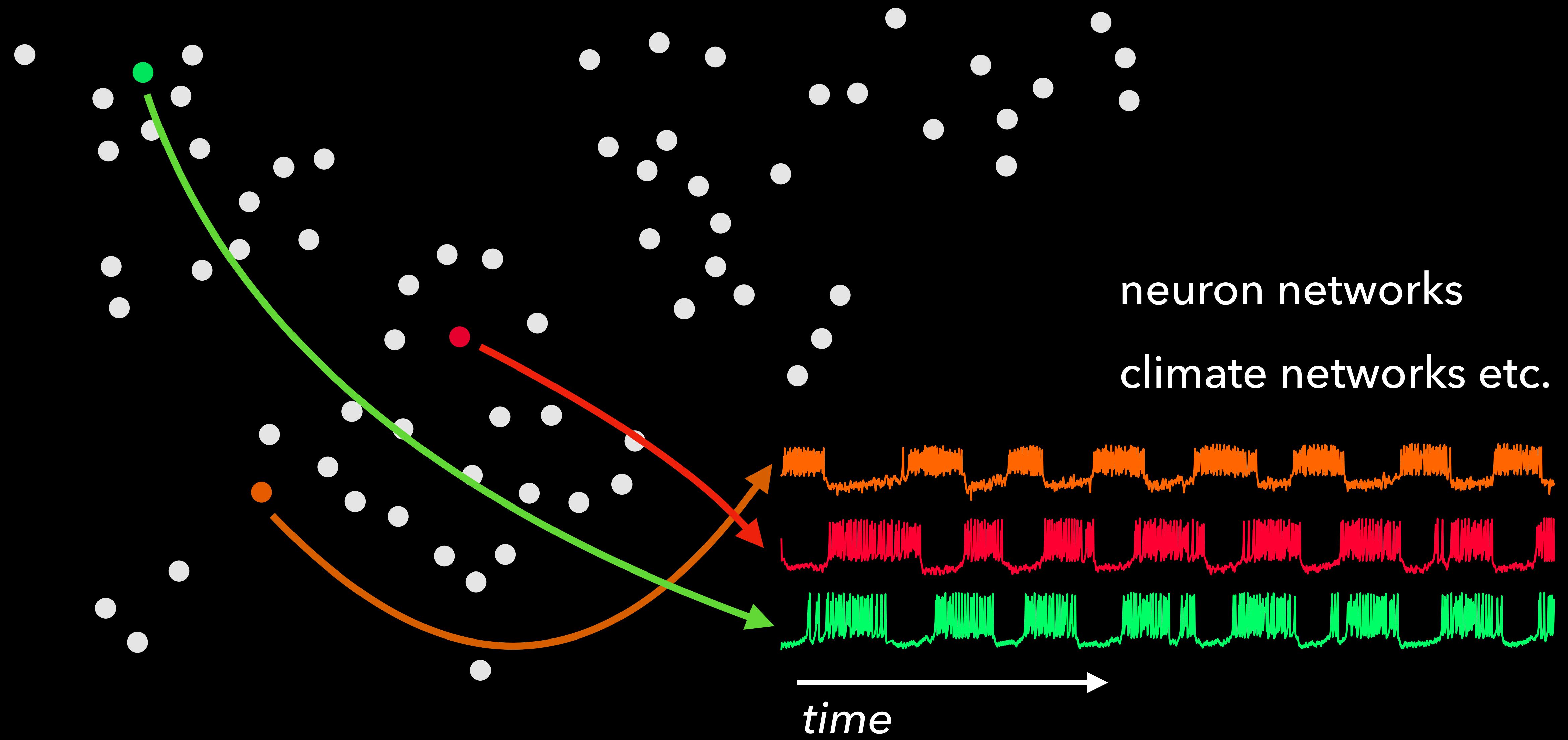


Deniz Eroglu

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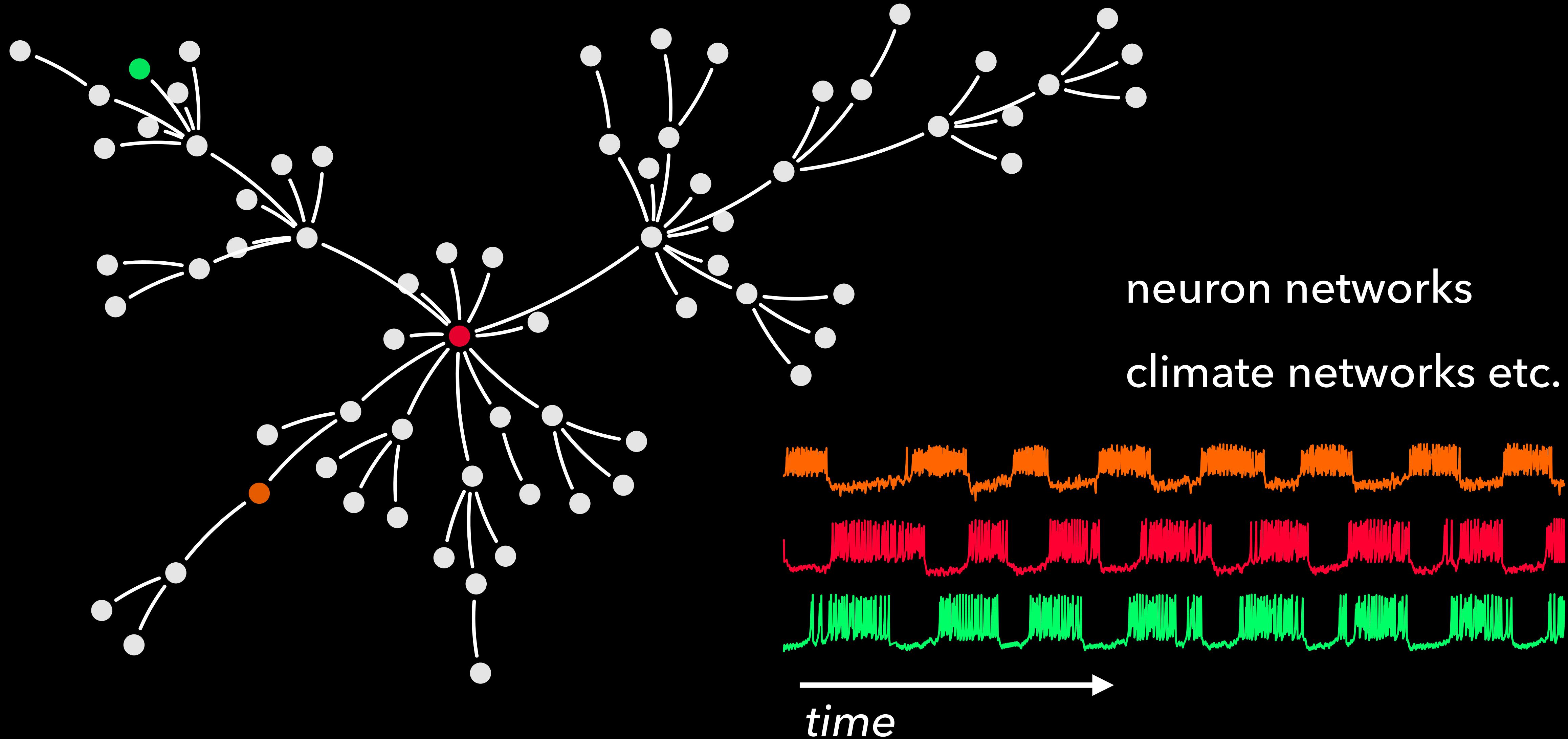
**DATA-DRIVEN NETWORK DYNAMICS RECONSTRUCTION AND PREDICTION**

**EMERGENT HIGHER-ORDER INTERACTIONS AND CRITICAL PHENOMENA**



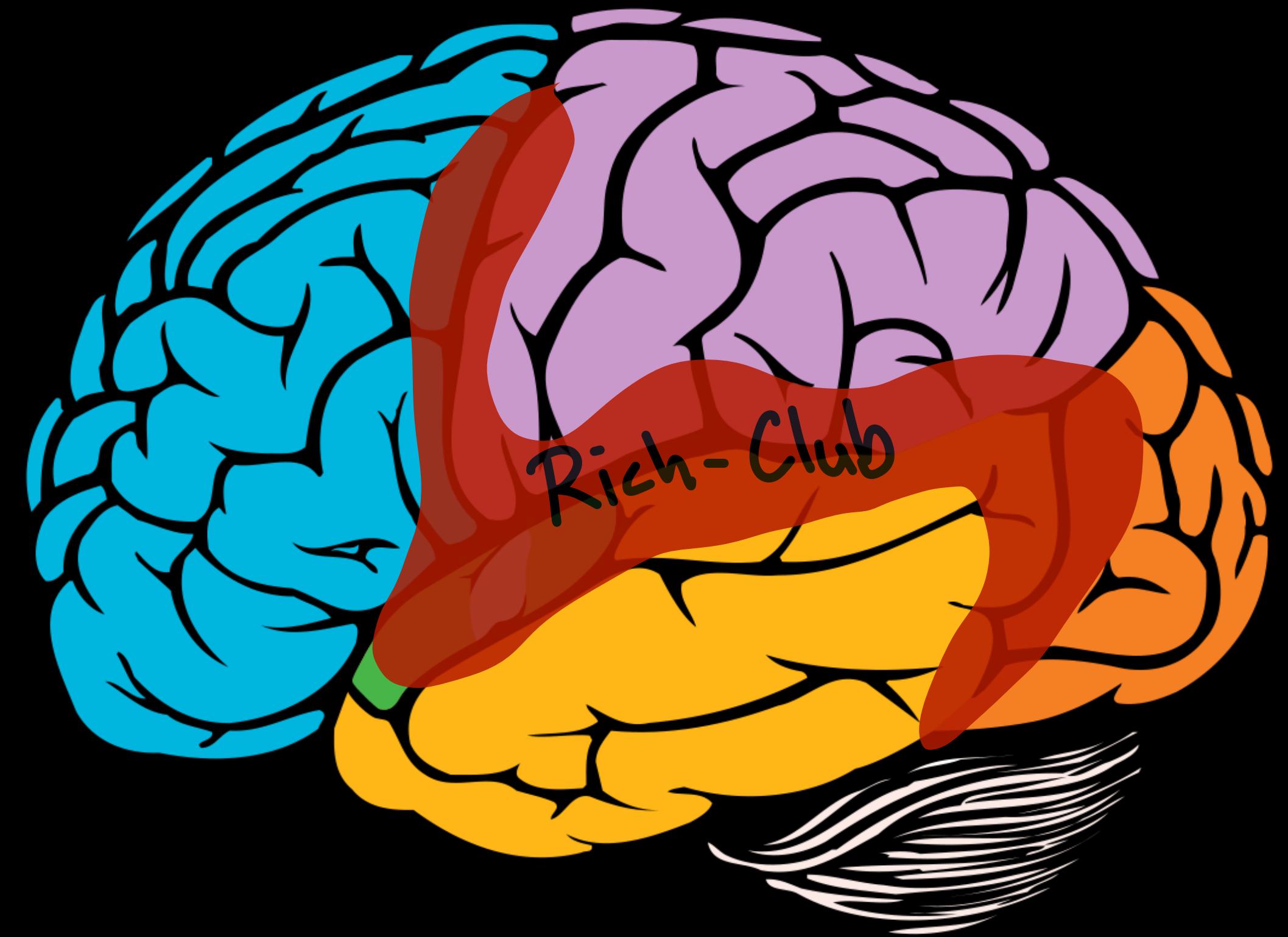
# INVERSE PROBLEM

*create an oscillator network with  $(G,f,H)$  to construct the data*



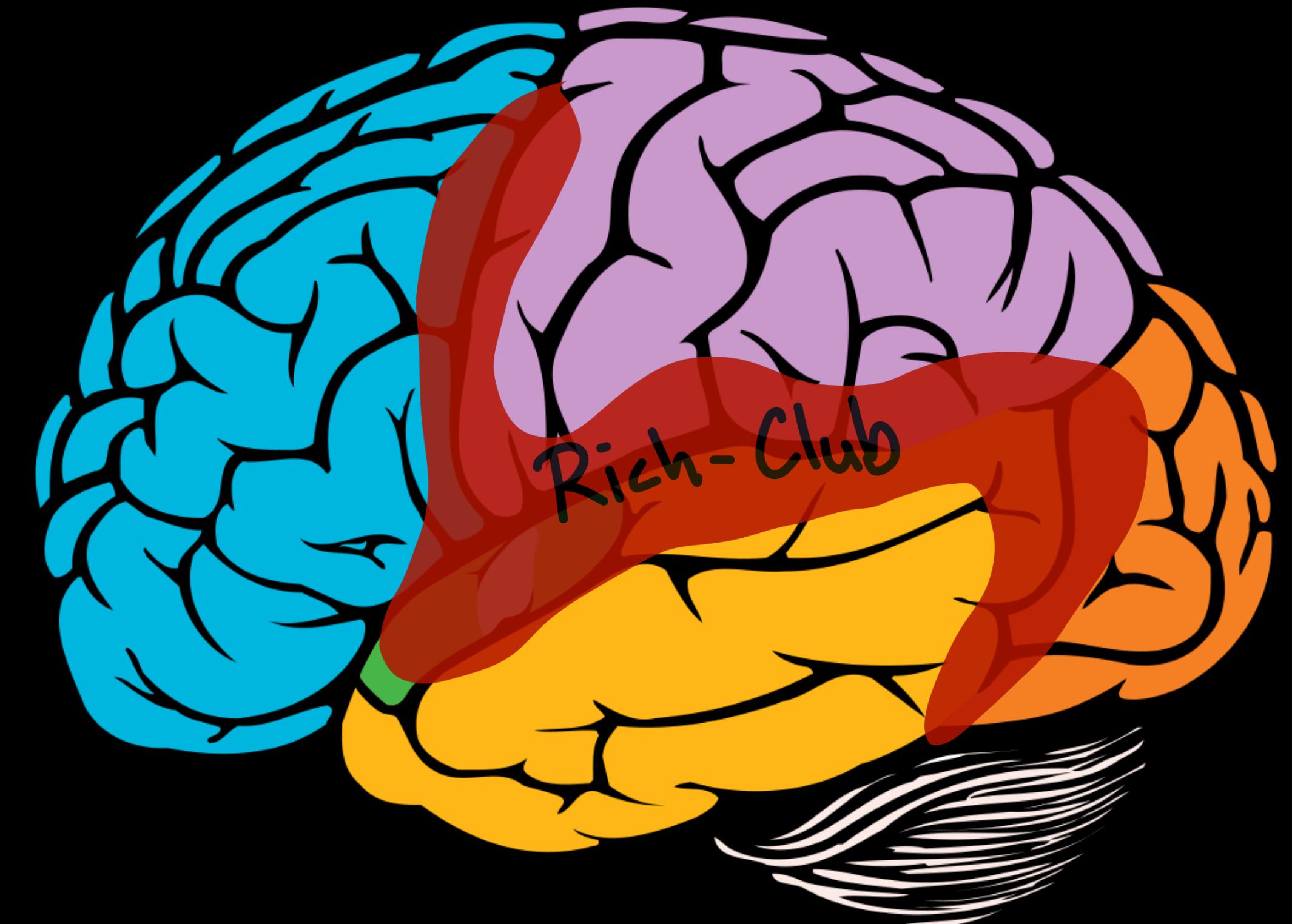
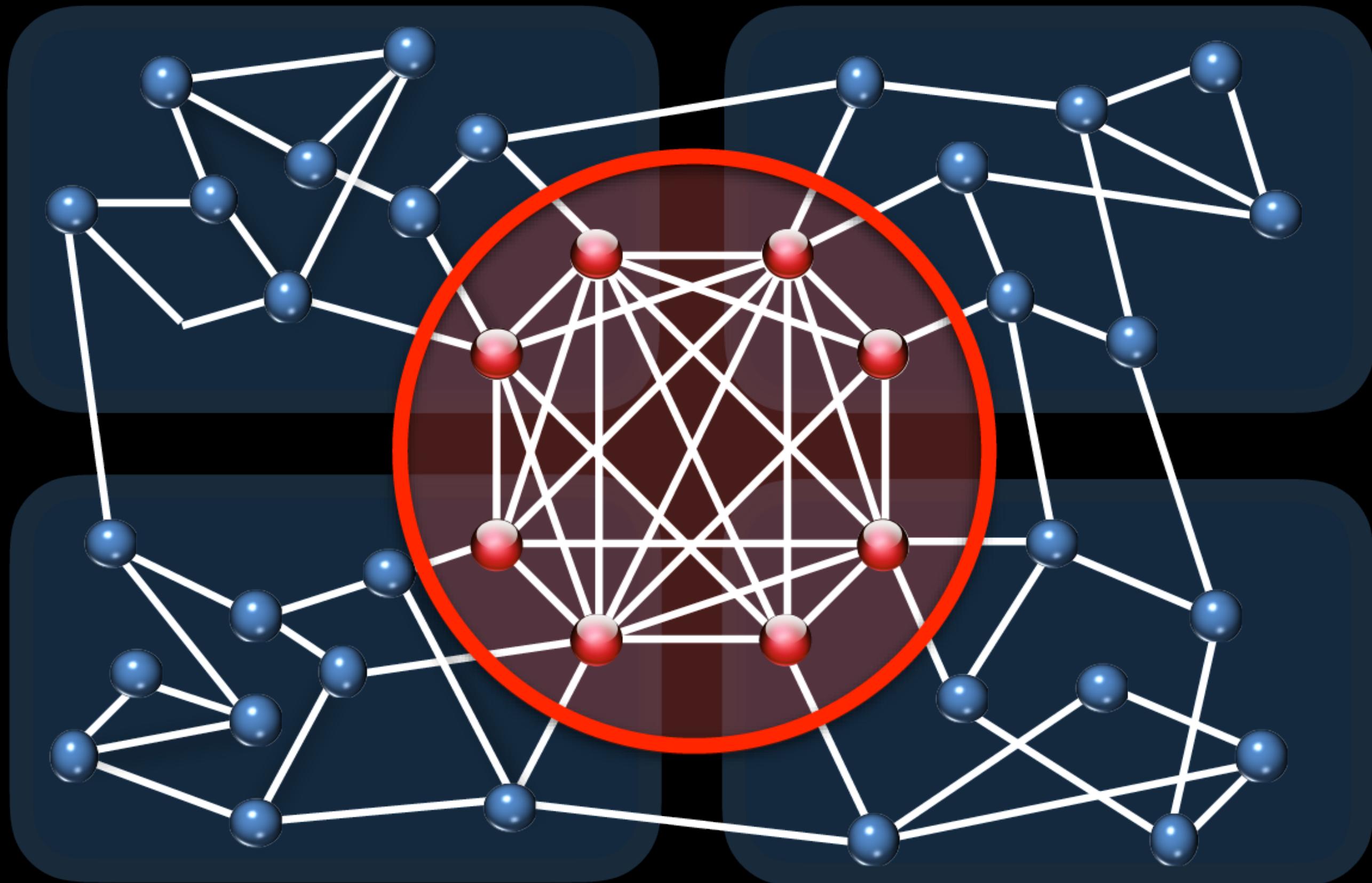
# BRAIN

*clusters*



# WHAT WE WANT TO SOLVE

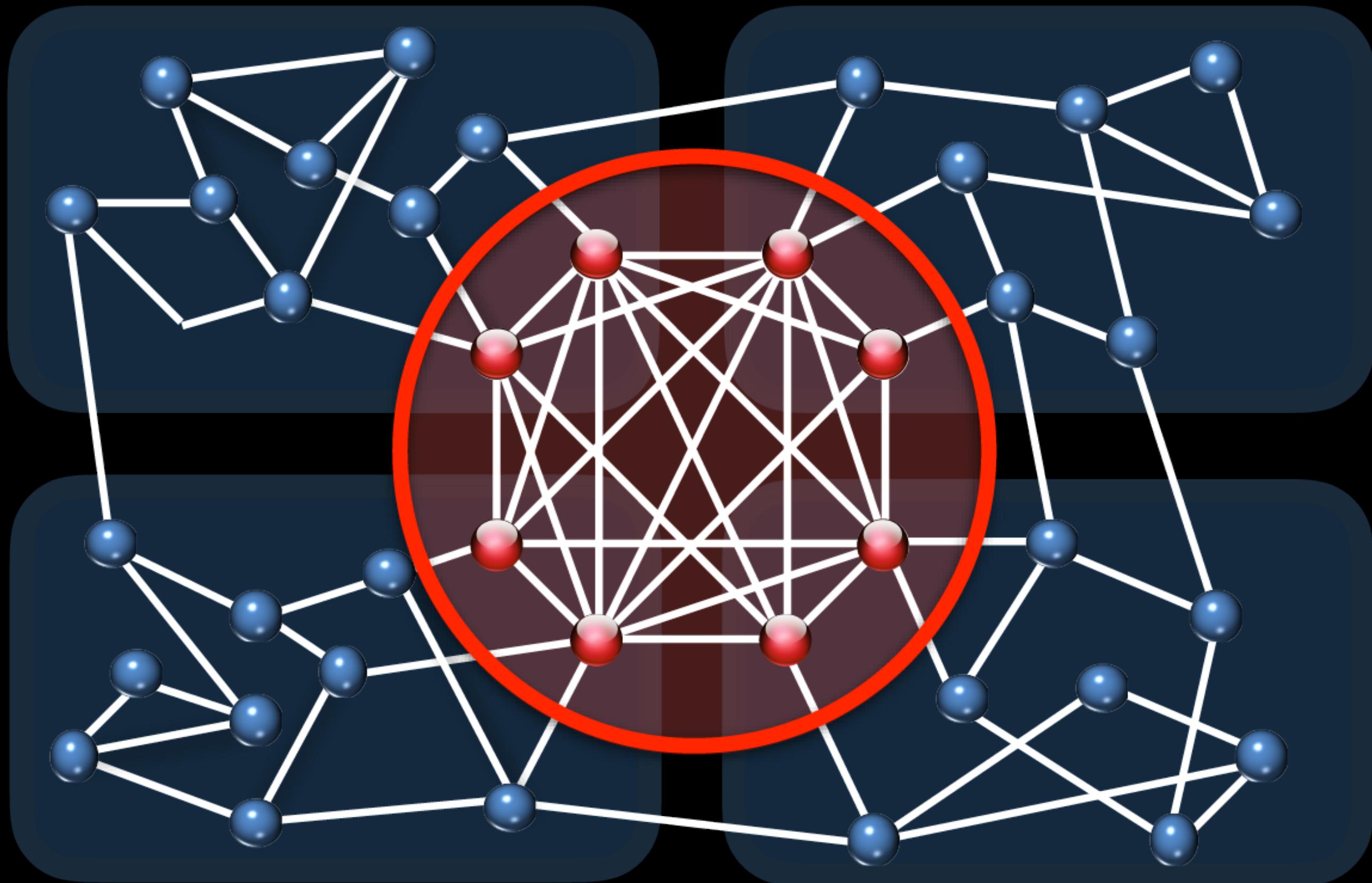
*rich-club structures*



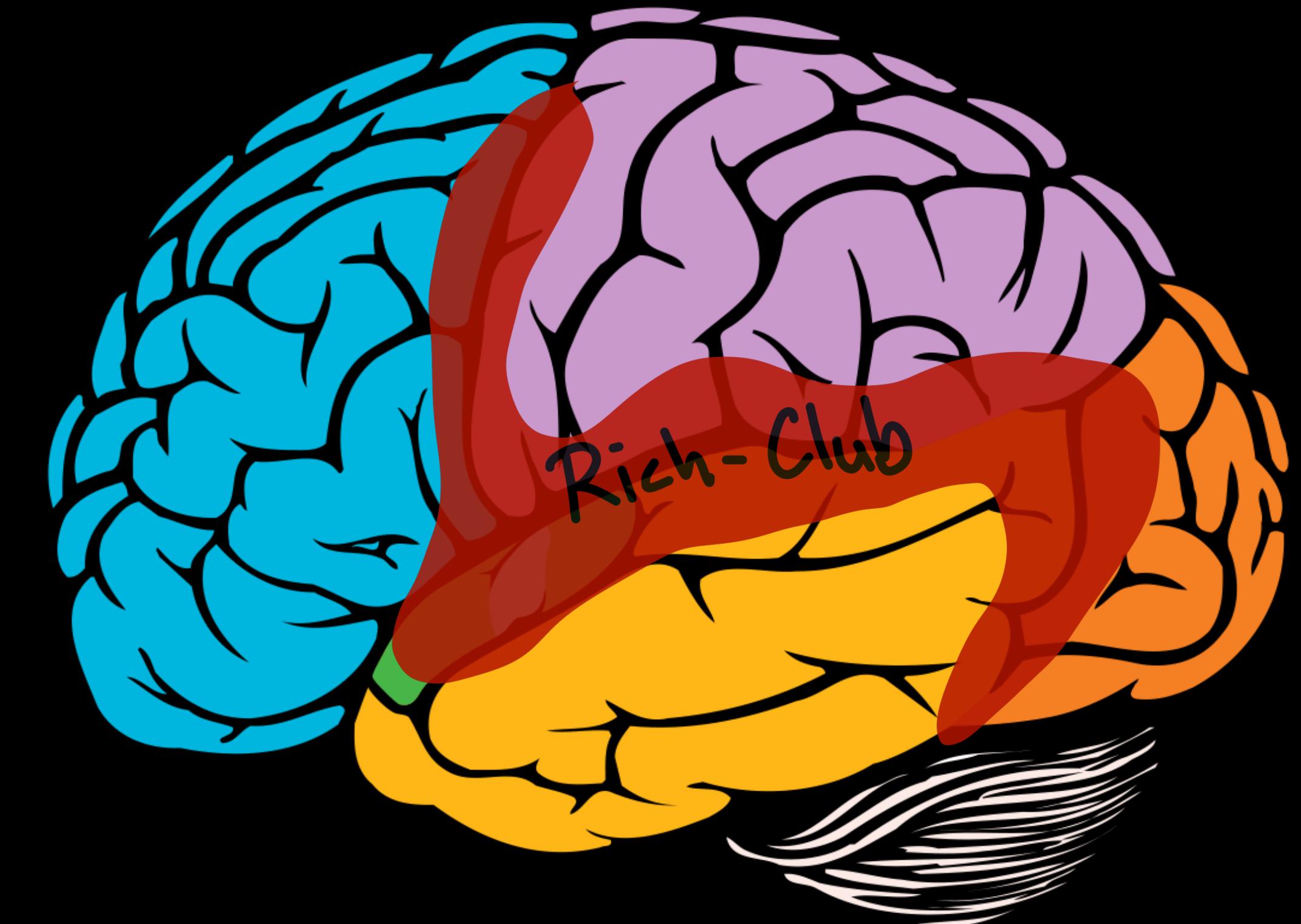
Scannell, J. W. & Young, M. P. Curr.(1993)  
Van Den Heuvel, M. P., & Sporns, O. (2011)

# WHAT WE WANT TO SOLVE

*rich-club structures*



time series = local dynamics + coupling

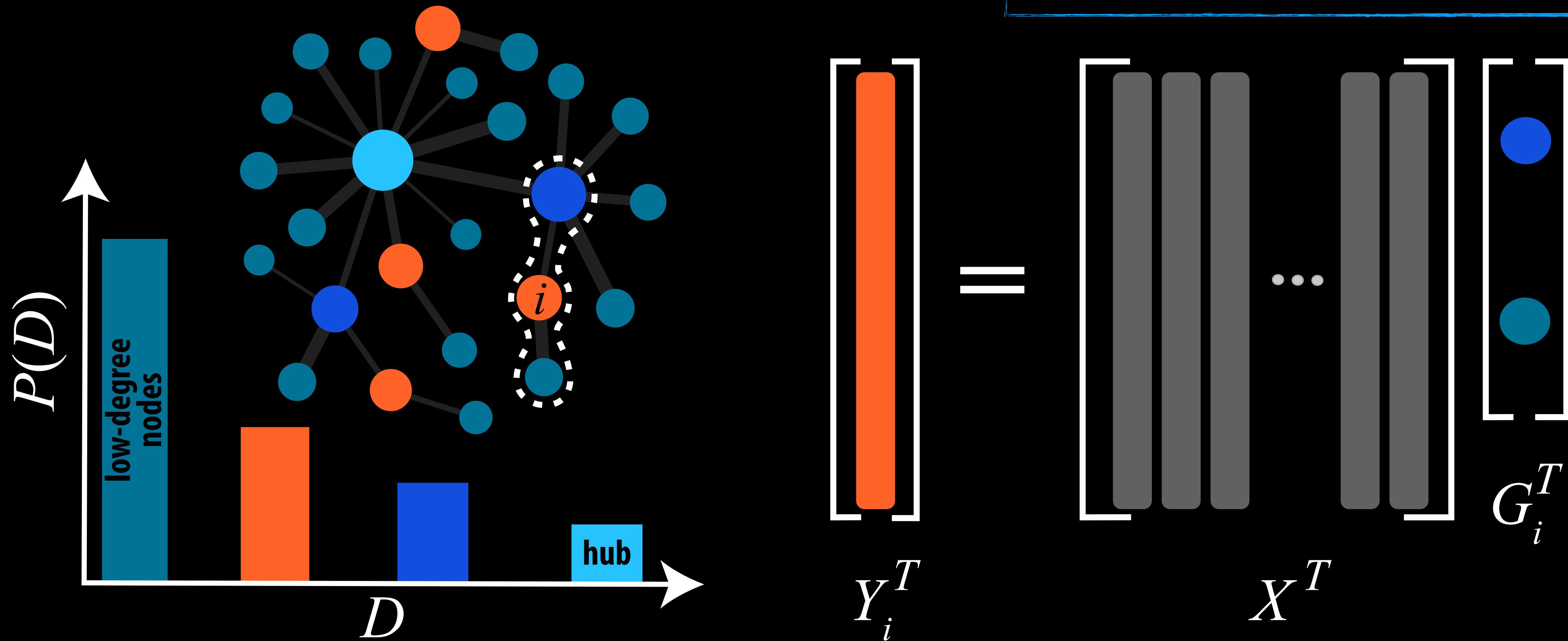


Scannell, J. W. & Young, M. P. Curr.(1993)

Van Den Heuvel, M. P., & Sporns, O. (2011)

# MICROSCOPIC INVESTIGATION

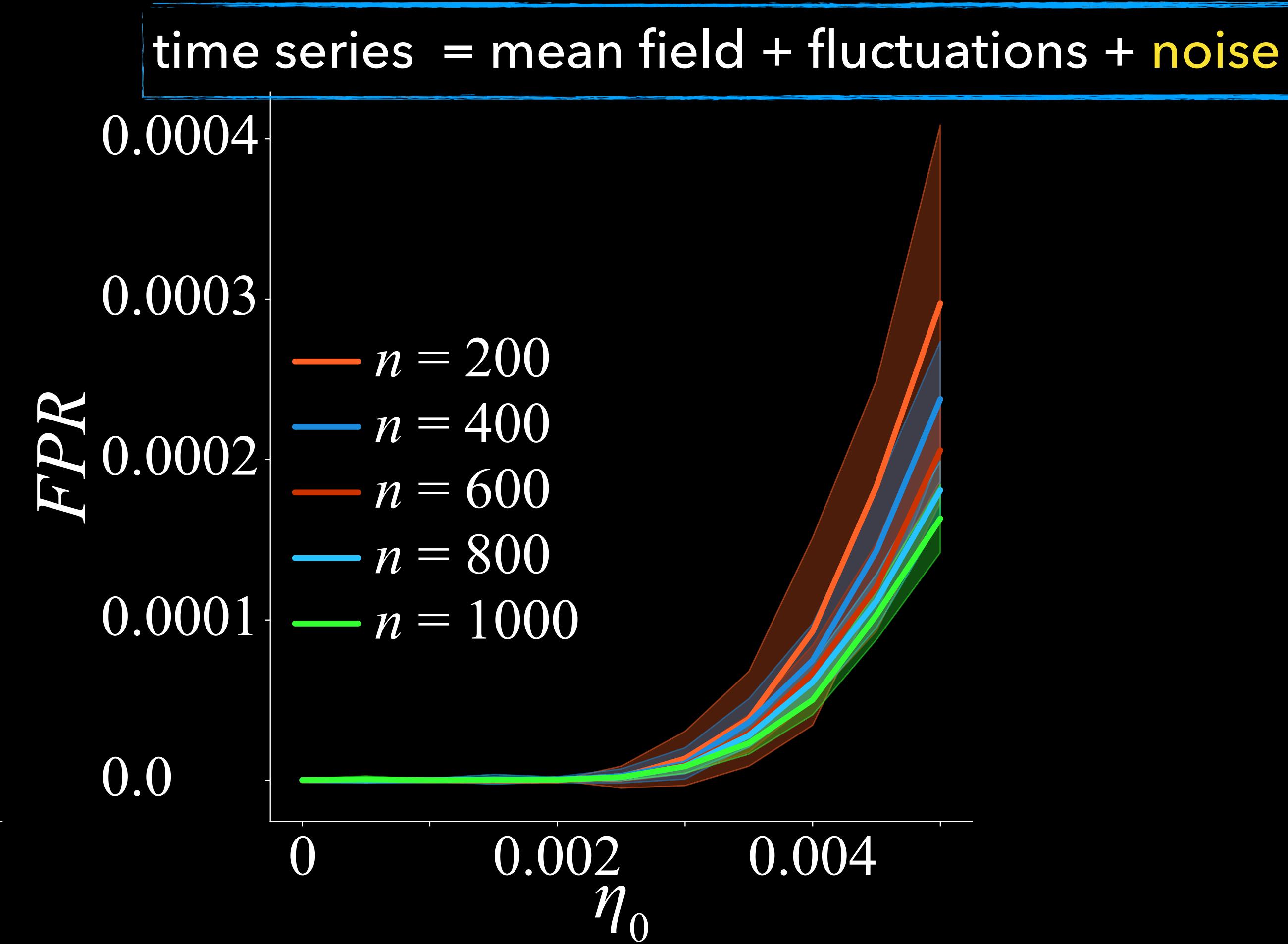
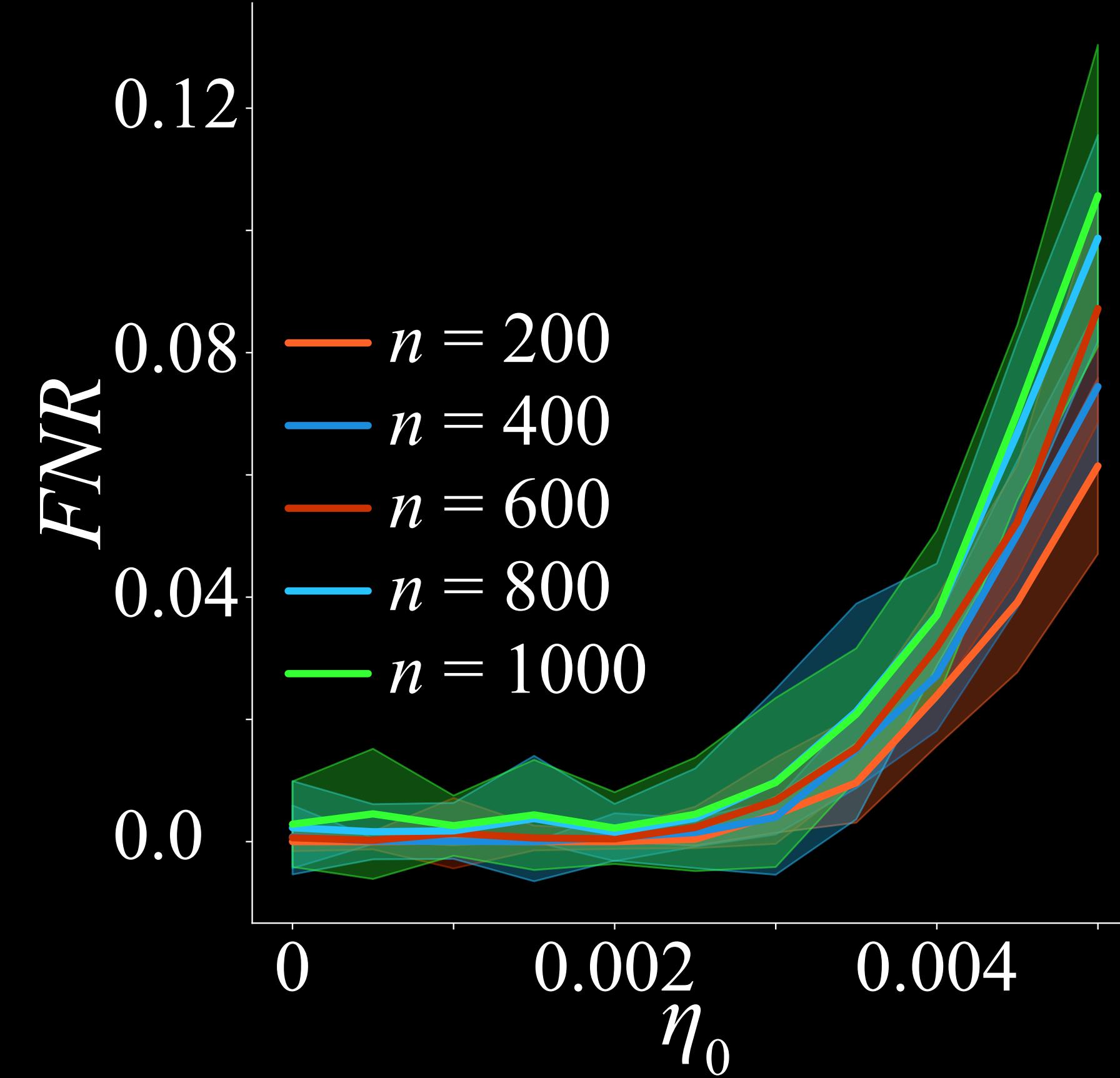
*reduction theorem and sparse regression*



Eroglu, D., Tanzi, M., van Strien, S., Pereira, T. *Physical Review X* 10 (2020)  
Candes, E J., Justin K. R, and Terence T., *Comm. Pure Appl. Math* 59 (2006)

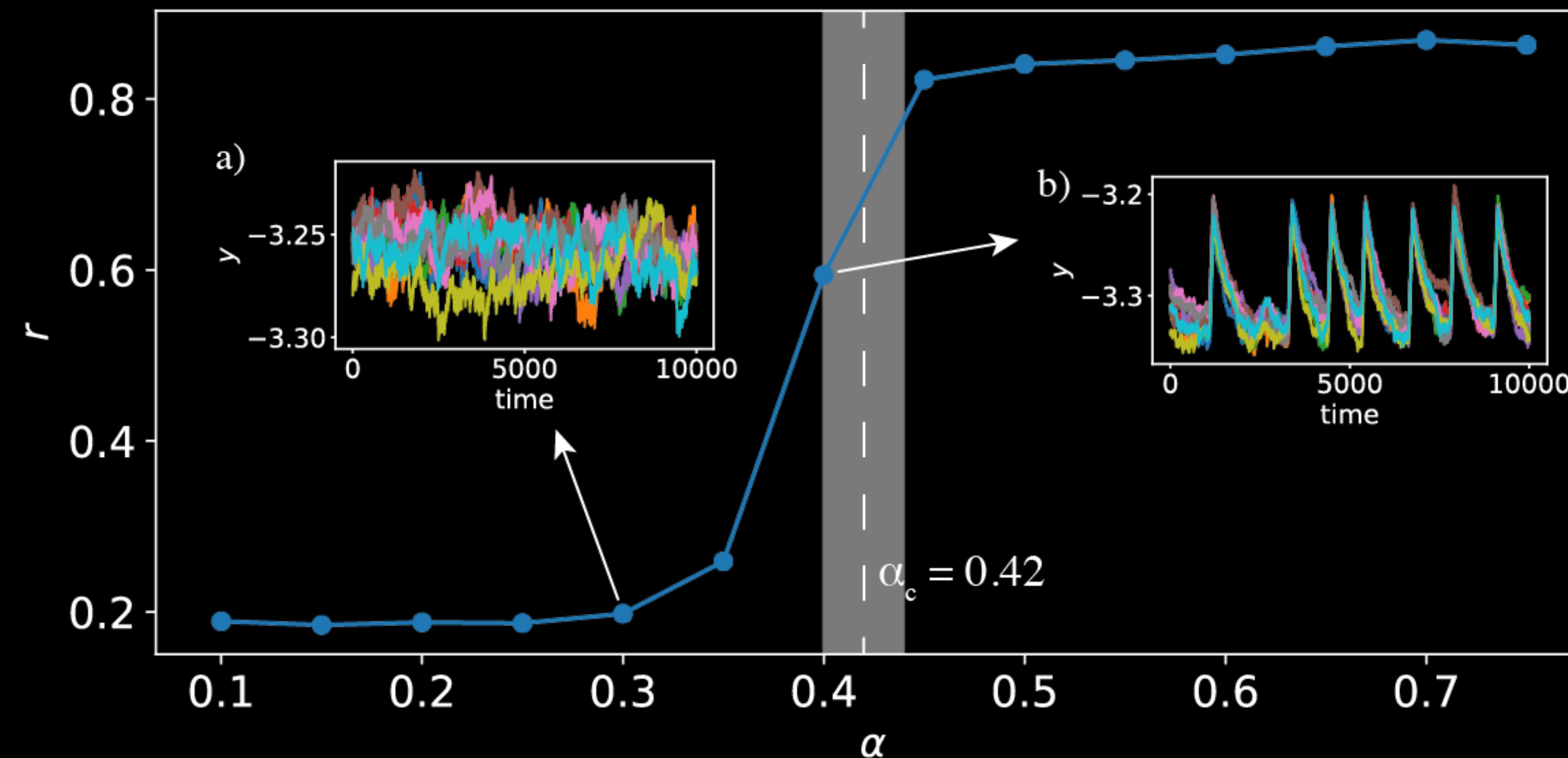
# ROBUST AGAINST NOISE

*better reconstruction, better prediction!*



# PREDICTION

*critical transitions*



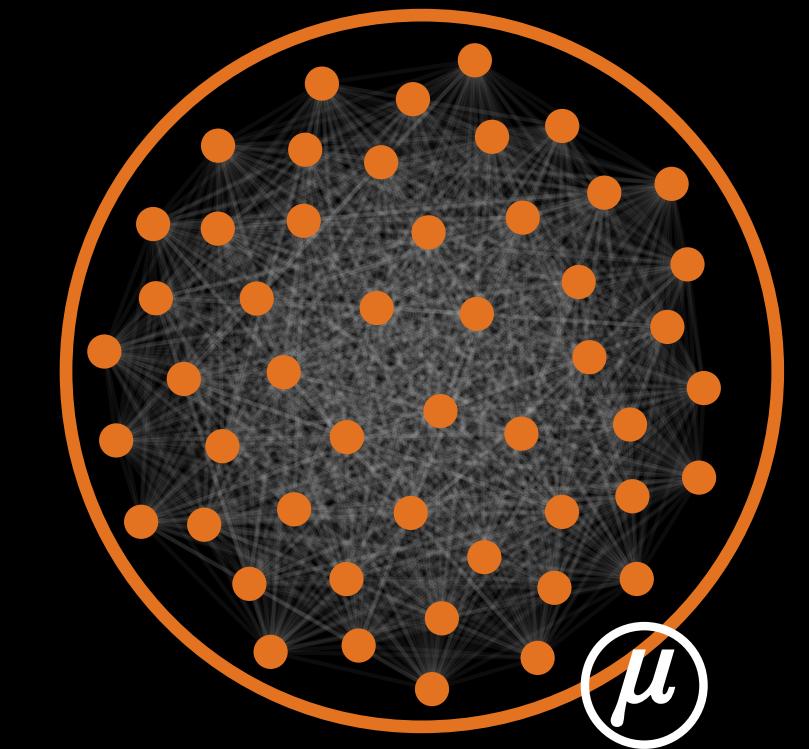
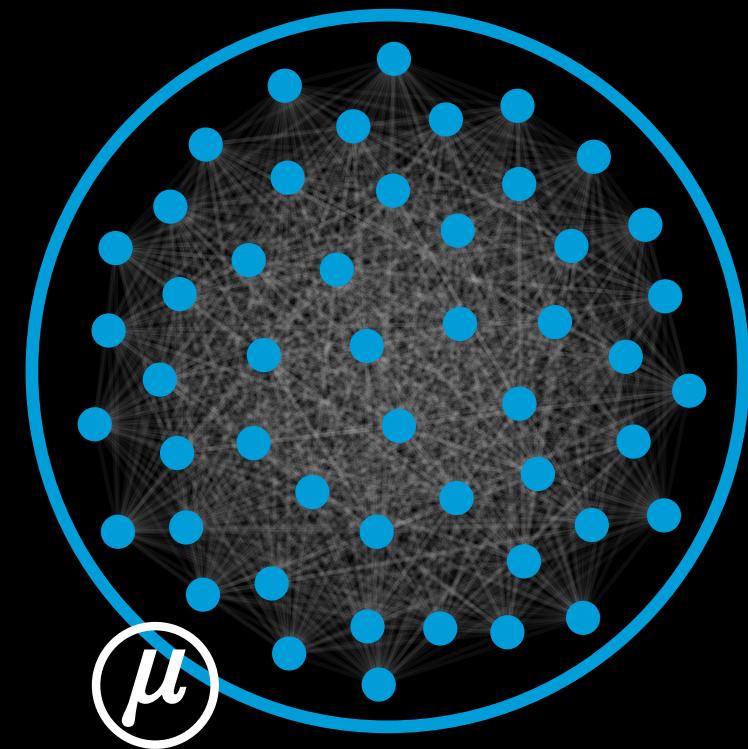
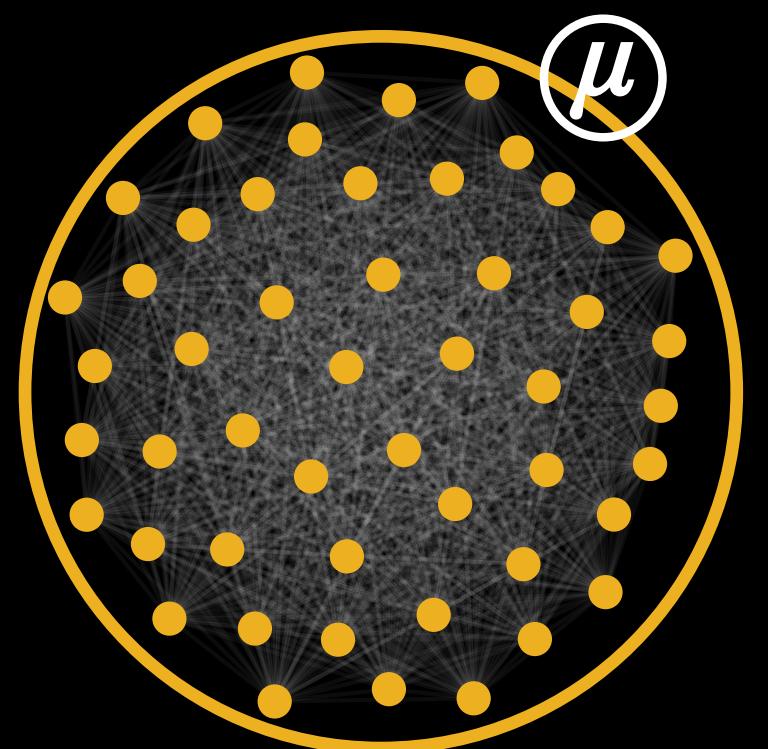
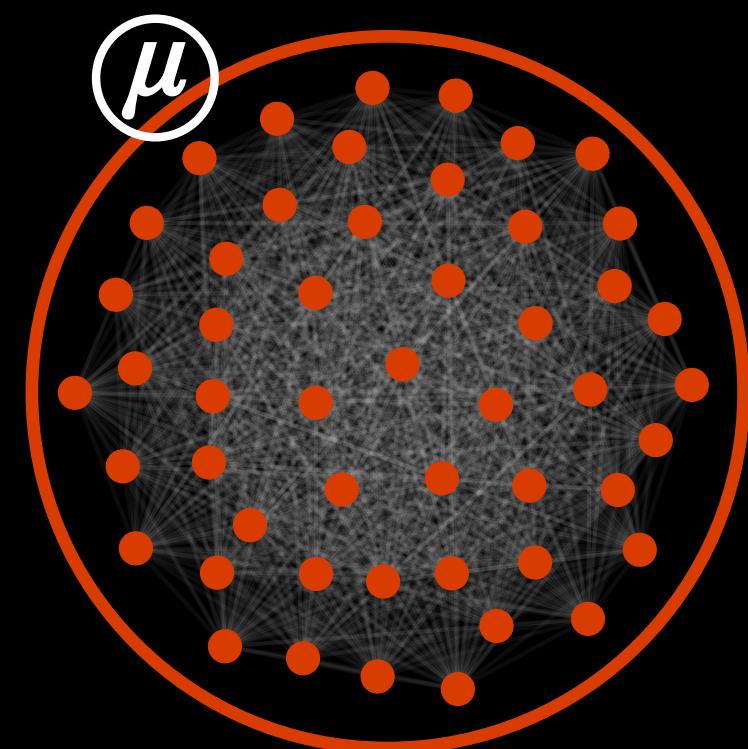
Eroglu, D., Lamb J., Pereira T. *Contemporary Physics* **58** 207 (2017)

Pereira, T., Eroglu, D., Bagci, GB., Tirnakli, U., Jensen, HJ.. *Physical Review Letters* 110 (2013)

Duan, C., Nishikawa, T., Eroglu, D., Motter, AE. *Science Advances* 8 (2022)

# PROBING DATA

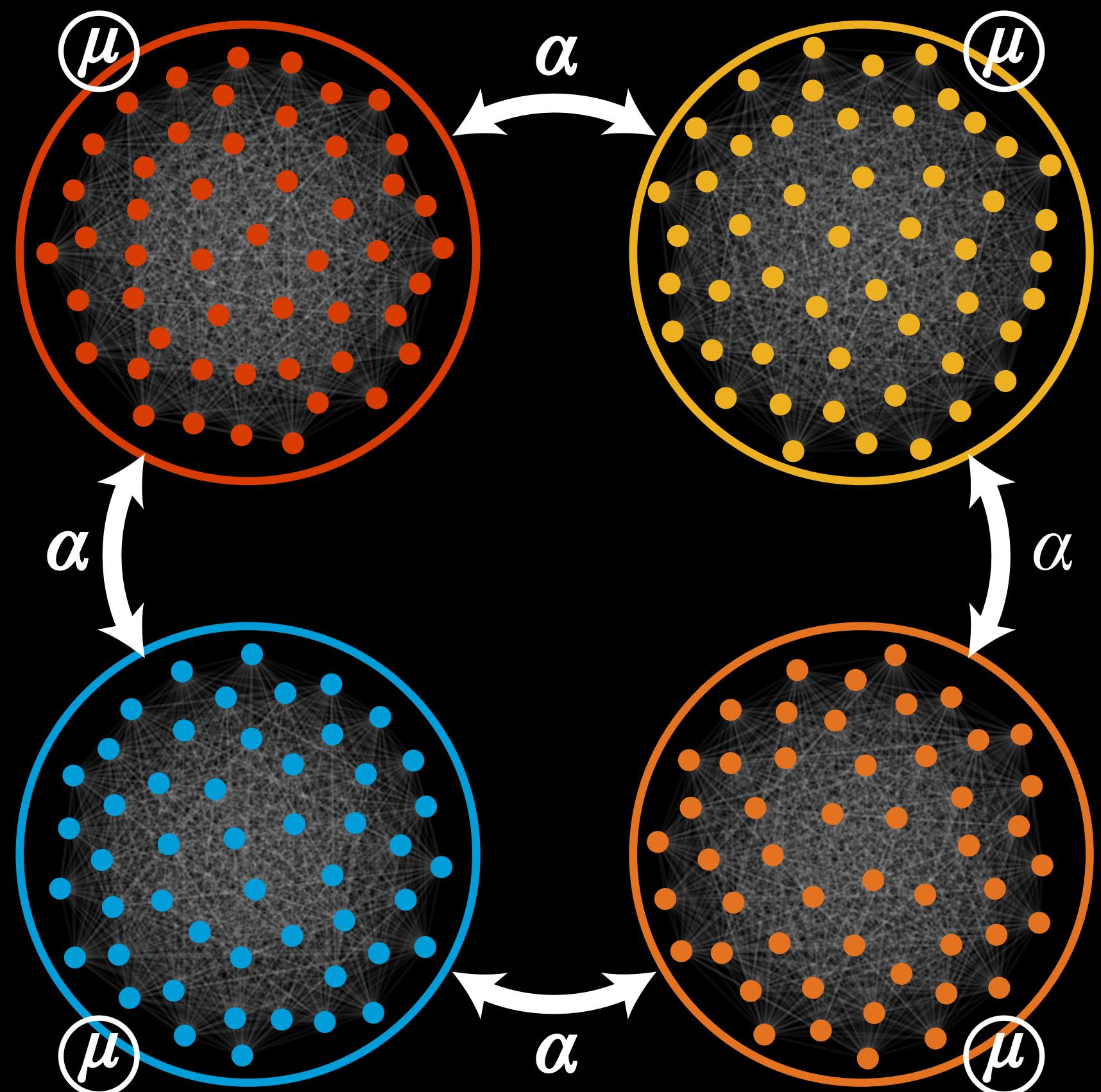
*mean-field measurements*



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N} \sum_{n=1}^N \sin(\psi_{kn} - \psi_{km})$$

# PROBING DATA

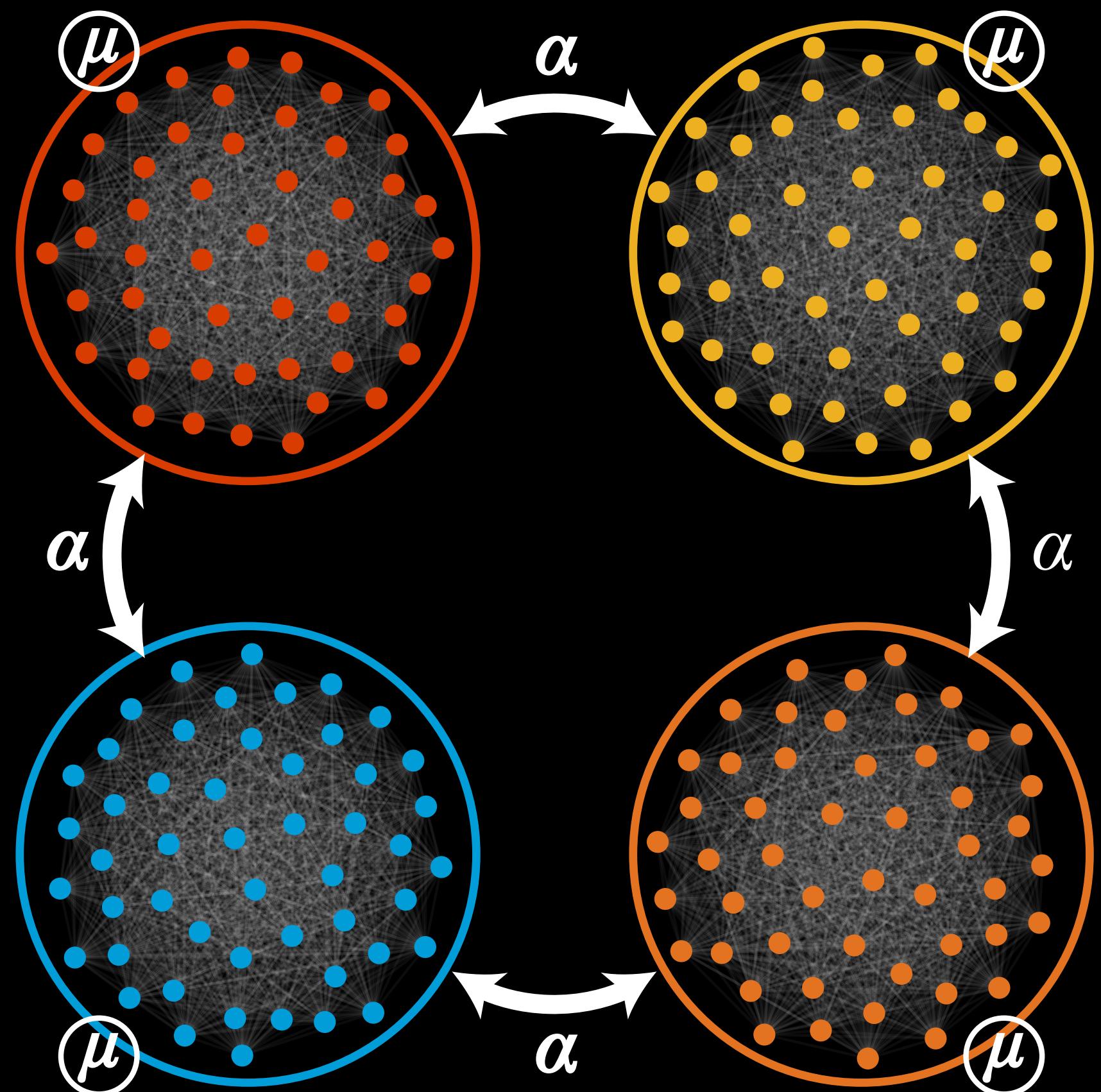
*mean-field measurements*



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N} \sum_{n=1}^N \sin(\psi_{kn} - \psi_{km}) + \sum_{\ell=1}^4 A_{k\ell} \left( \frac{\alpha}{N} \sum_{n=1}^N \sin(\psi_{\ell n} - \psi_{km}) \right)$$

# PROBING DATA

*mean-field measurements*



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N} \sum_{n=1}^N \sin(\psi_{kn} - \psi_{km}) + \sum_{\ell=1}^4 A_{k\ell} \left( \frac{\alpha}{N} \sum_{n=1}^N \sin(\psi_{\ell n} - \psi_{km}) \right)$$

In terms of mean-fields

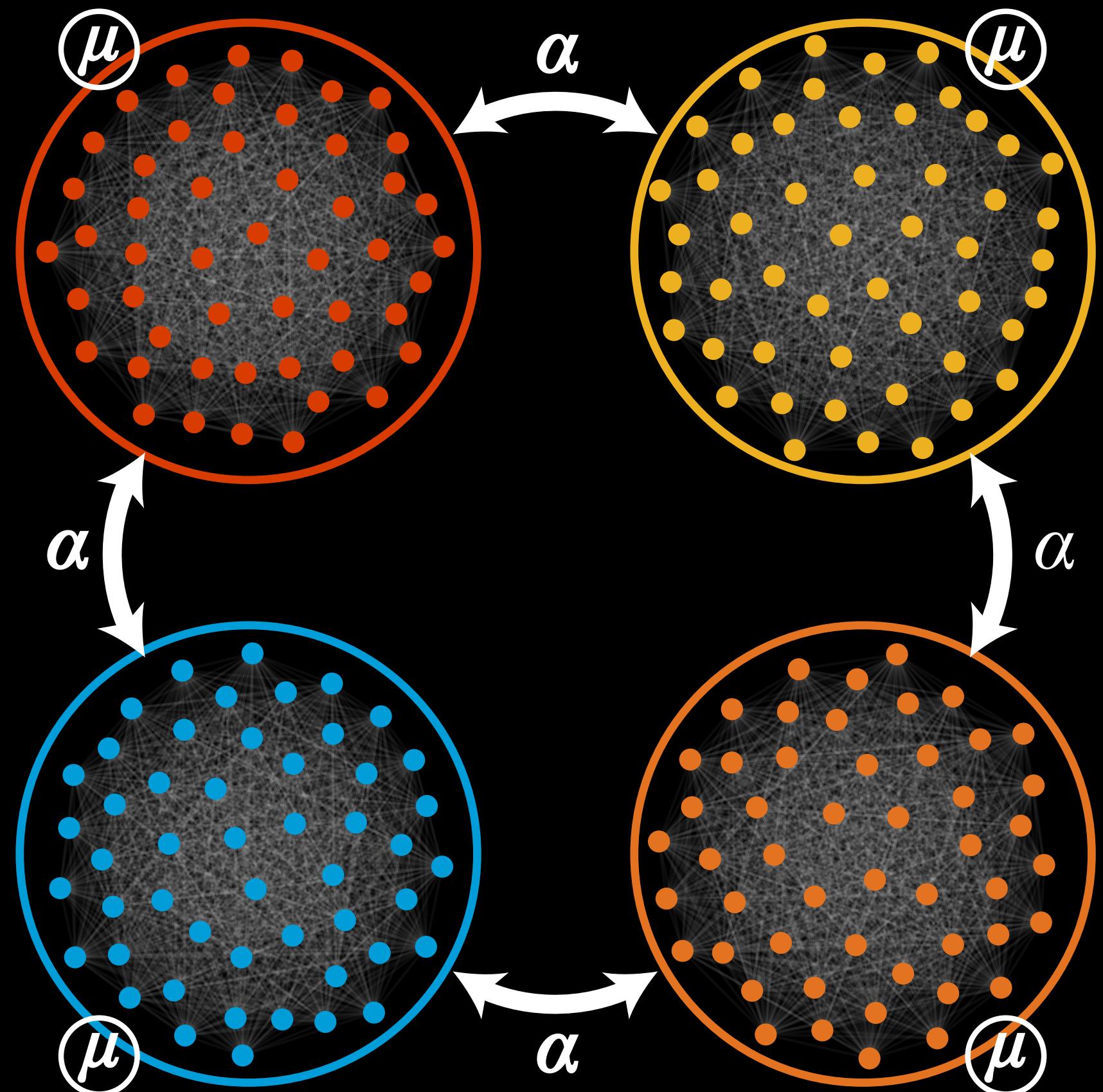
$$\dot{\psi}_{km} = \omega_{km} + \text{Im} \left( \mu z_k + \alpha \sum A_{kl} z_\ell \right) e^{-i\psi_{km}}$$

where

$$z_k = \frac{1}{N} \sum_{m=1}^N e^{i\psi_{km}}$$

# PROBING DATA

*assume infinitely many neurons*

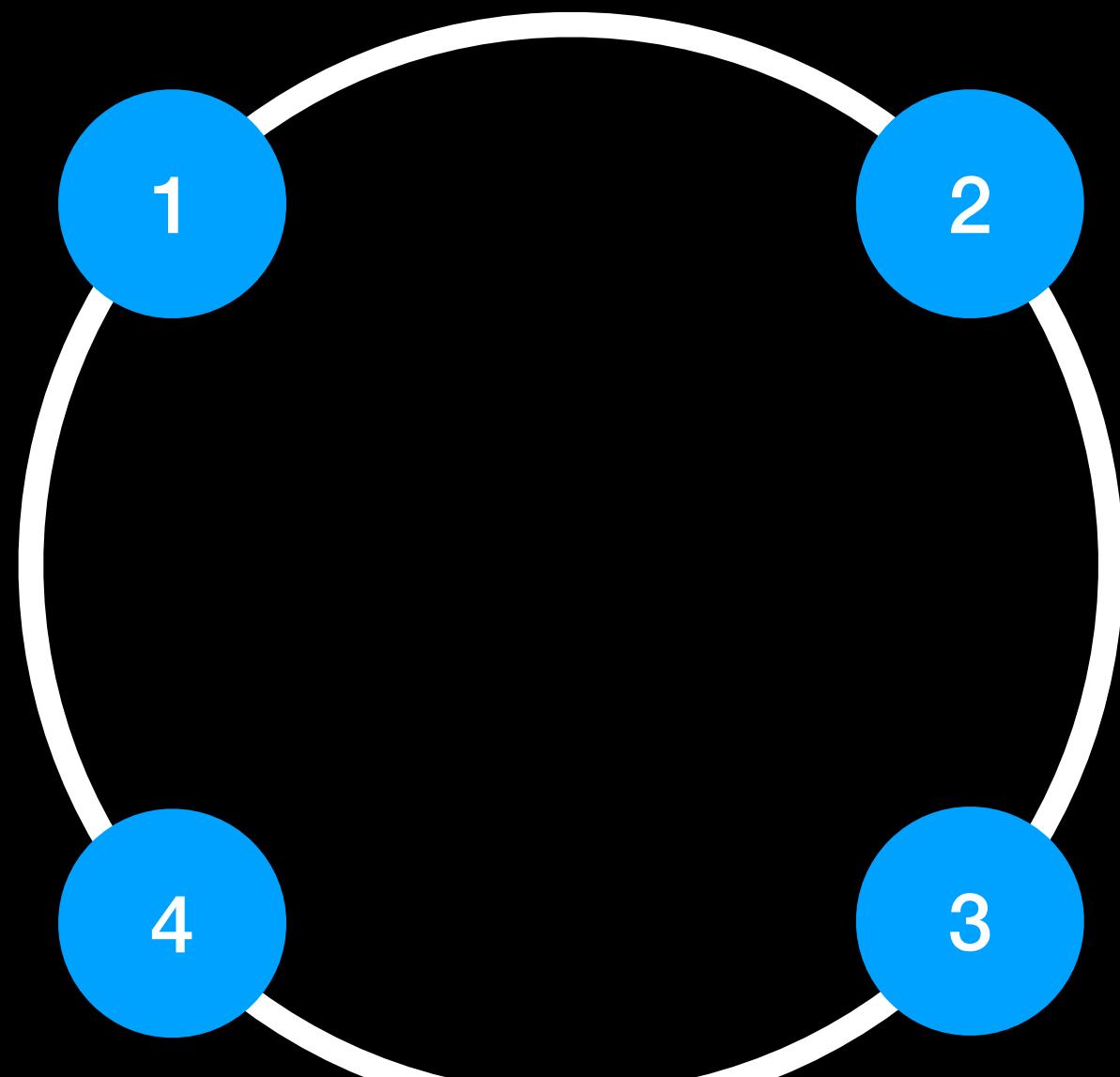


Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

# RING GRAPH

*undirected and cubic polynomial interaction*



Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

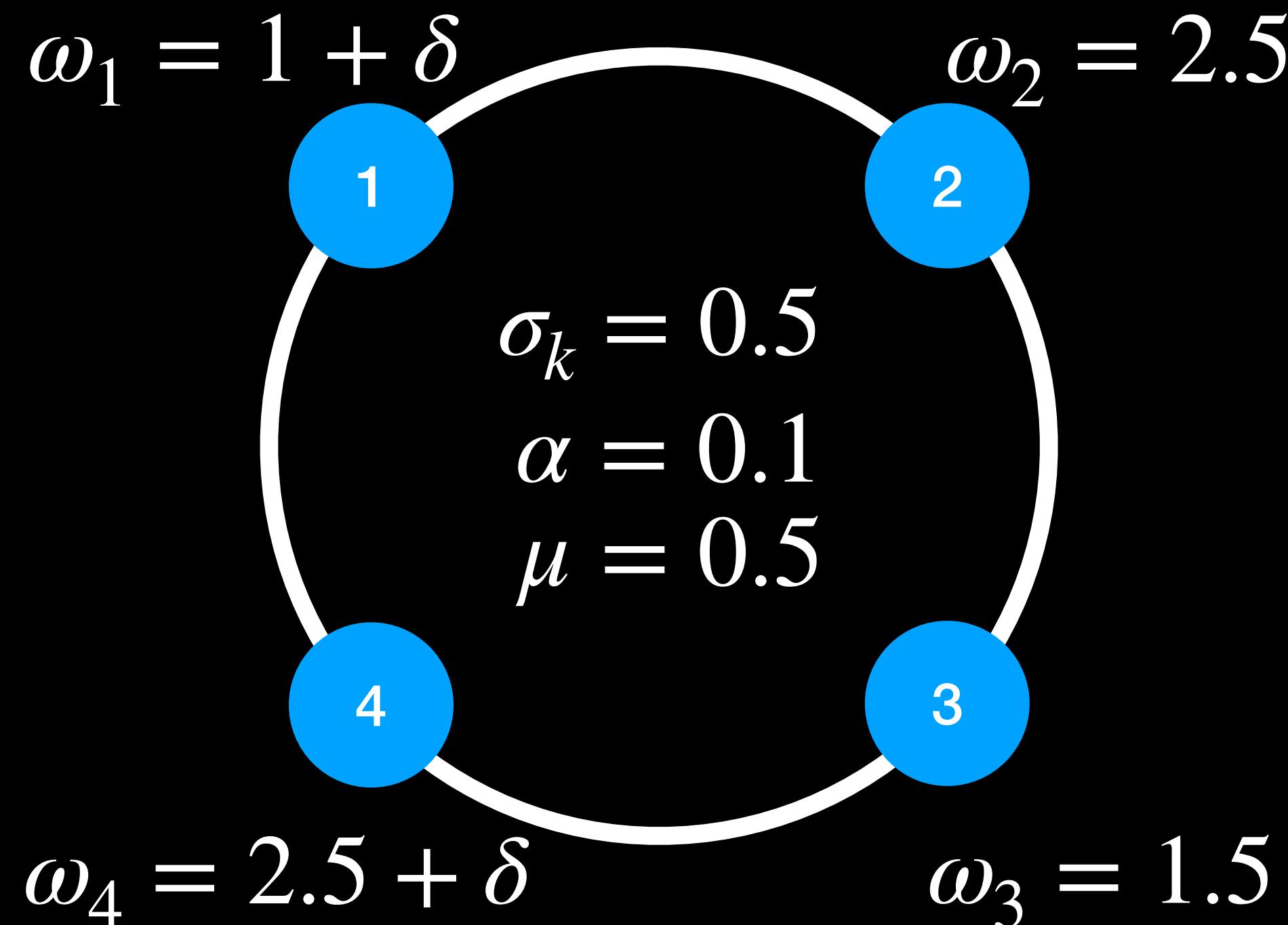
where

$$f_k(z_k) = \gamma_k z_k + \beta_k z_k - z_k^2; \quad \gamma_k = (i\Omega_k + \mu - \sigma_k)$$

$$\beta_k = -\mu; \quad h(z_k, z_\ell) = \alpha z_\ell + \alpha \bar{z}_\ell z_k^2$$

# RING GRAPH

*undirected and cubic polynomial interaction*



Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^4 A_{k\ell} h(z_k, z_\ell)$$

where

$$f_k(z_k) = \gamma_k z_k + \beta_k z_k - z_k^2; \quad \gamma_k = (i\Omega_k + \mu - \sigma_k)$$

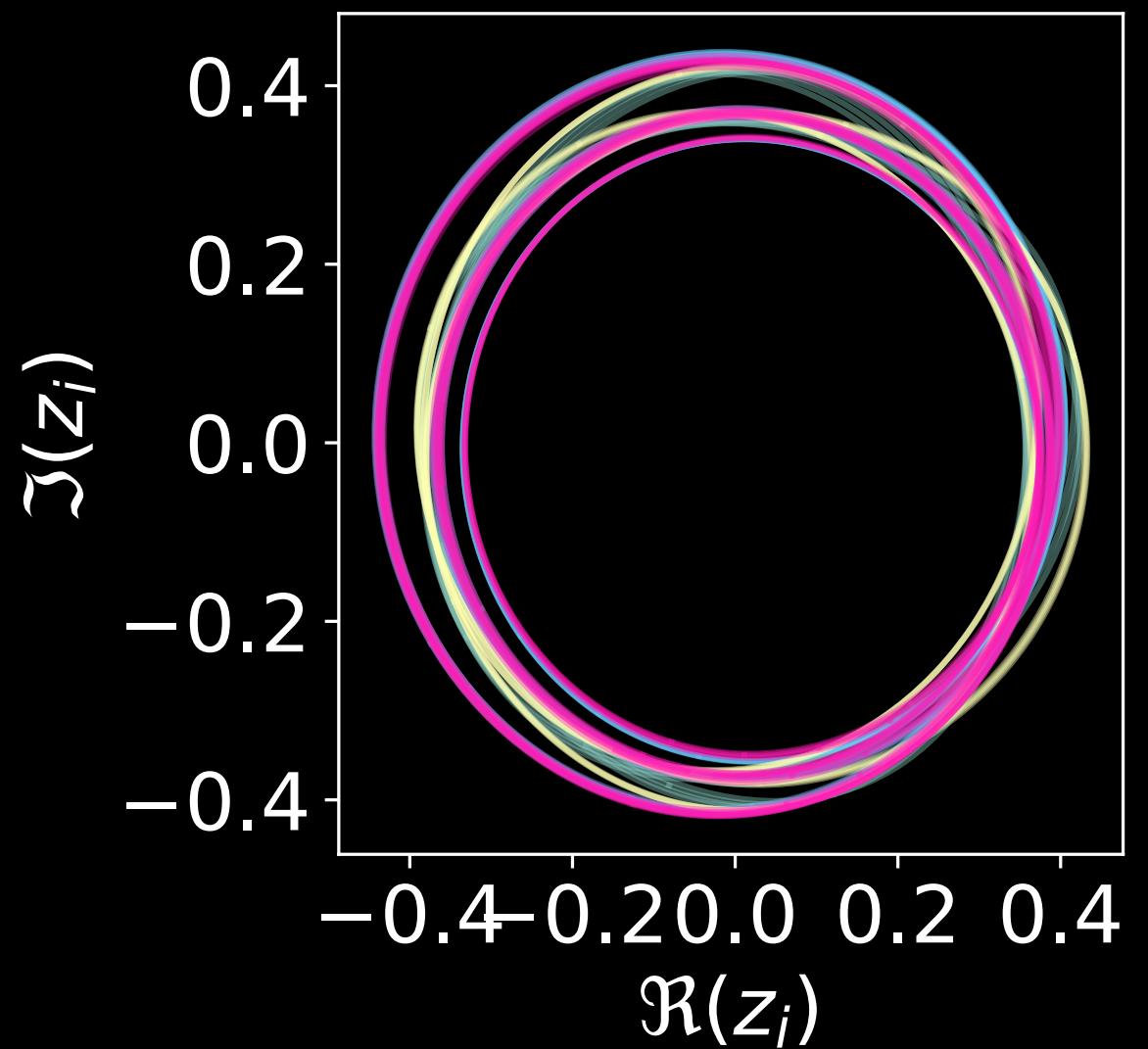
$$\beta_k = -\mu; \quad h(z_k, z_\ell) = \alpha z_\ell + \alpha \bar{z}_\ell z_k^2$$

Resonance satisfying condition:  $\omega_2 = \omega_1 + \omega_3$  &  $\omega_4 = \omega_1 + \omega_3$

# RING GRAPH

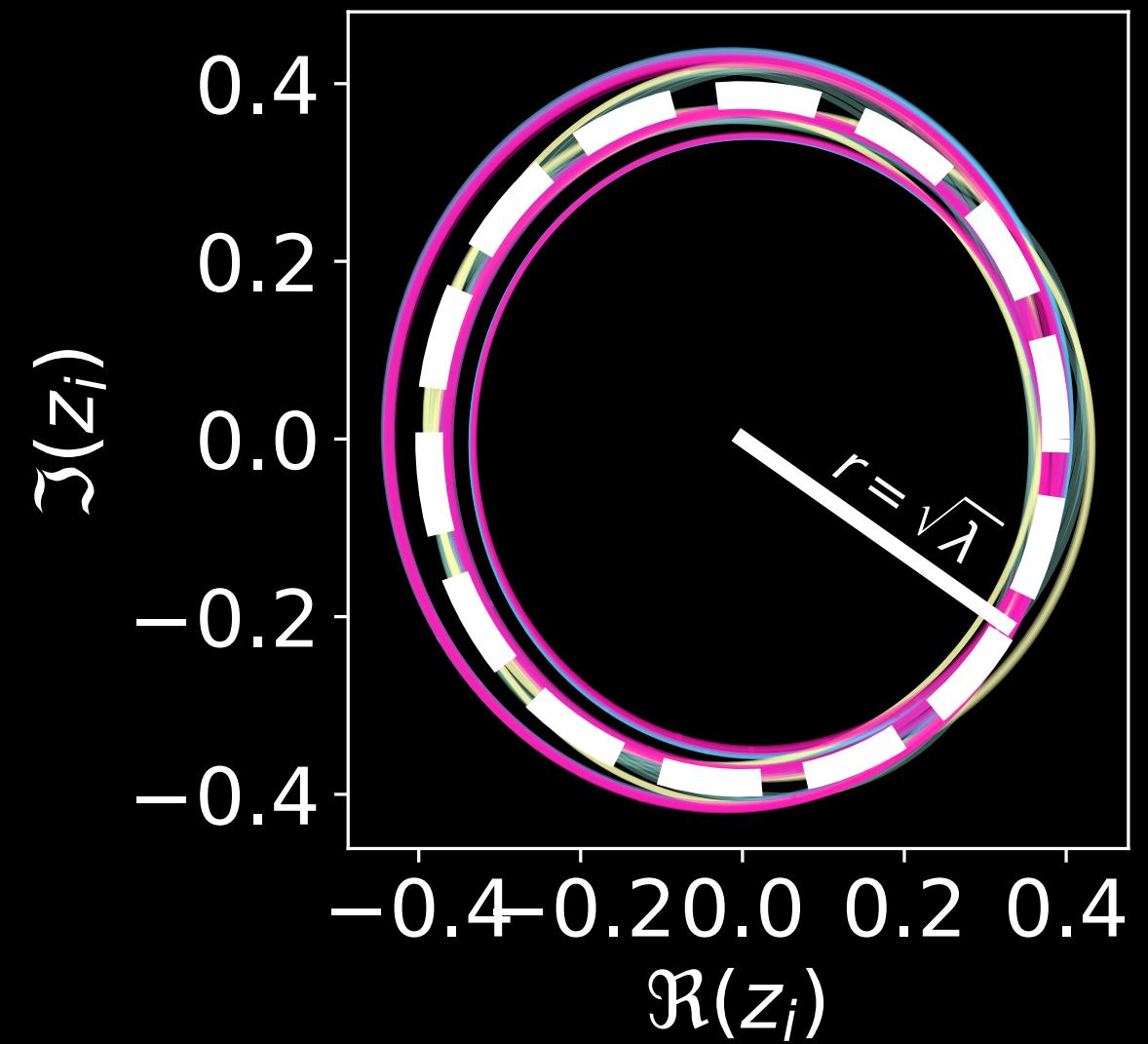
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*simulation*



# RING GRAPH

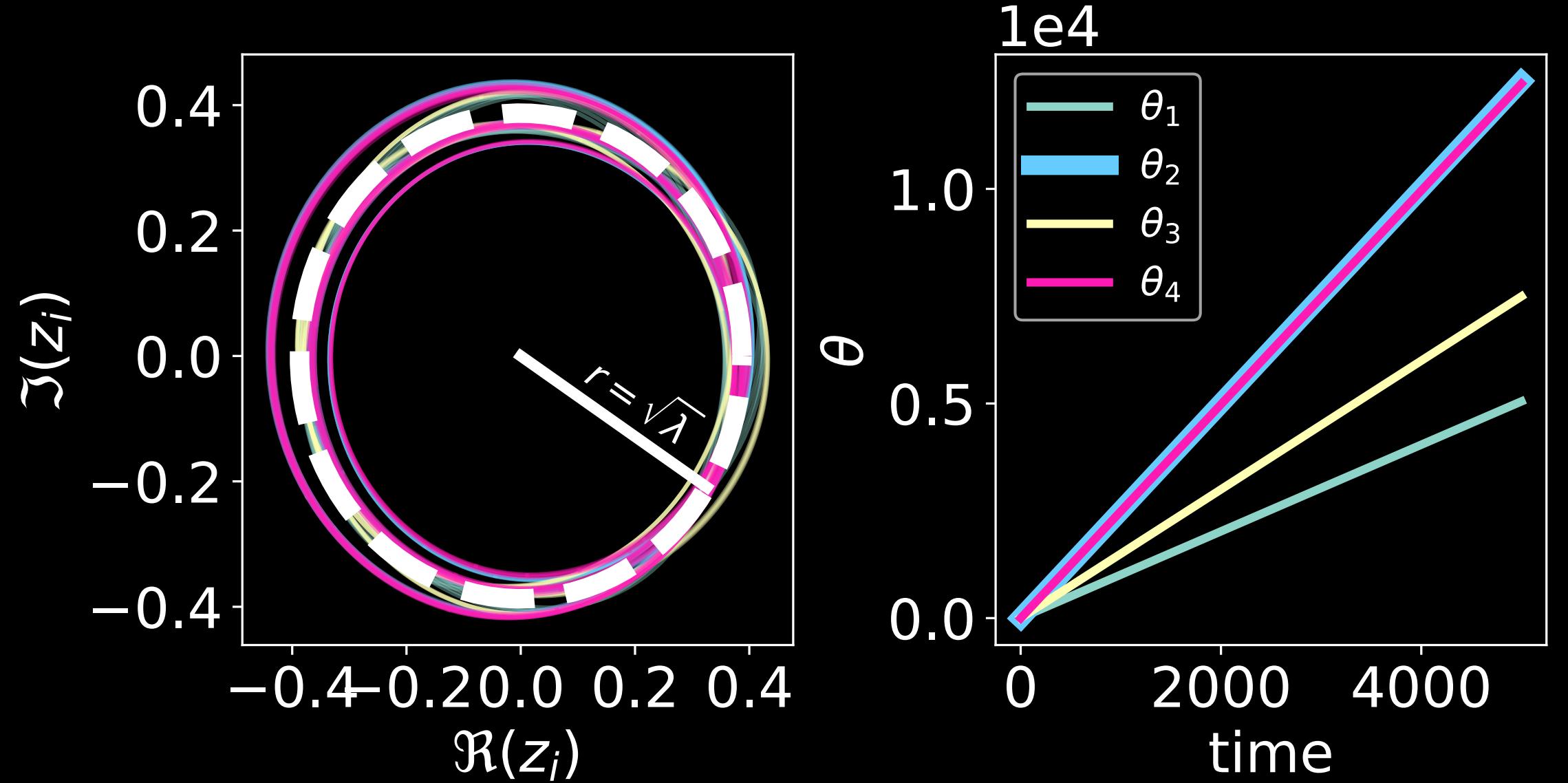
*simulation*



$$\lambda_k = \frac{\mu - \sigma_k}{\mu}$$

# RING GRAPH

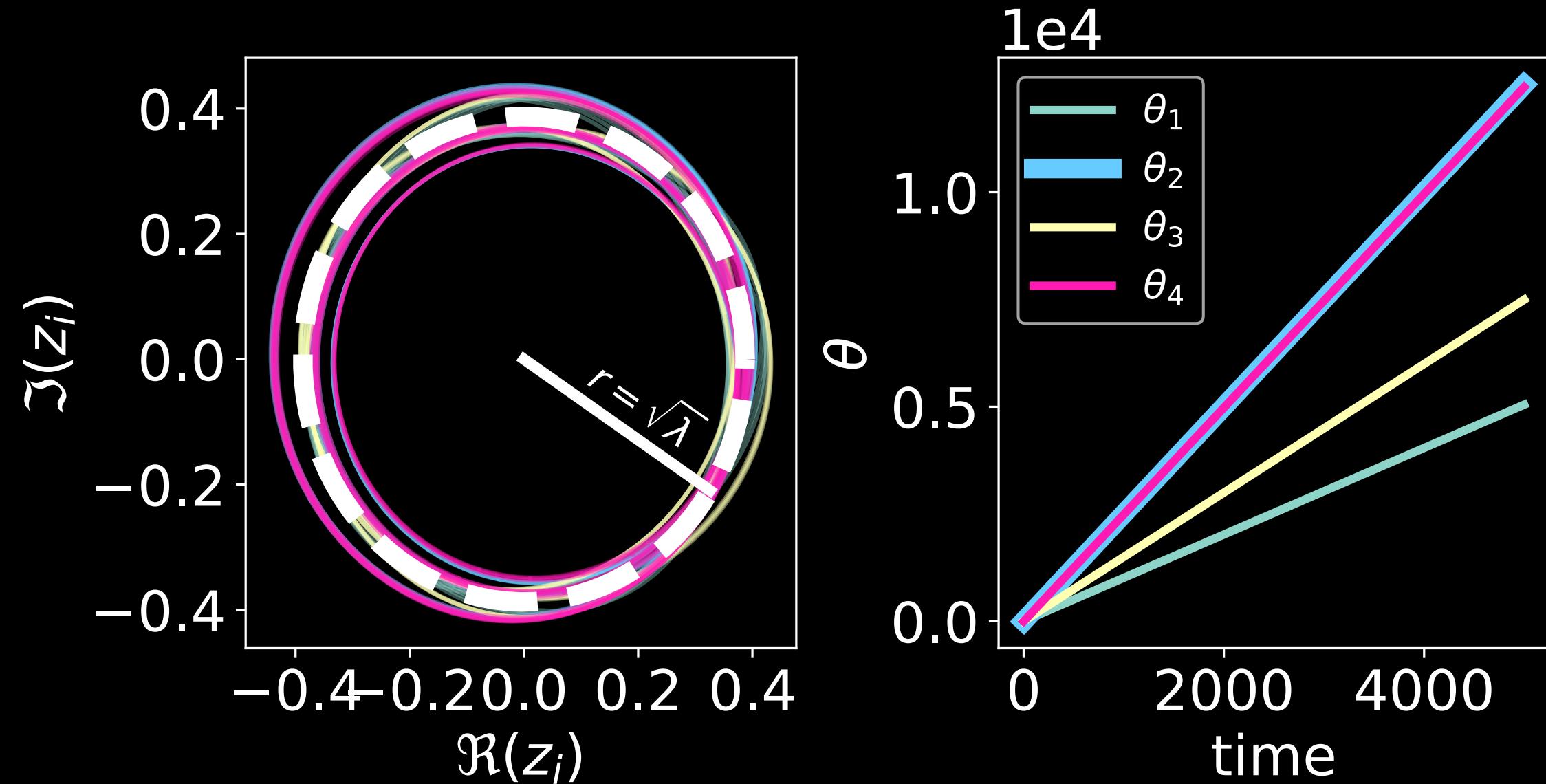
*defining phases*



$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$

# RING GRAPH

*defining slow variables*

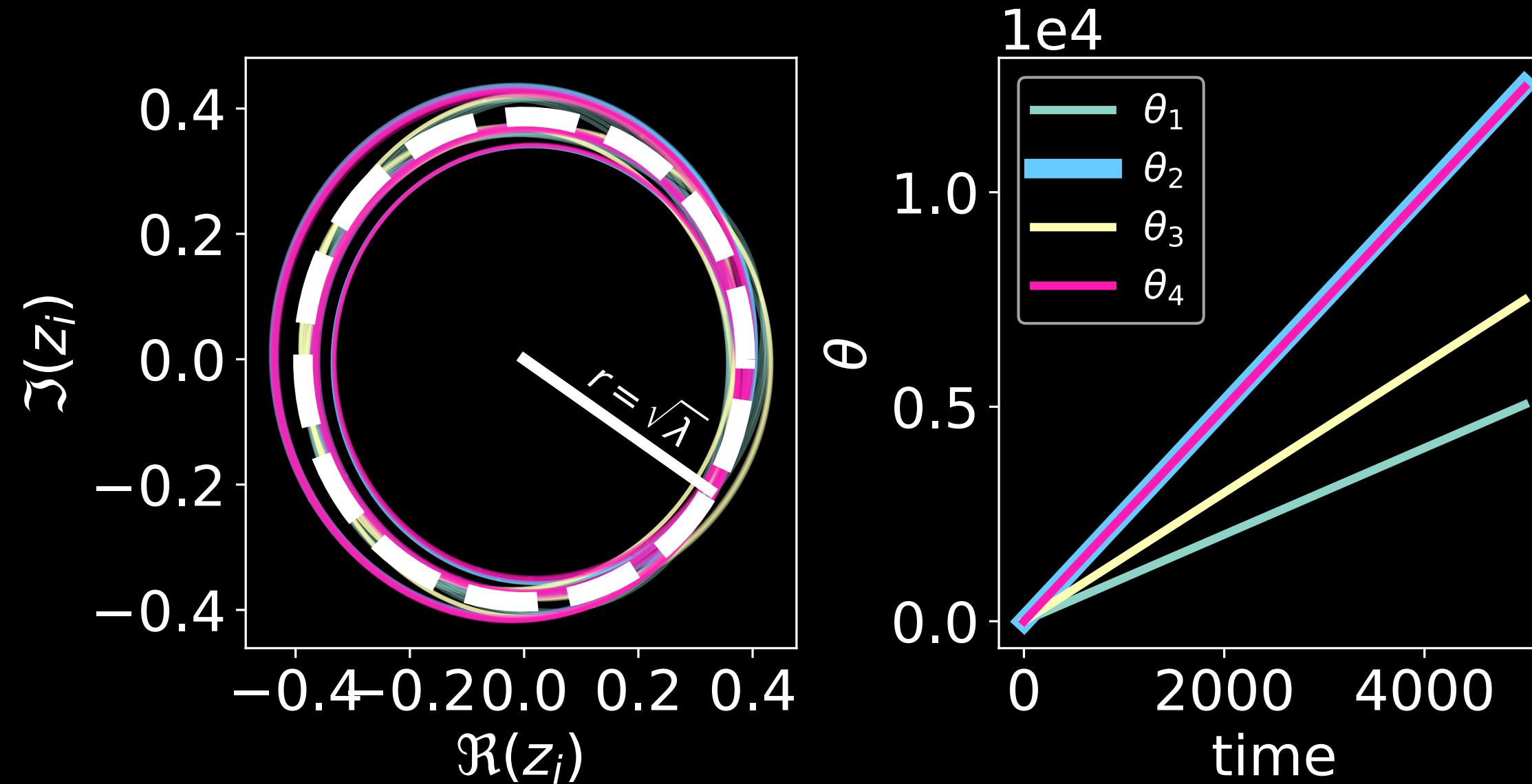


$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$
$$\vartheta_k(t) = \theta_k(t) - \Omega_k t$$

Then we would like to reconstruct  $\vartheta_i$  from data

# RING GRAPH

*defining slow variables*



$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$
$$\vartheta_k(t) = \theta_k(t) - \Omega_k t$$

Then we would like to reconstruct  $\vartheta_i$  from data

$$\dot{\vartheta} = \Theta(\vartheta)\Xi$$

# COMPRESSED SENSING

Candes, E J., Justin K. R, and Terence T. , 2006

*model reconstruction from data*

$$\begin{bmatrix} \vartheta_1 & \vartheta_2 & \vartheta_3 & \vartheta_4 \end{bmatrix} = \Theta(\vartheta) \begin{bmatrix} \sin(\vartheta_1) & \sin(\vartheta_4) & \sin(\vartheta_1, \vartheta_2) & \sin(\vartheta_1, \vartheta_2, \vartheta_3) \\ 1 & \cos(\vartheta_1) & \cos(\vartheta_4) & \cos(\vartheta_1, \vartheta_2) \\ & & & \cos(\vartheta_1, \vartheta_2, \vartheta_3) \\ \dots & \dots & \dots & \dots \\ & & & \dots \\ \Sigma_1 & \Sigma_2 & \Sigma_3 & \Sigma_4 \end{bmatrix}$$

$\vartheta$        $\Theta(\vartheta)$        $\Sigma$

# COMPRESSED SENSING

Candes, E J., Justin K. R, and Terence T. , 2006

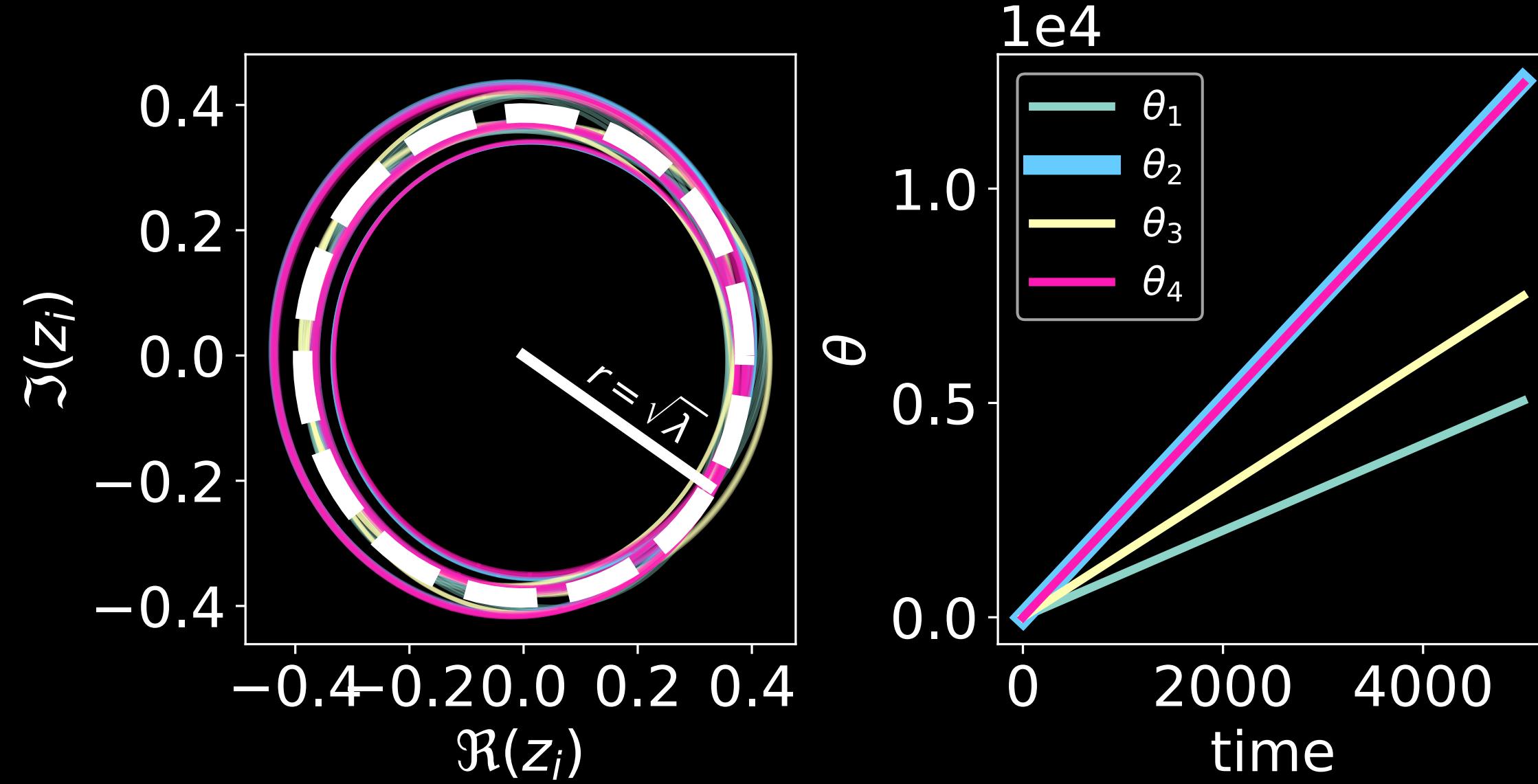
*model reconstruction from data*

$$\begin{bmatrix} \vartheta_1 & \vartheta_2 & \vartheta_3 & \vartheta_4 \end{bmatrix} = \Theta(\vartheta) \begin{bmatrix} \sin(\vartheta_1) & \sin(\vartheta_4) & \sin(\vartheta_1, \vartheta_2) & \sin(\vartheta_1, \vartheta_2, \vartheta_3) \\ 1 & \cos(\vartheta_1) & \cos(\vartheta_4) & \cos(\vartheta_1, \vartheta_2) \\ \dots & \dots & \dots & \dots \\ \Sigma_1 & \Sigma_2 & \Sigma_3 & \Sigma_4 \end{bmatrix}$$

The diagram illustrates the compressed sensing model reconstruction process. On the left, a vector  $\vartheta$  is shown as a column of four blue bars. An equals sign follows, indicating the reconstruction process. To the right of the equals sign is the matrix  $\Theta(\vartheta)$ , which is a tall column of colored bars representing basis functions. The first row of  $\Theta(\vartheta)$  contains the terms  $1, \sin(\vartheta_1), \cos(\vartheta_1), \sin(\vartheta_4), \cos(\vartheta_4), \sin(\vartheta_1, \vartheta_2), \cos(\vartheta_1, \vartheta_2), \sin(\vartheta_1, \vartheta_2, \vartheta_3), \cos(\vartheta_1, \vartheta_2, \vartheta_3)$ . Subsequent rows are indicated by ellipses. To the right of  $\Theta(\vartheta)$  is a matrix  $\Sigma$  represented as a column of four gray bars. Blue dots on these bars indicate the non-zero entries, corresponding to the columns of  $\Theta(\vartheta)$  that are active for each component of  $\vartheta$ .

# RING GRAPH

*reconstruction of slow phases*



$$\dot{\theta}_1 = 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_4 + \theta_3)$$

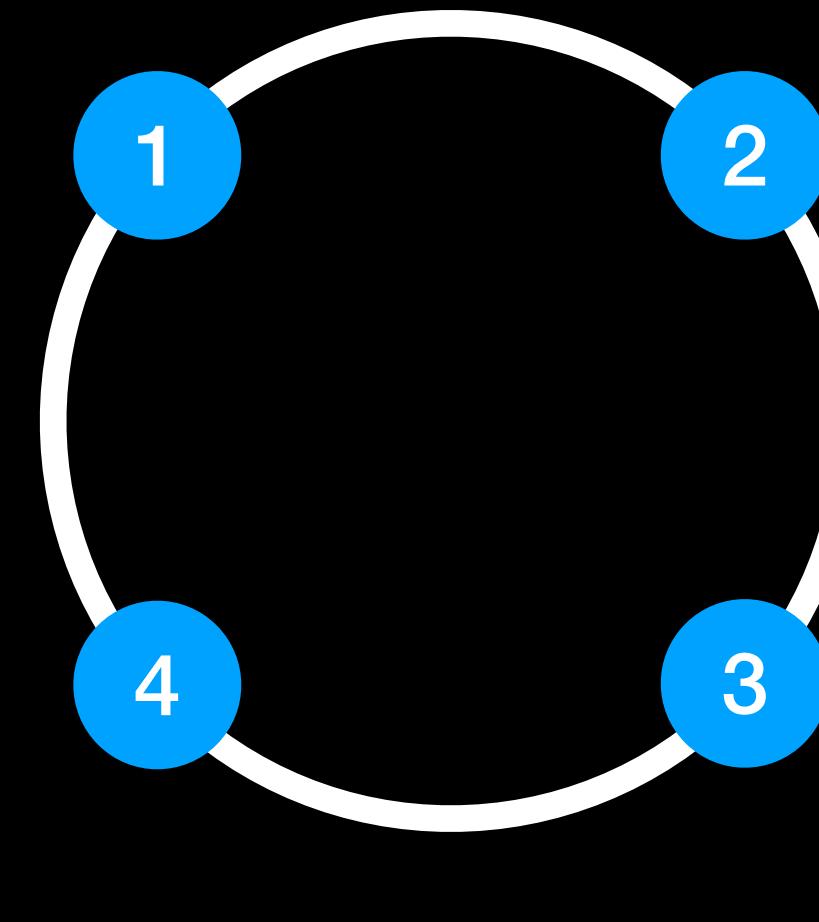
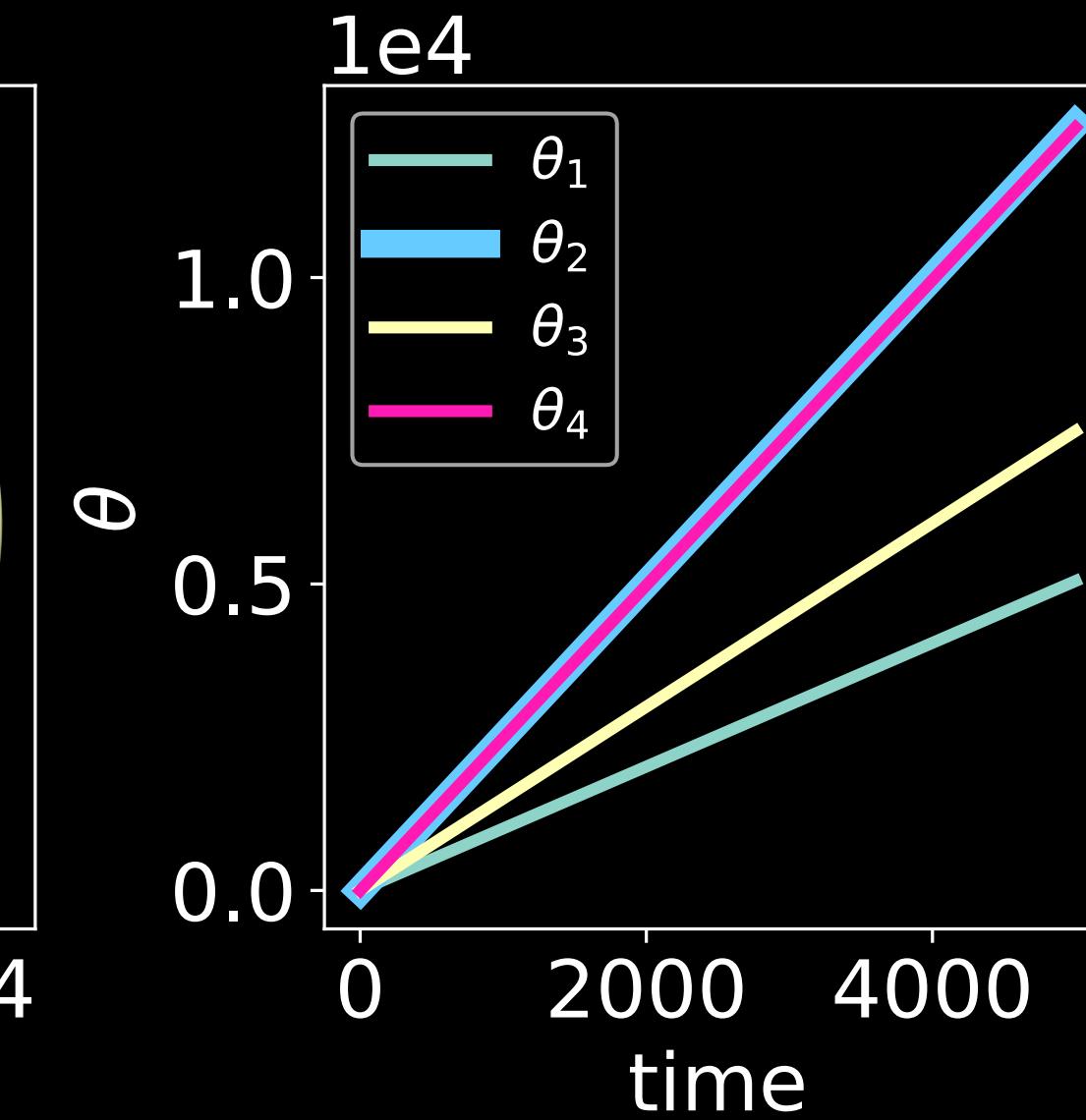
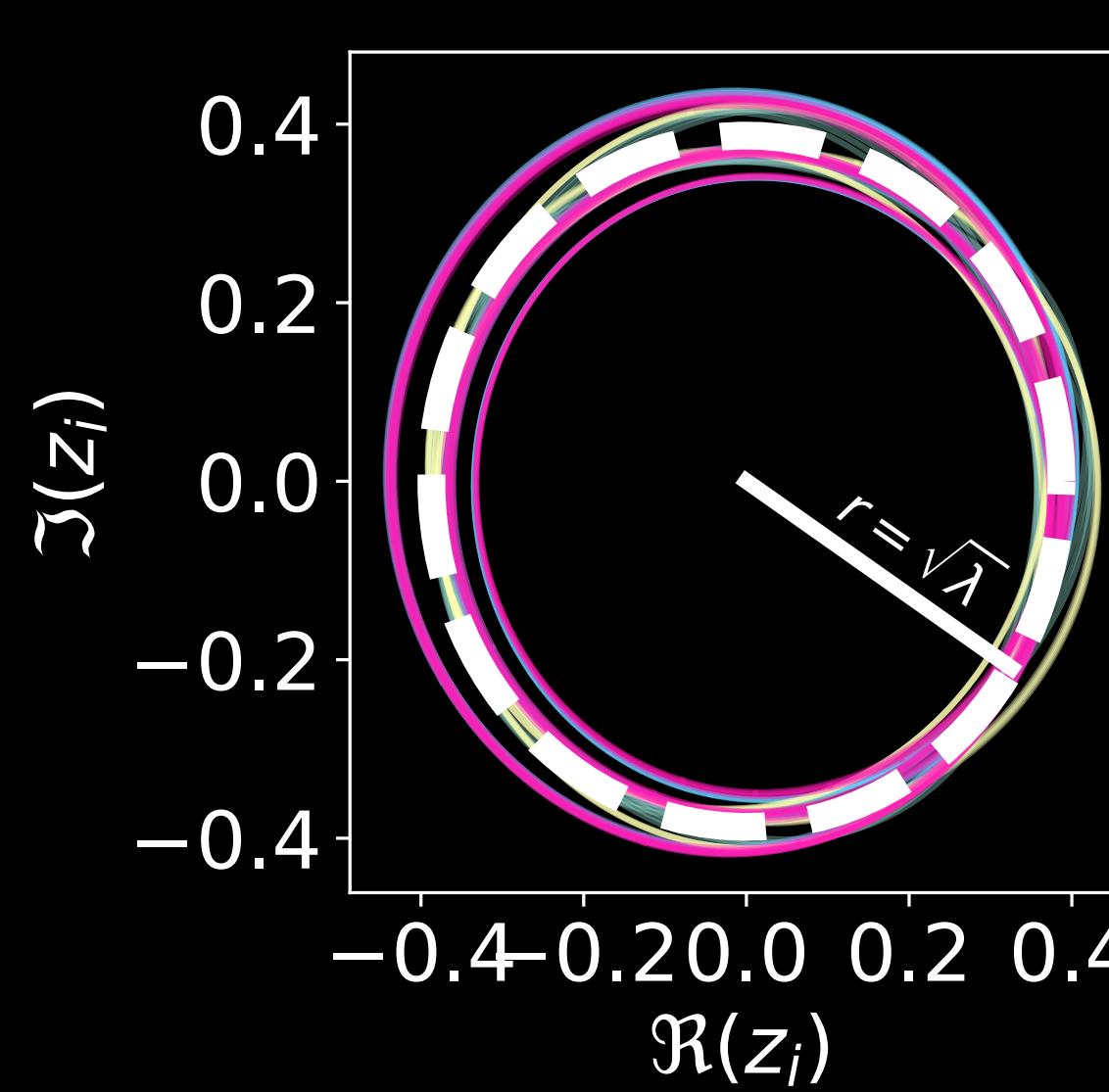
$$\dot{\theta}_2 = 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3)$$

$$\dot{\theta}_3 = 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_4 + \theta_3)$$

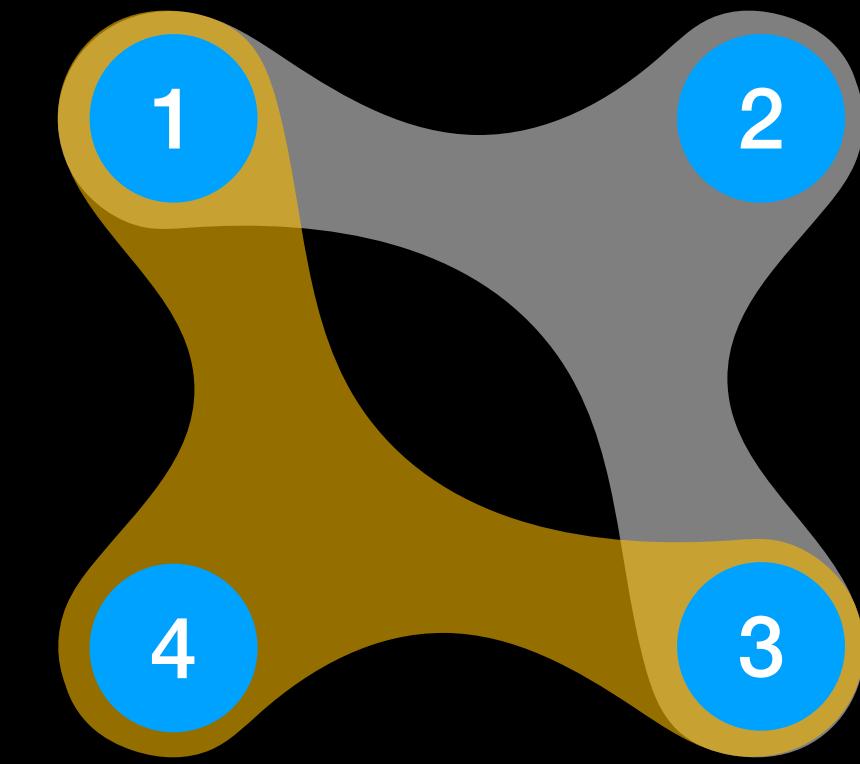
$$\dot{\theta}_4 = 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3)$$

# RING GRAPH

*emergent hypergraphs*



original  
network



reconstructed  
hypernetwork

$$\dot{\theta}_1 = 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_4 + \theta_3)$$

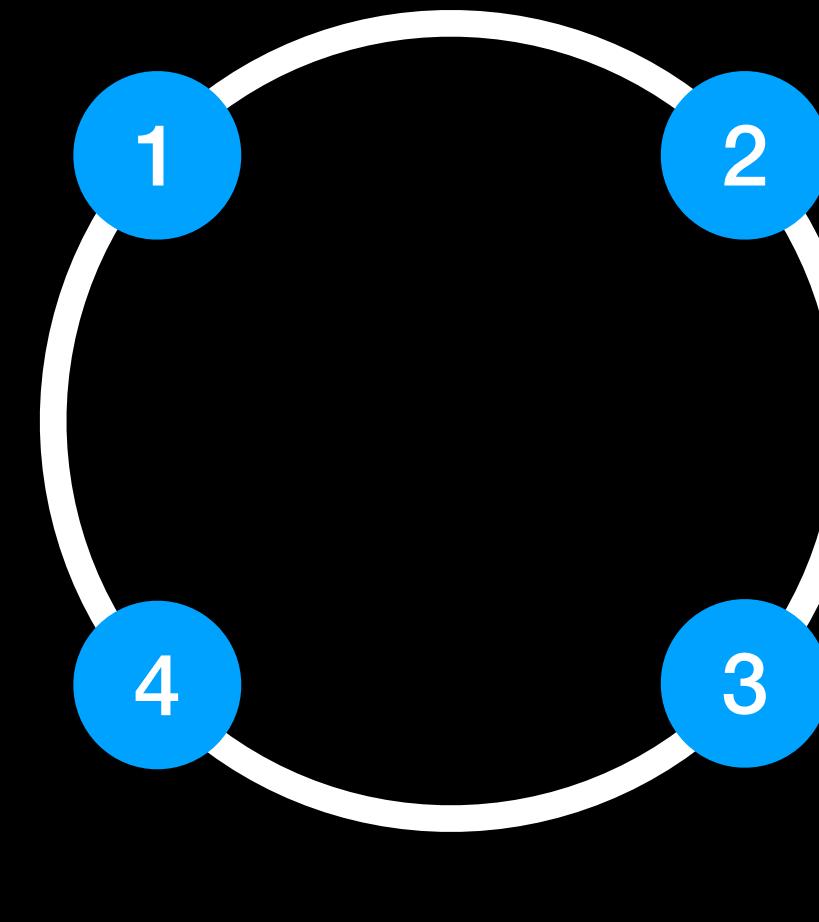
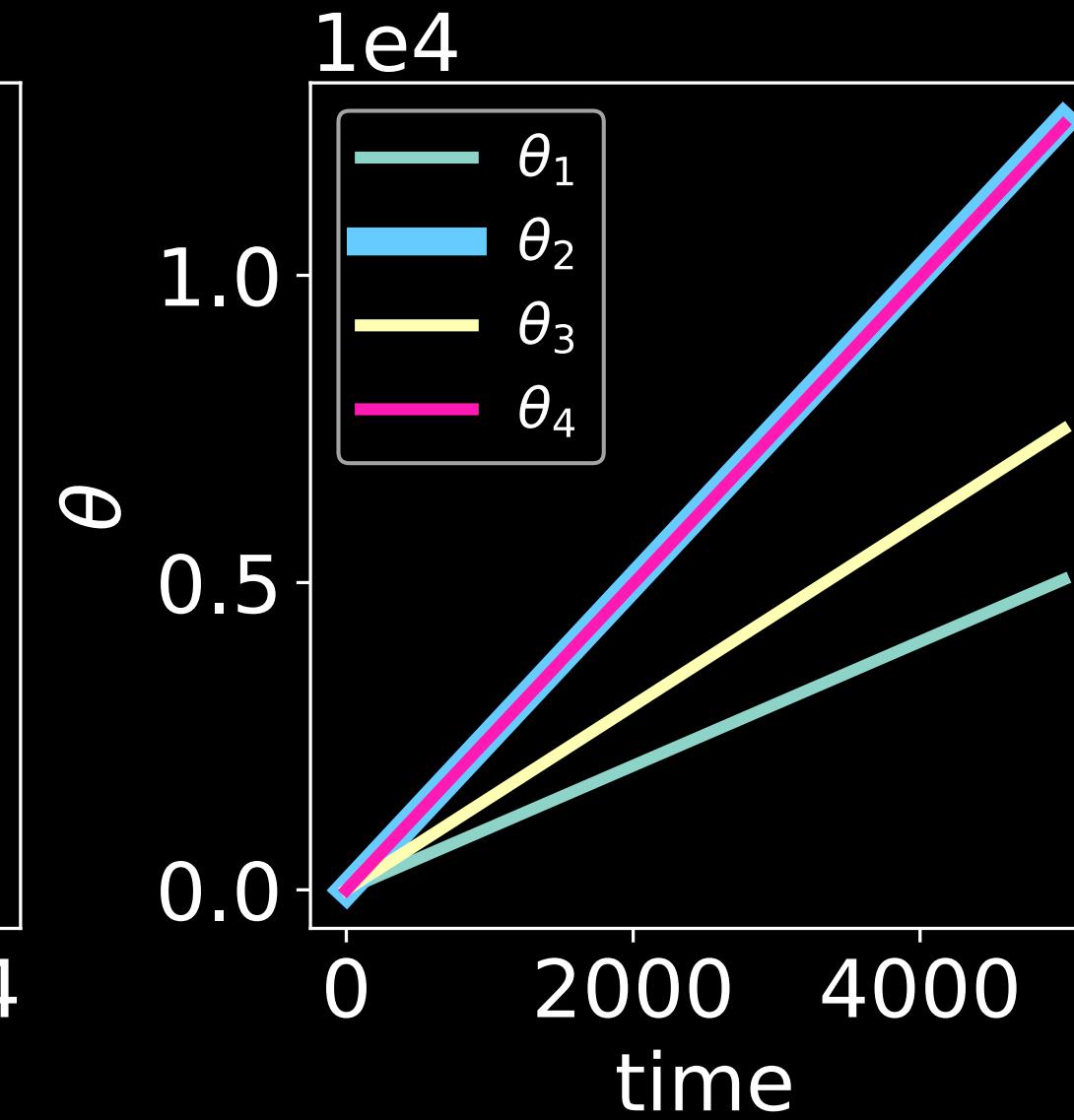
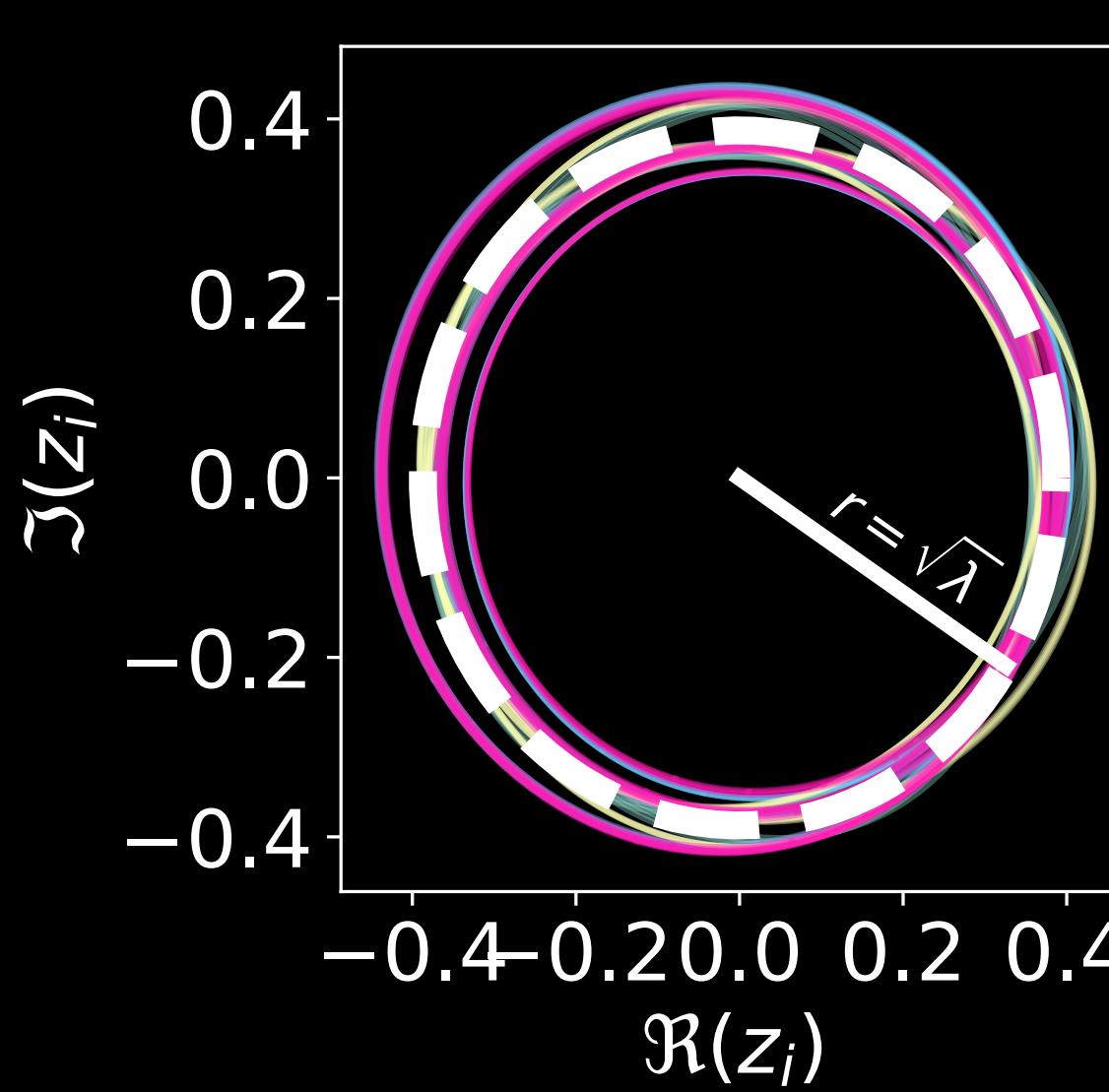
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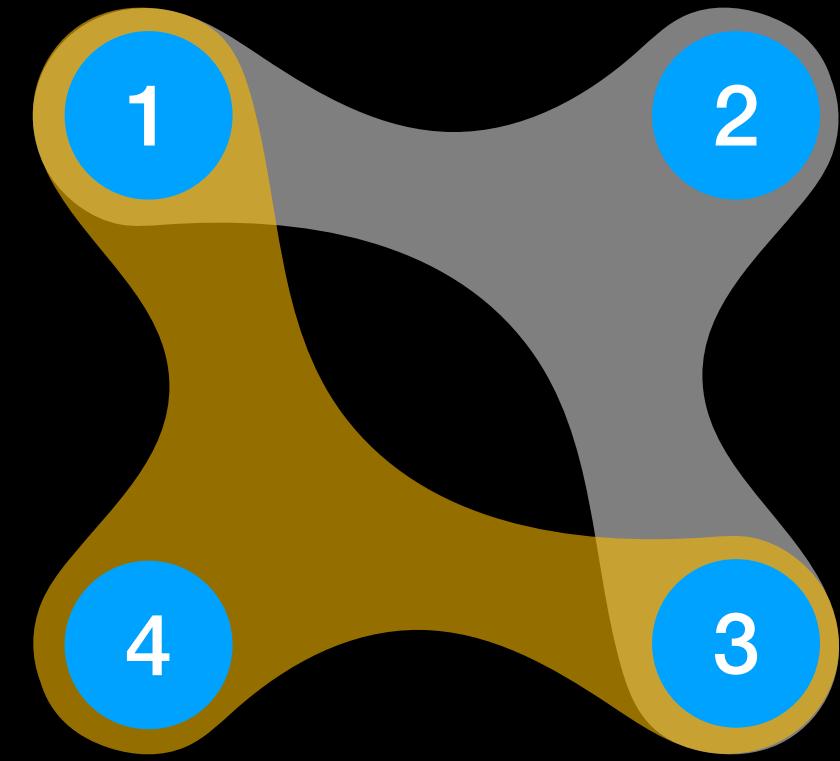
$$\dot{\theta}_4 = 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3)$$

# RING GRAPH

*emergent hypergraphs*



original  
network



reconstructed  
hypernetwork

$$\dot{\theta}_1 = 1.010 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_4 + \theta_3)$$

$$\dot{\theta}_2 = 2.489 - 0.005 \cos(\theta_1 - \theta_2 + \theta_3)$$

$$\dot{\theta}_3 = 1.499 + 0.001 \cos(\theta_1 - \theta_2 + \theta_3) - 0.001 \cos(\theta_1 - \theta_4 + \theta_3)$$

$$\dot{\theta}_4 = 2.508 + 0.005 \cos(\theta_1 - \theta_4 + \theta_3)$$

WHY?

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$

a coordinate transformation of the form  $w_k = z_k - \alpha P_k(z)$

$$\text{for some polynomials } P_k(z) = \sum_{\ell=1}^n A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$$

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$

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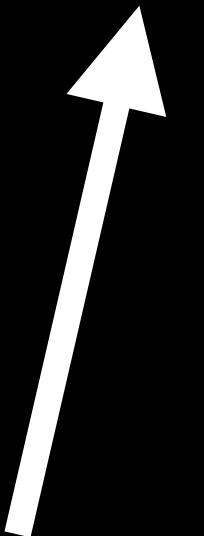
$$\text{for some polynomials } P_k(z) = \sum_{\ell=1}^n A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$$

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$



transformation generates additional  
undesired terms from the isolated dynamics

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$

second coordinate transformation of the form  $u_k = w_k - \alpha Q_k(w)$

# NORMAL FORM THEORY

---

*looking for the sparsest solution*

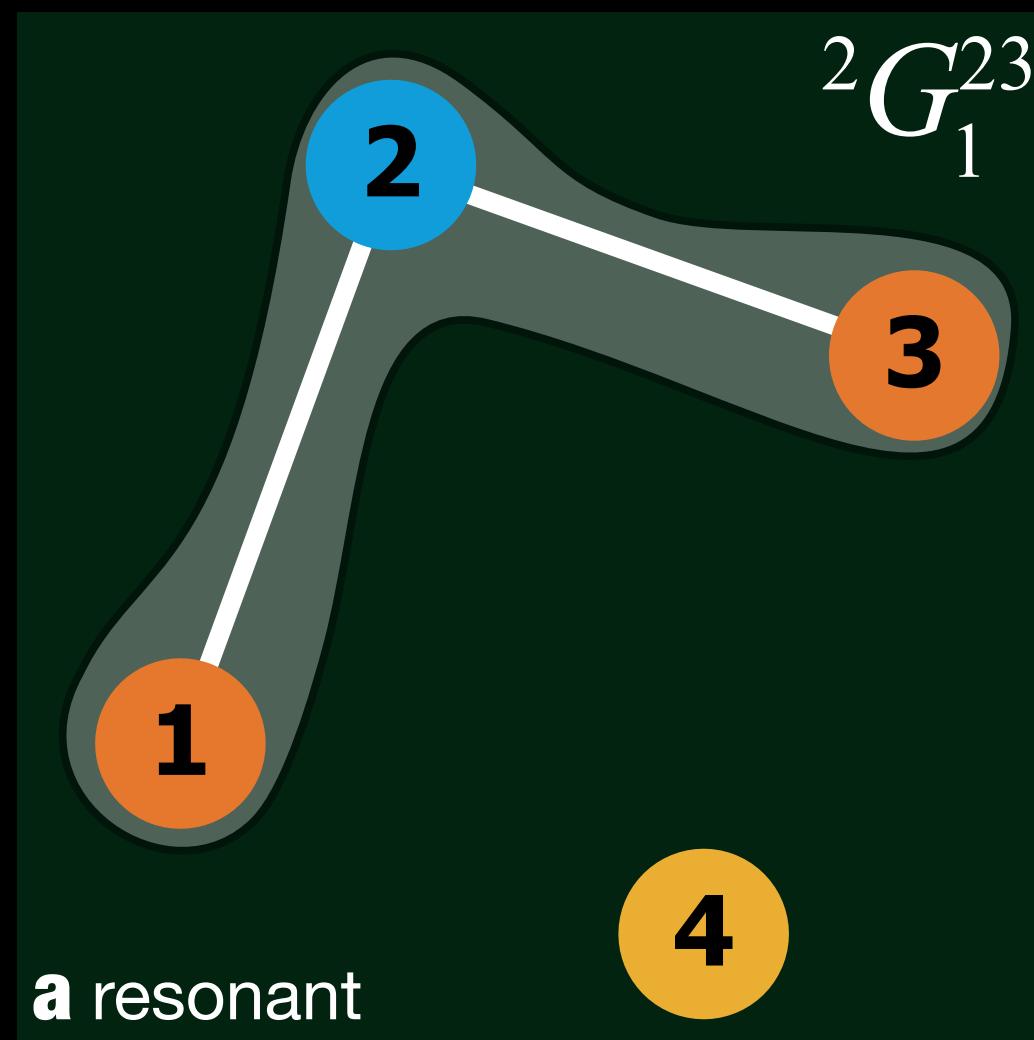
$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^n A_{k\ell} h_k(z_k, z_\ell)$$

second coordinate transformation of the form  $u_k = w_k - \alpha Q_k(w)$

nontrivial combinatorial problem tackled by introducing a special bracket  $[\bullet||\bullet]$  on the space of polynomials.

# NORMAL FORM THEORY

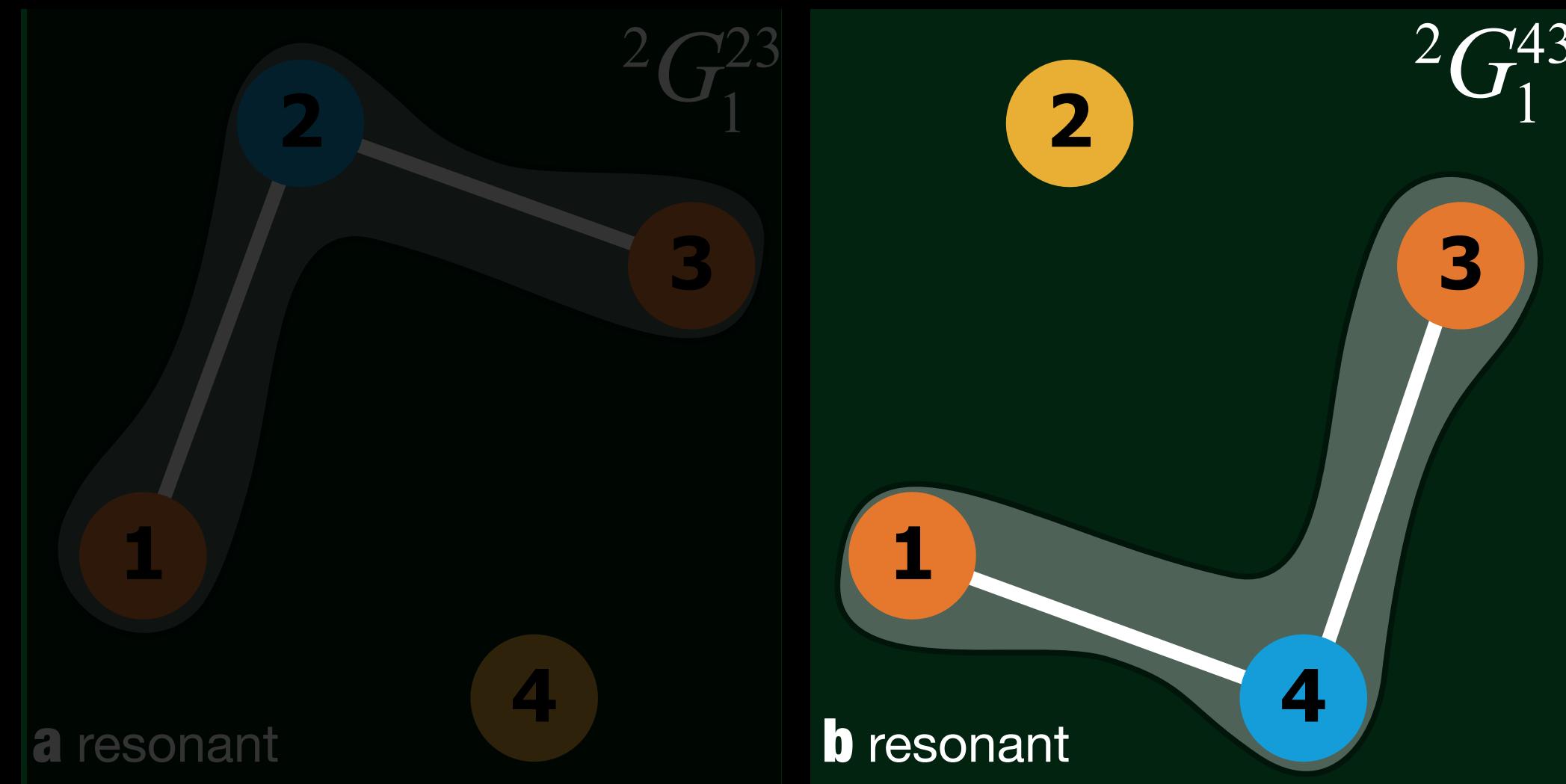
*rules for resonant and nonresonant terms*



Checking only for node-1

# NORMAL FORM THEORY

*rules for resonant and nonresonant terms*



Checking only for node-1

# NORMAL FORM THEORY

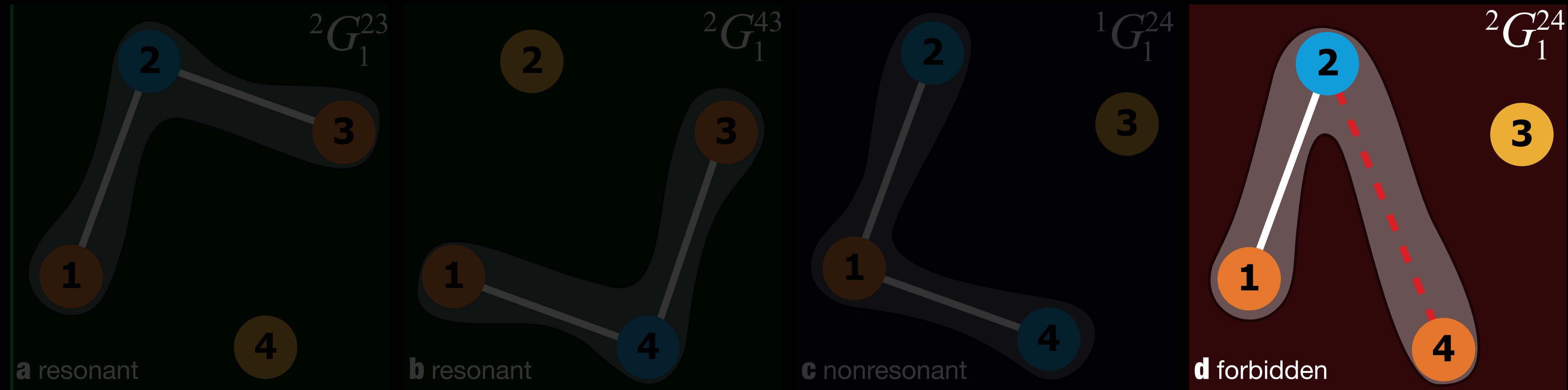
*rules for resonant and nonresonant terms*



Checking only for node-1

# NORMAL FORM THEORY

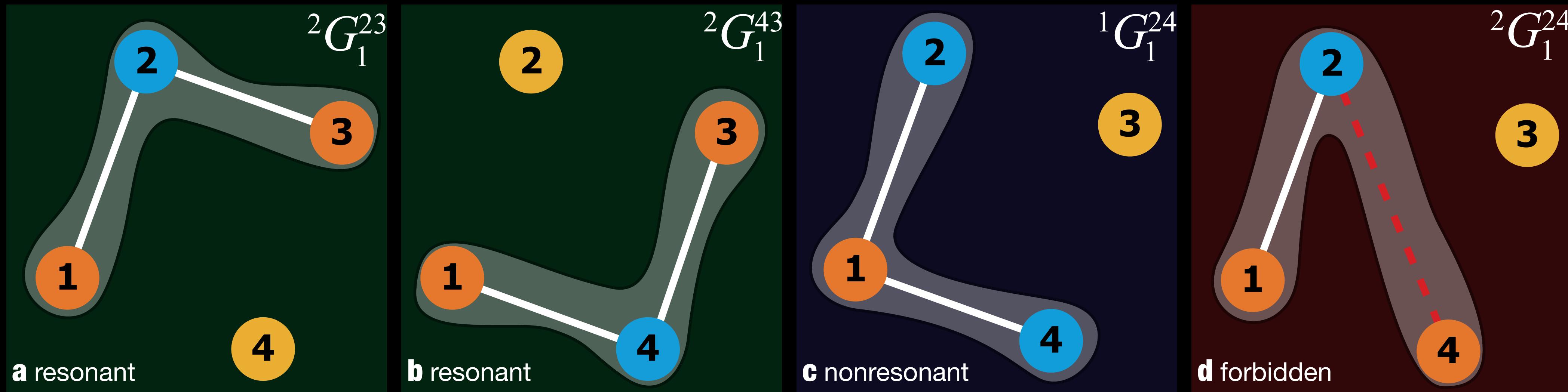
*rules for resonant and nonresonant terms*



Checking only for node-1

# NORMAL FORM THEORY

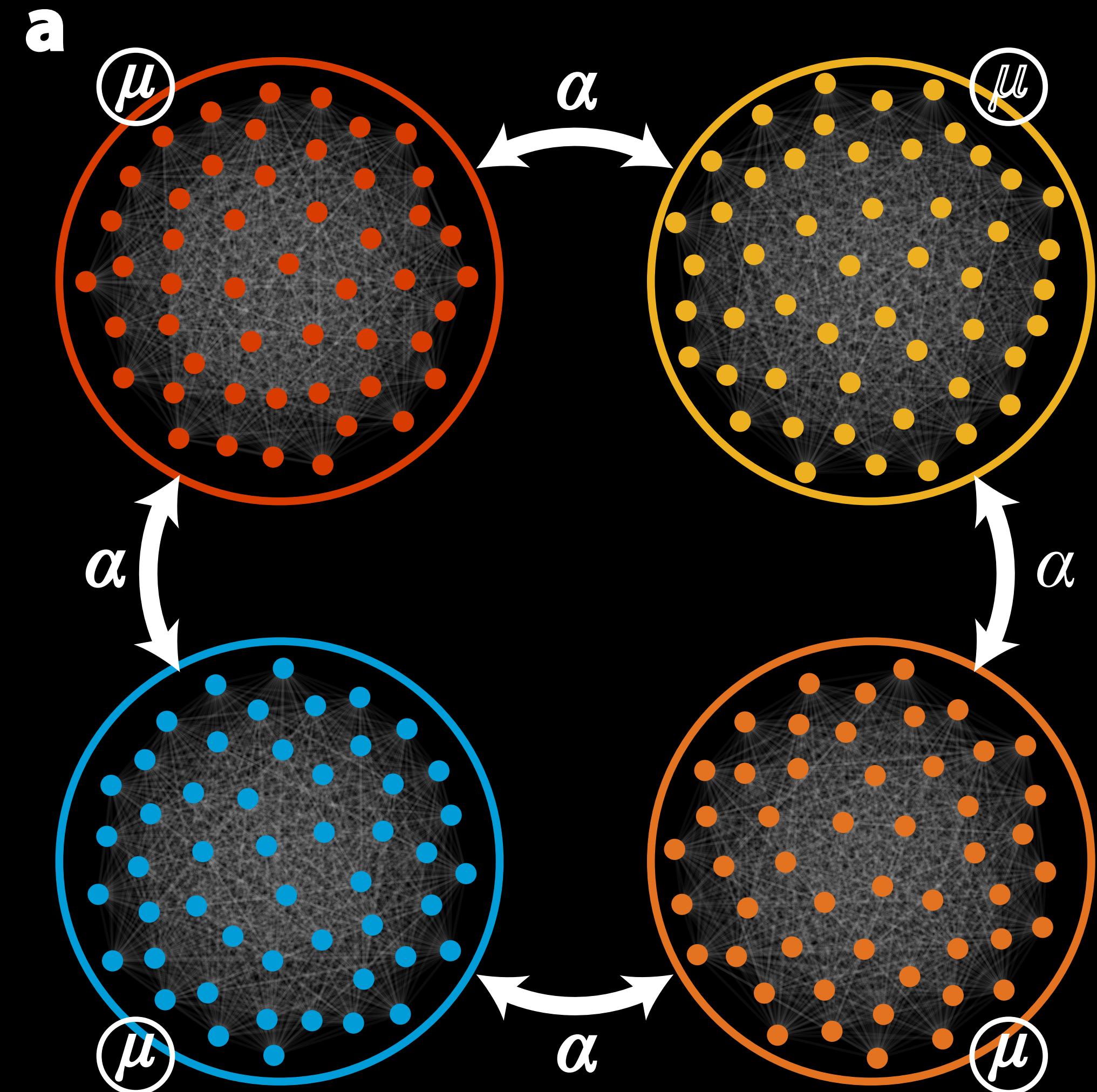
*rules for resonant and nonresonant terms*



Checking only for node-1

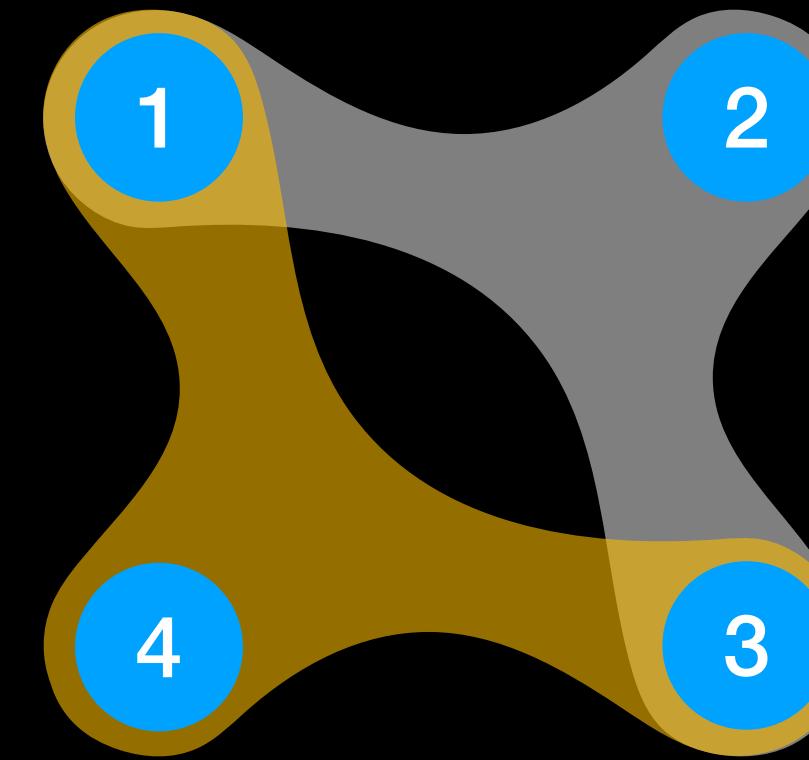
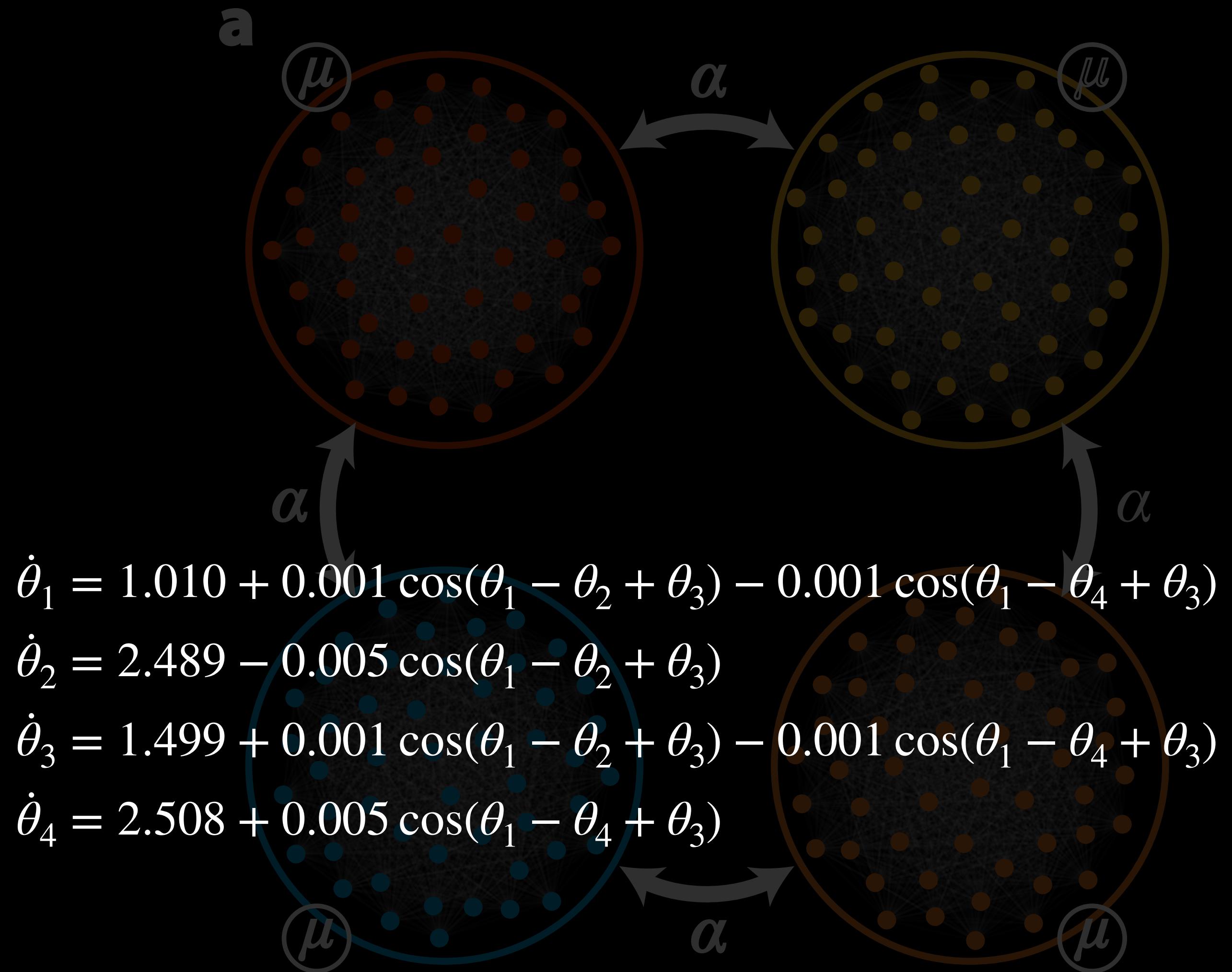
# EMERGENT HYPERNETWORK

*mean-field pops nonlinearity*



# EMERGENT HYPERNETWORK

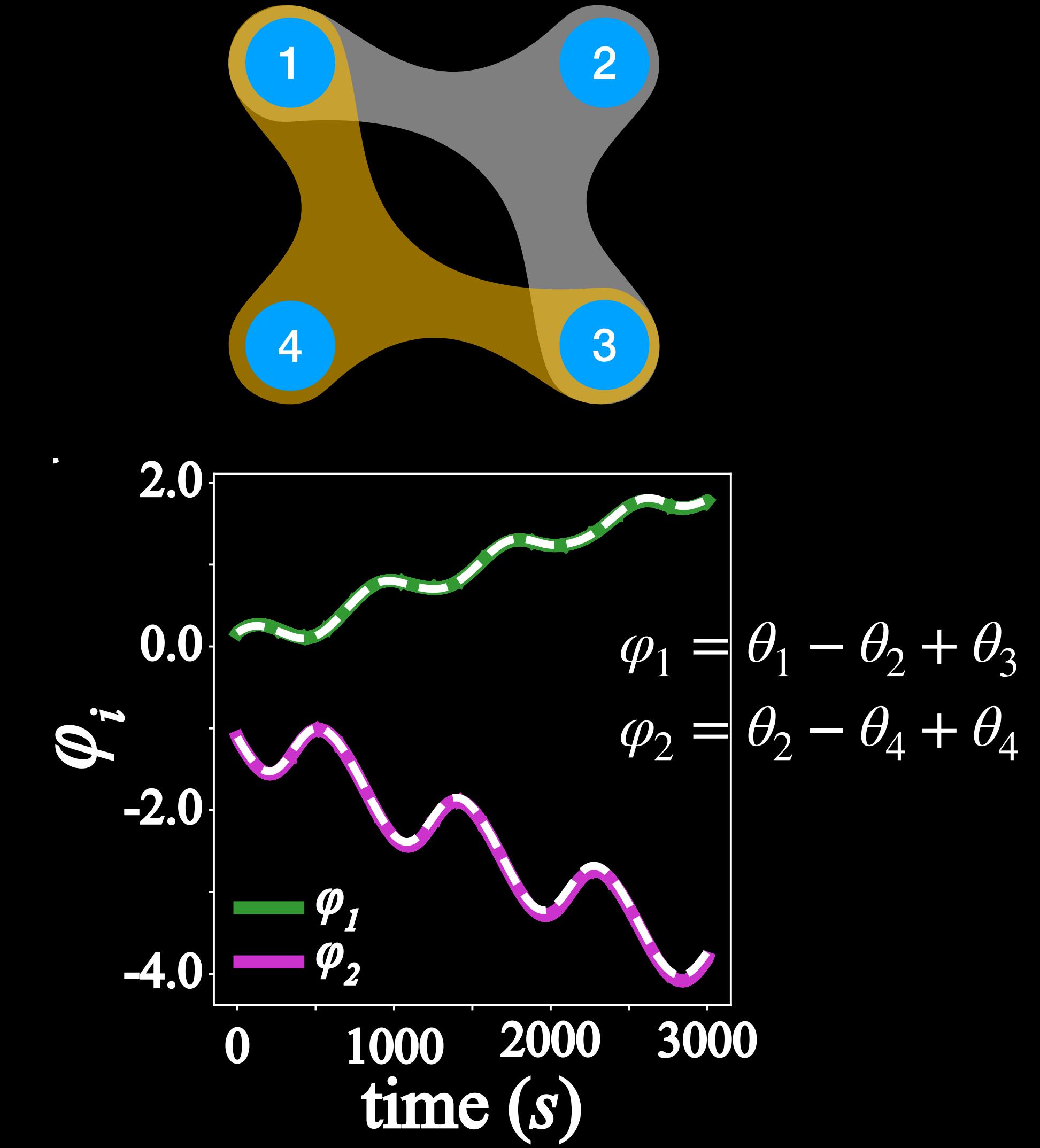
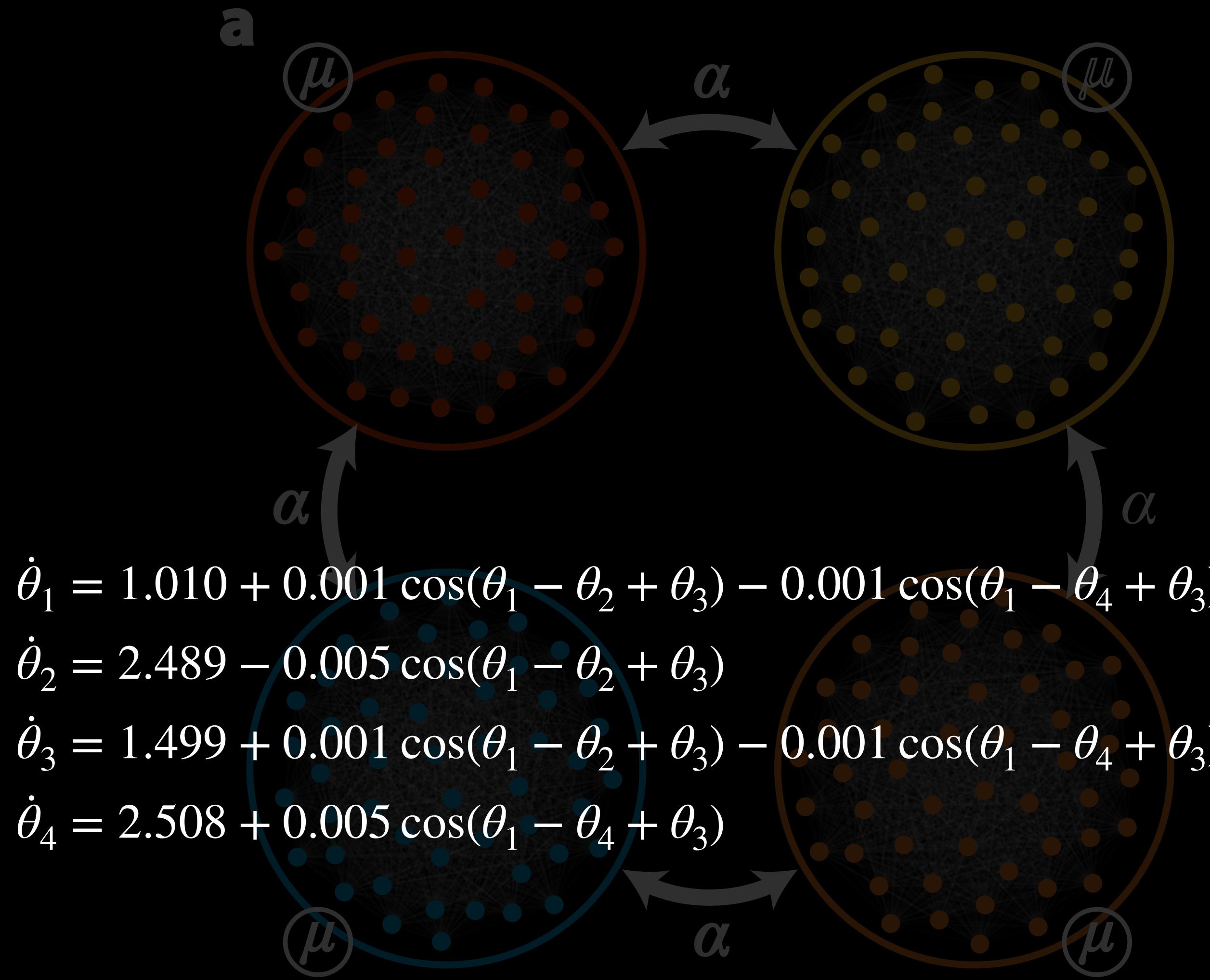
*mean-field pops nonlinearity*



$$\varphi_1 = \theta_1 - \theta_2 + \theta_3$$
$$\varphi_2 = \theta_2 - \theta_4 + \theta_4$$

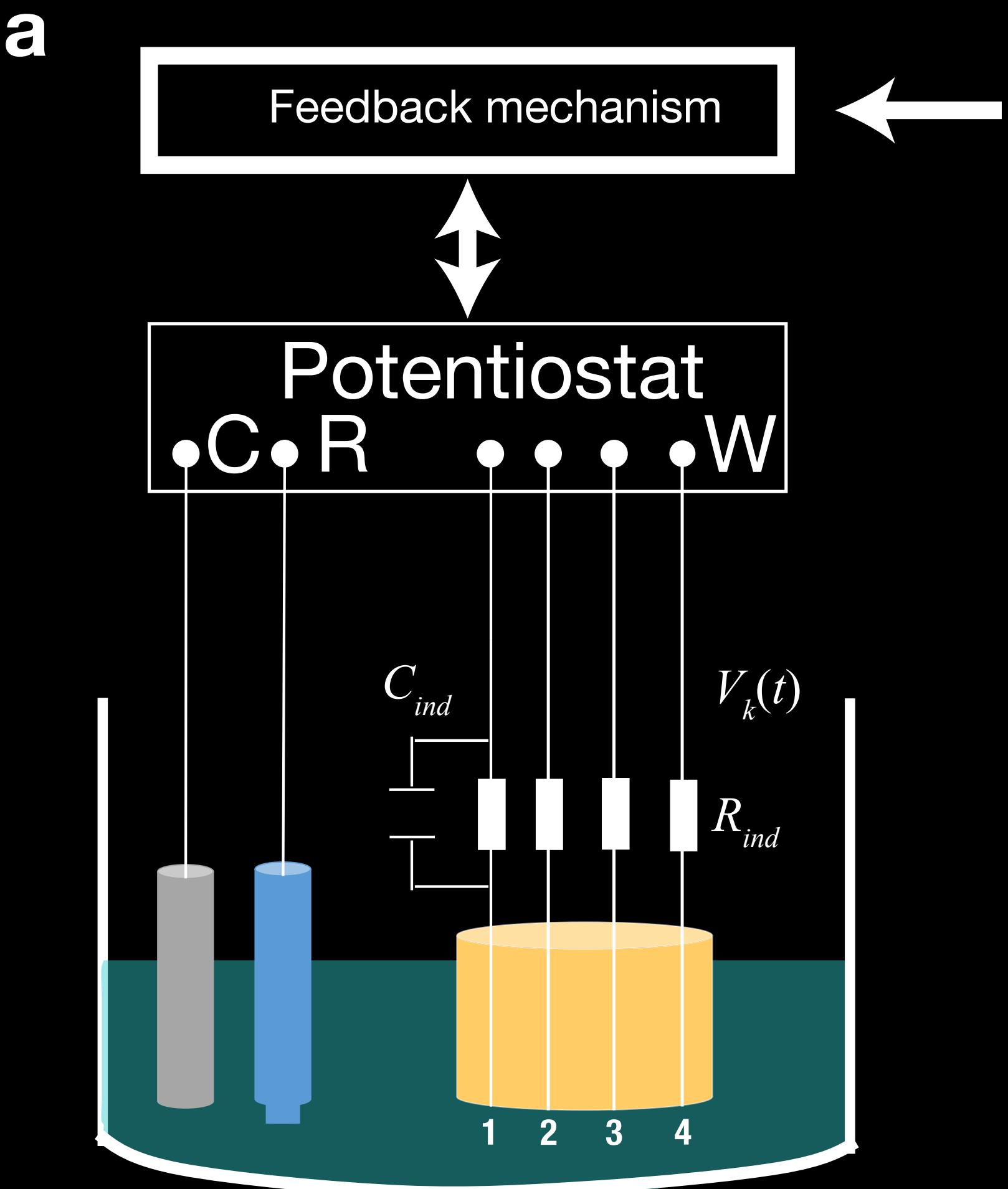
# EMERGENT HYPERNETWORK

*mean-field pops nonlinearity*



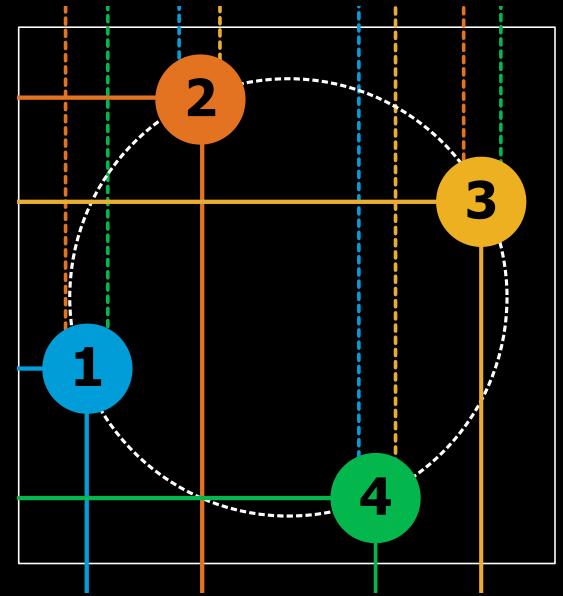
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



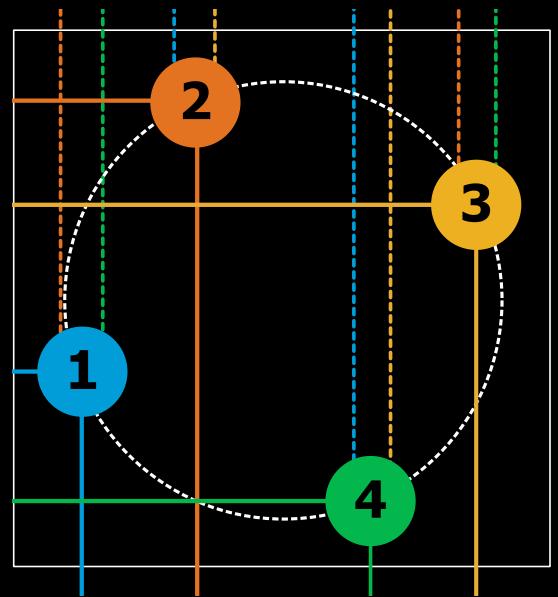
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



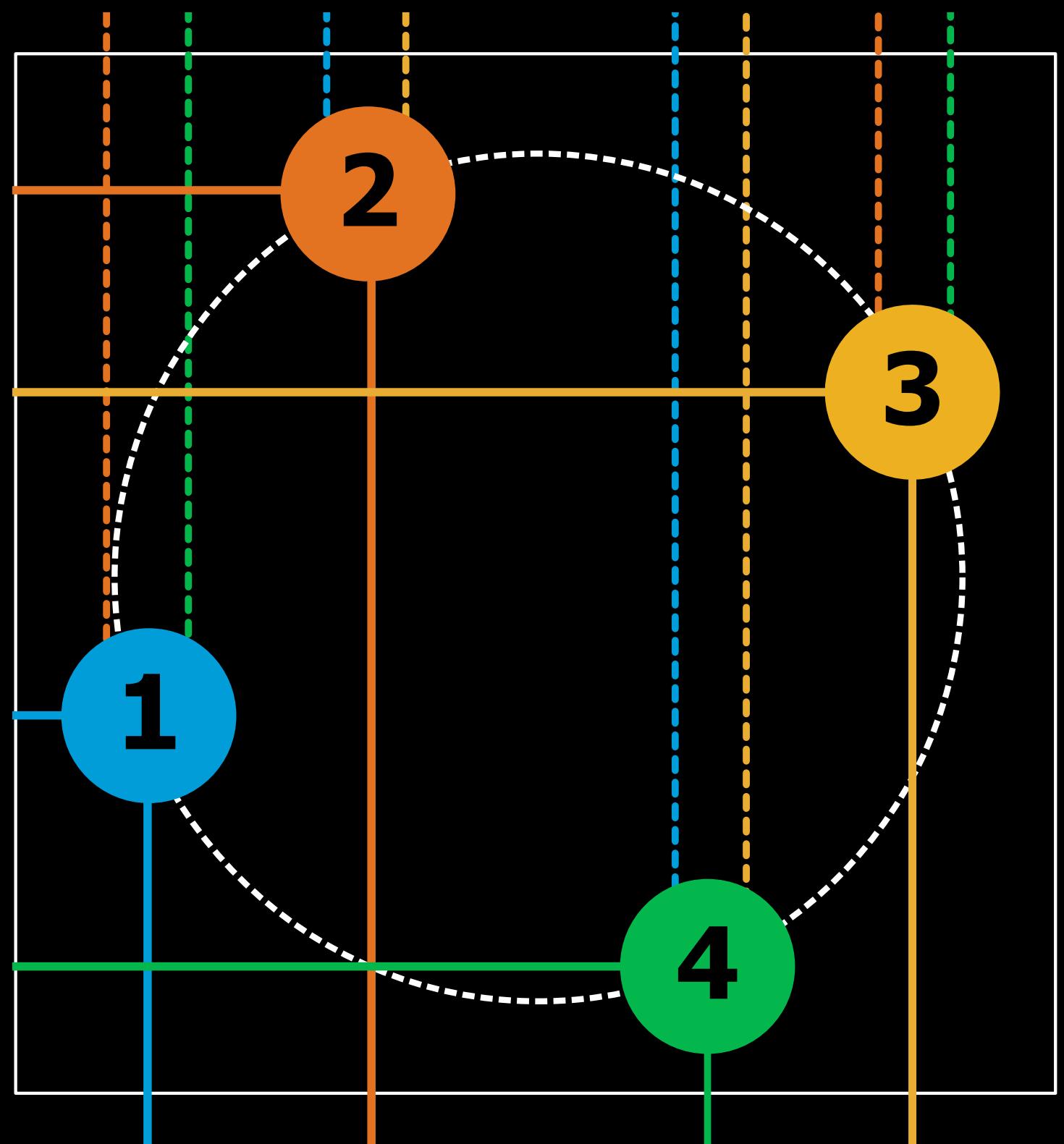
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



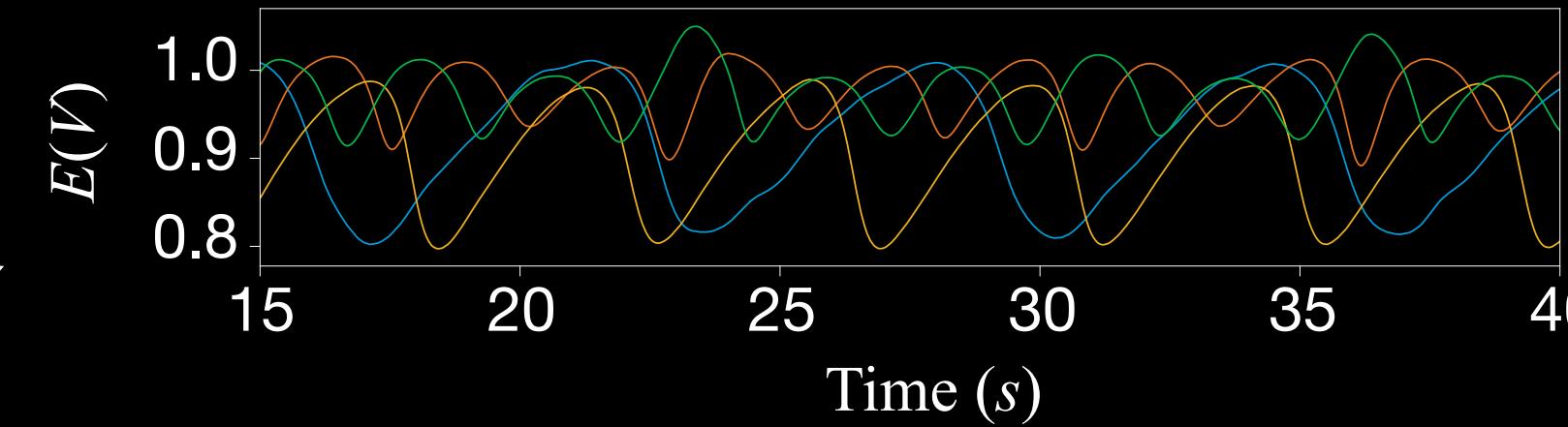
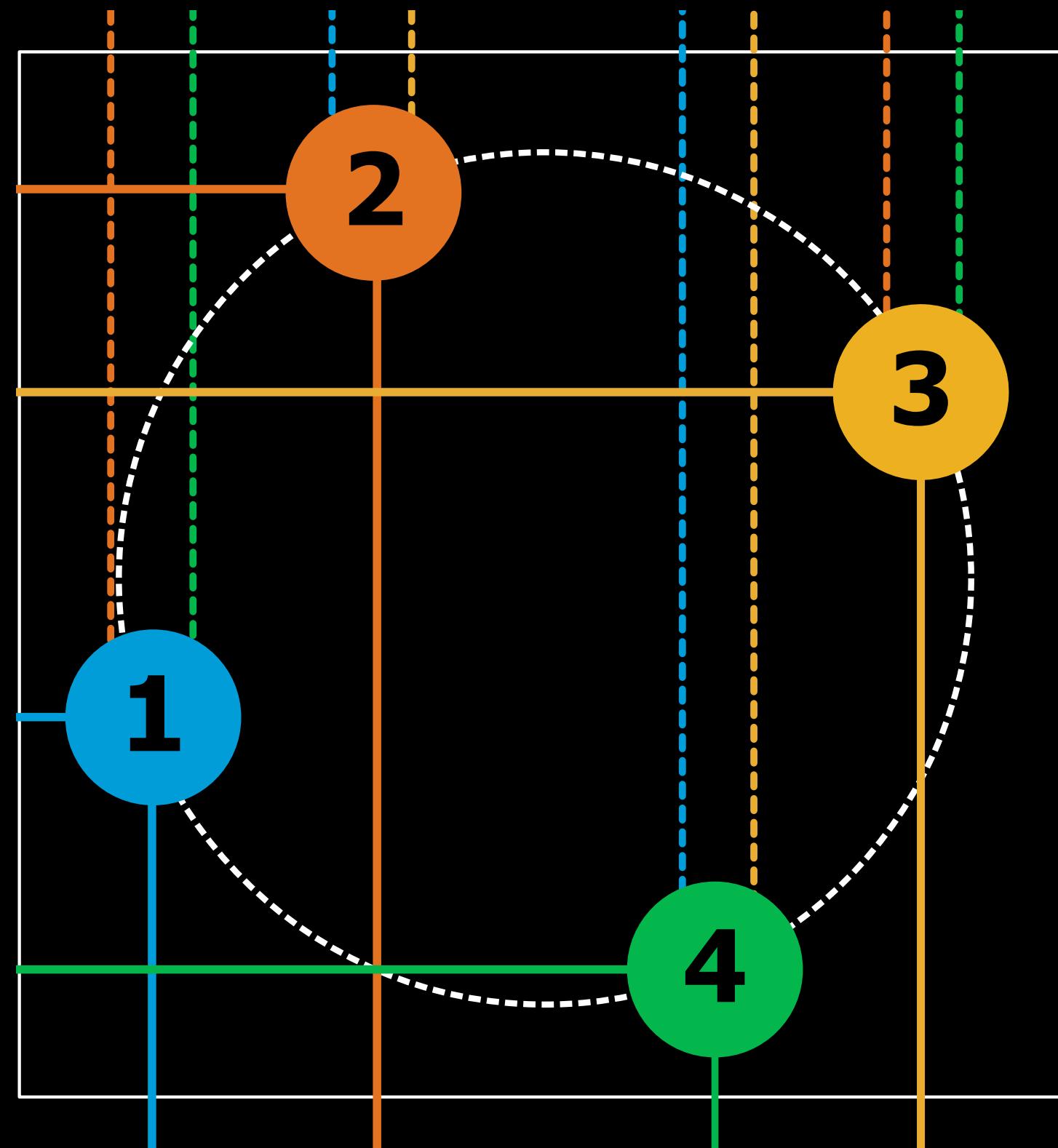
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



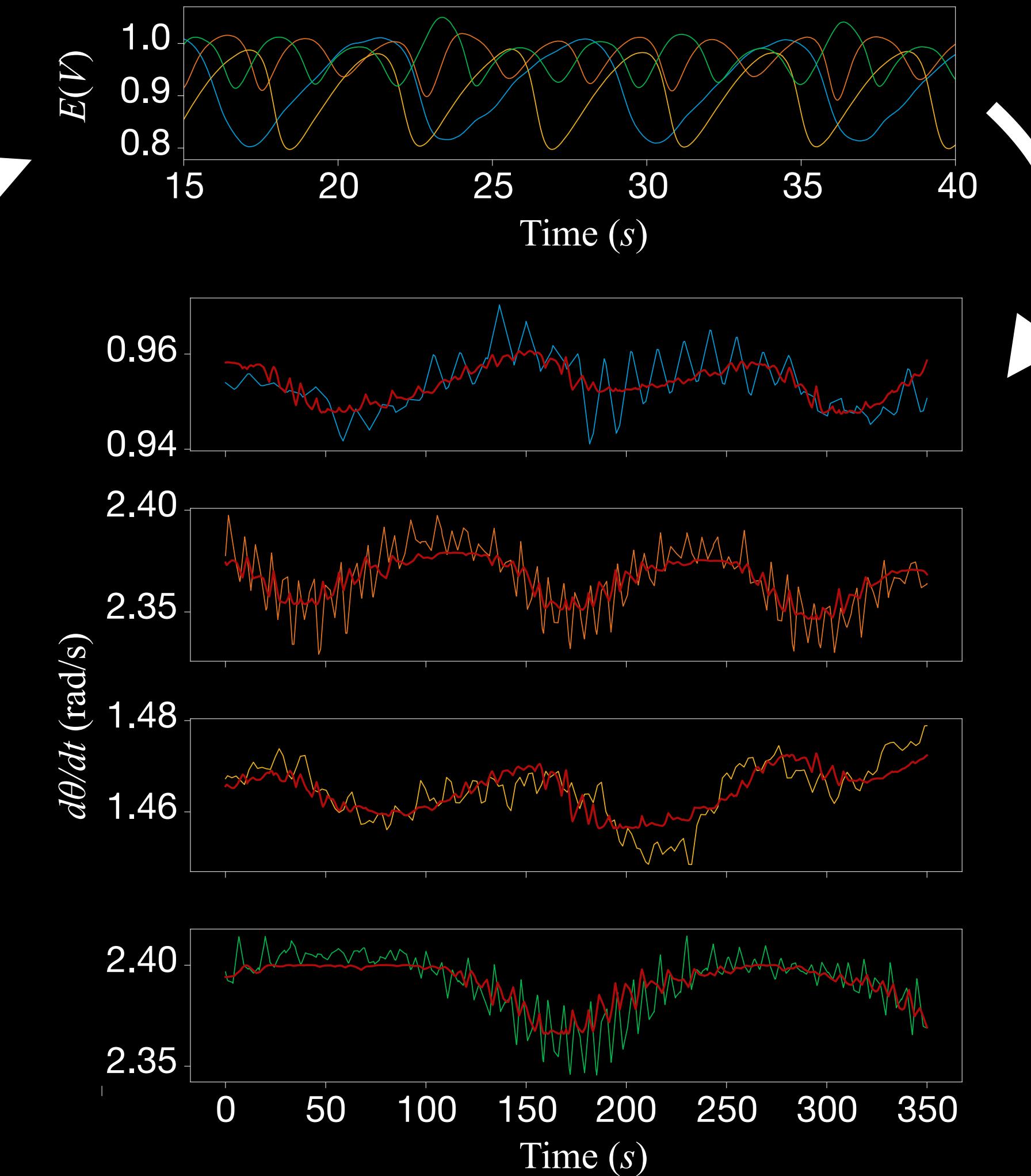
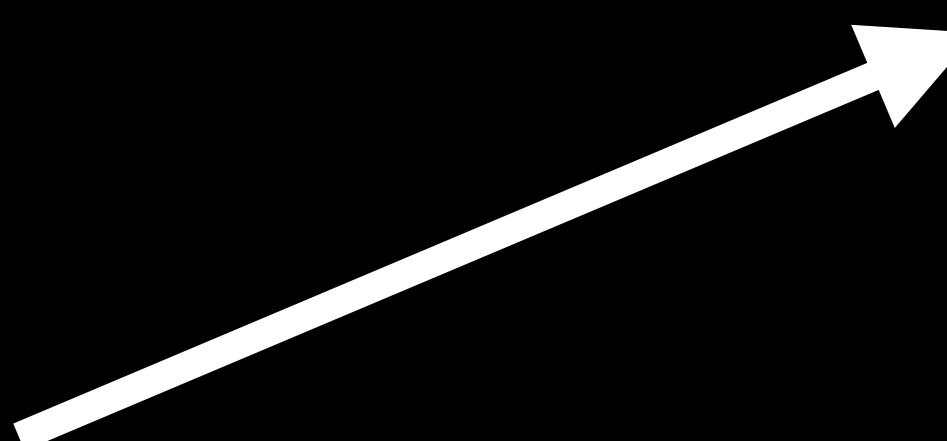
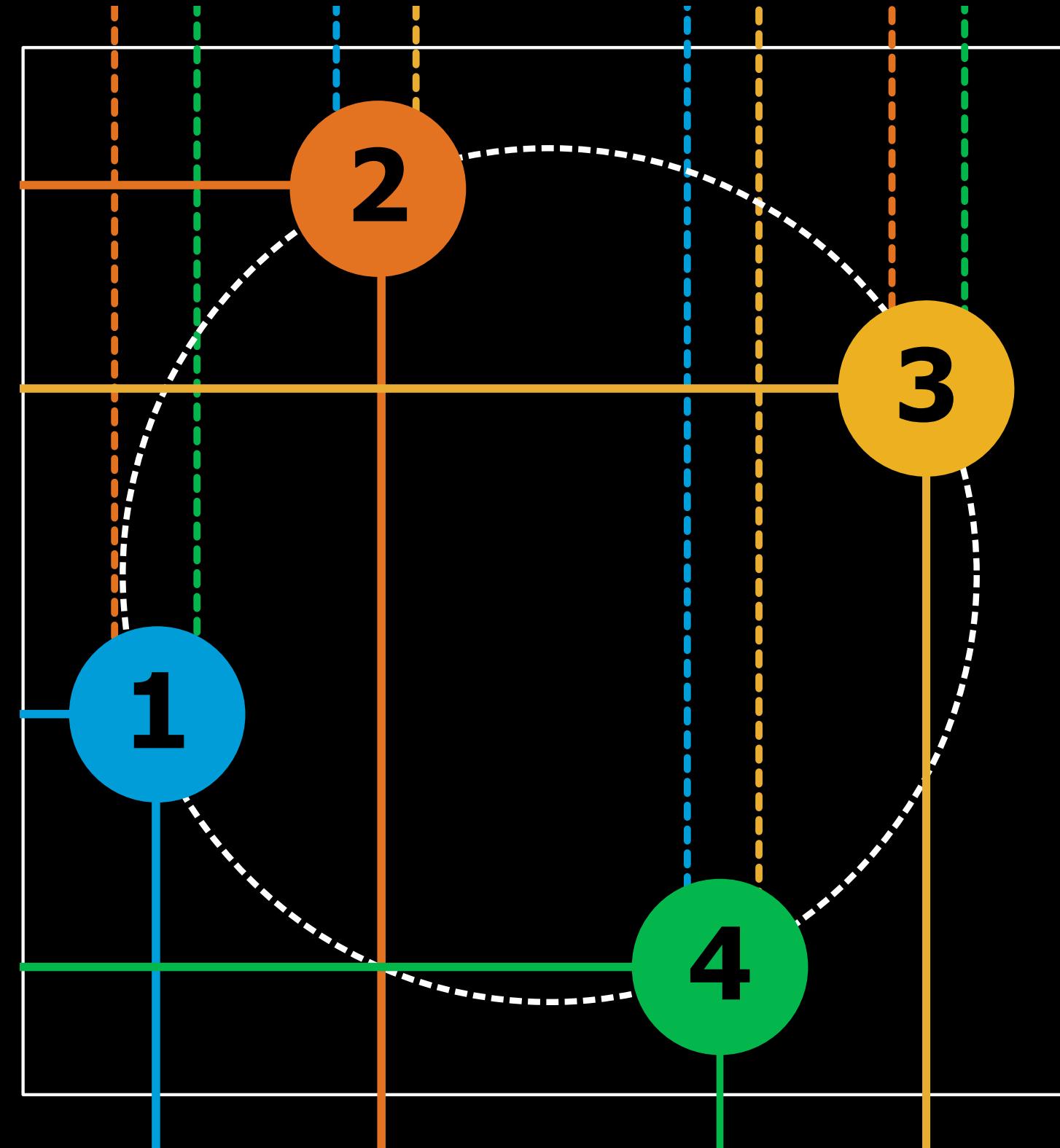
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



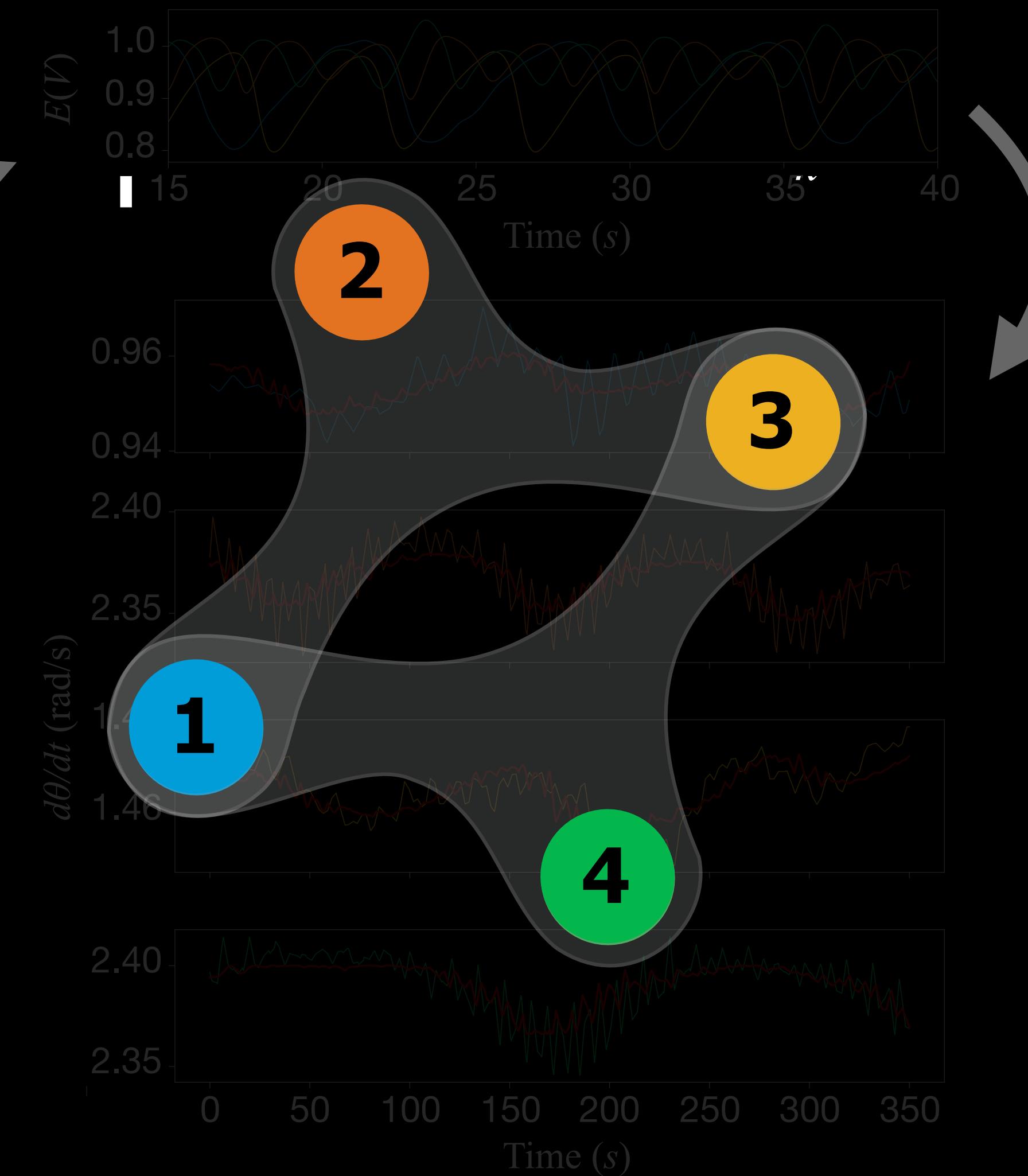
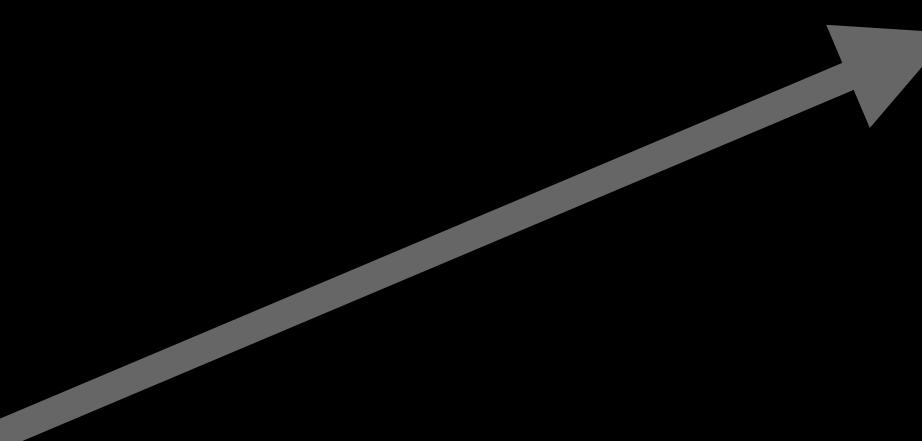
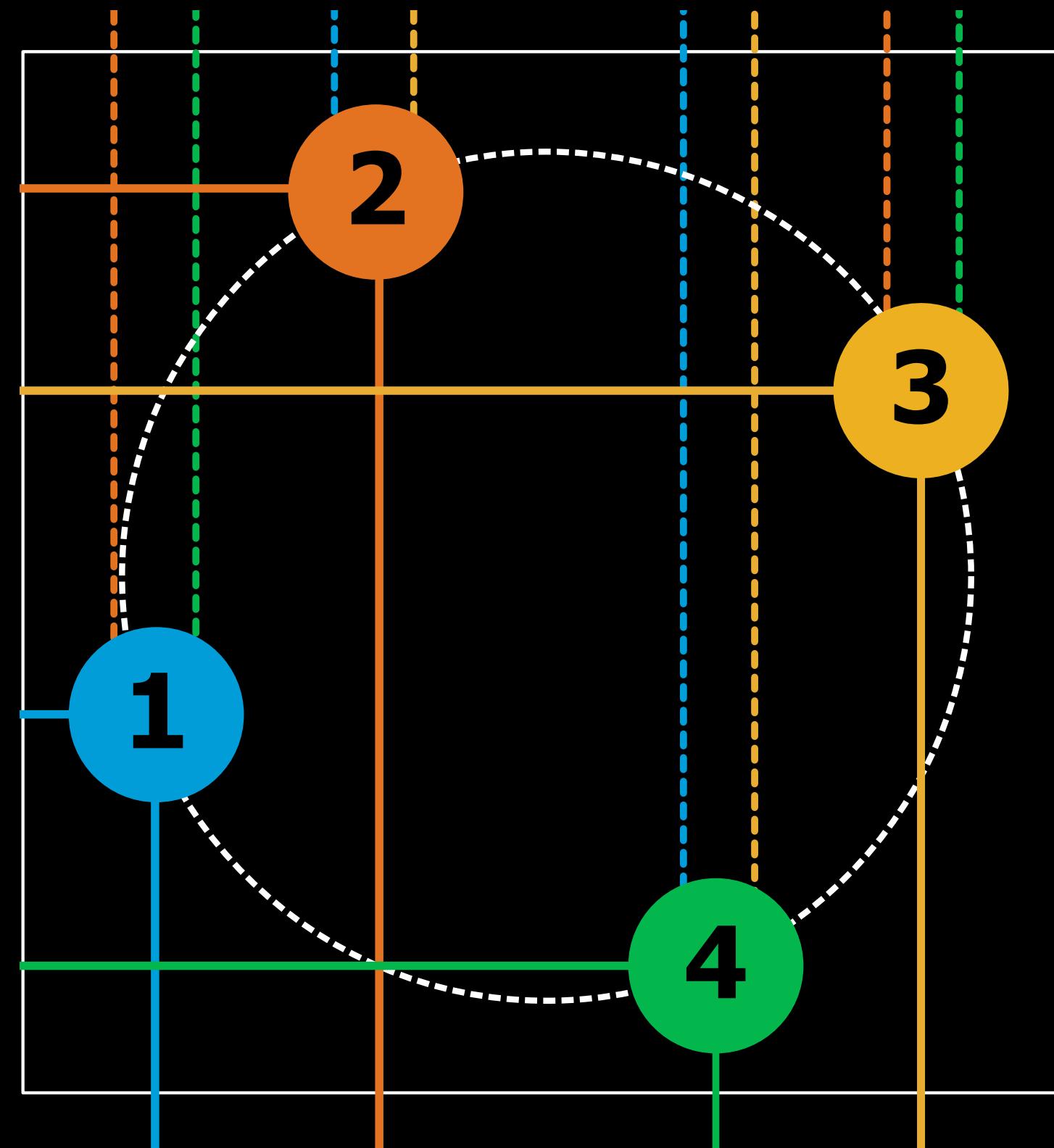
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



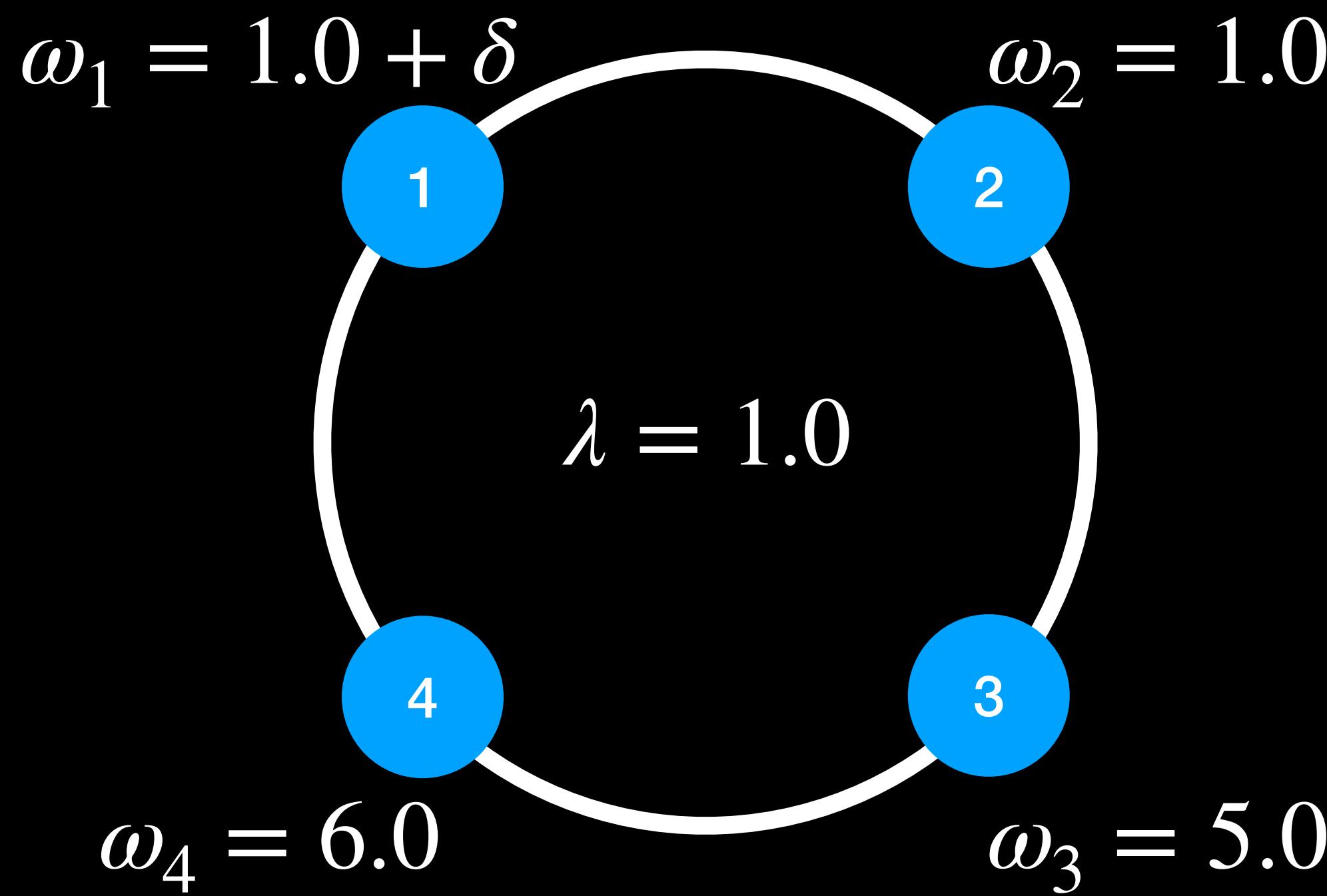
# REAL-WORLD EXPERIMENT

*electrochemical oscillators*



# SURPRISING PREDICTIONS!

*ring topology with quadratic coupling*



$$h(z, w) = z\bar{w}$$

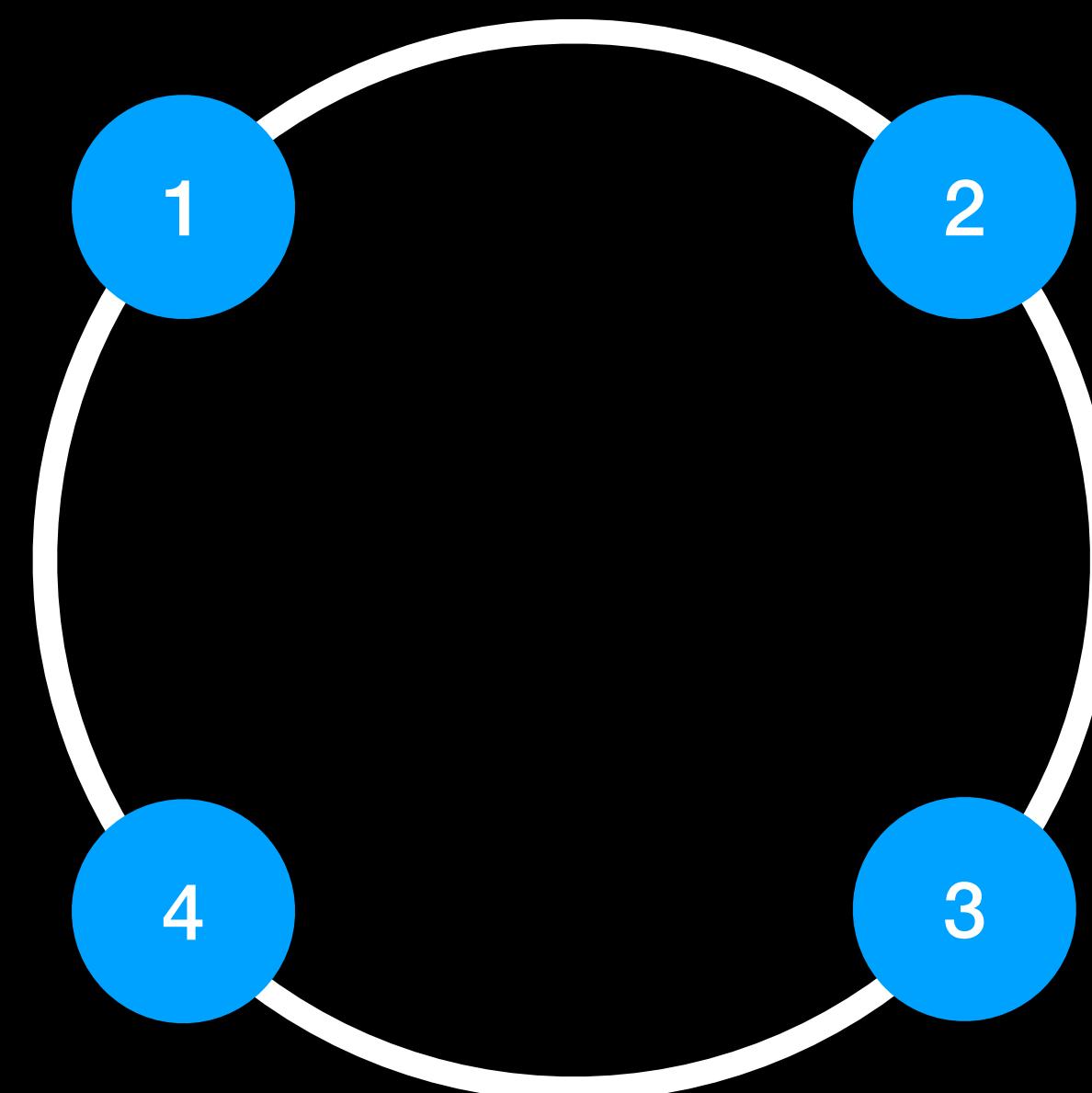
pairwise coupling function

Jacobian vanishes at the origin

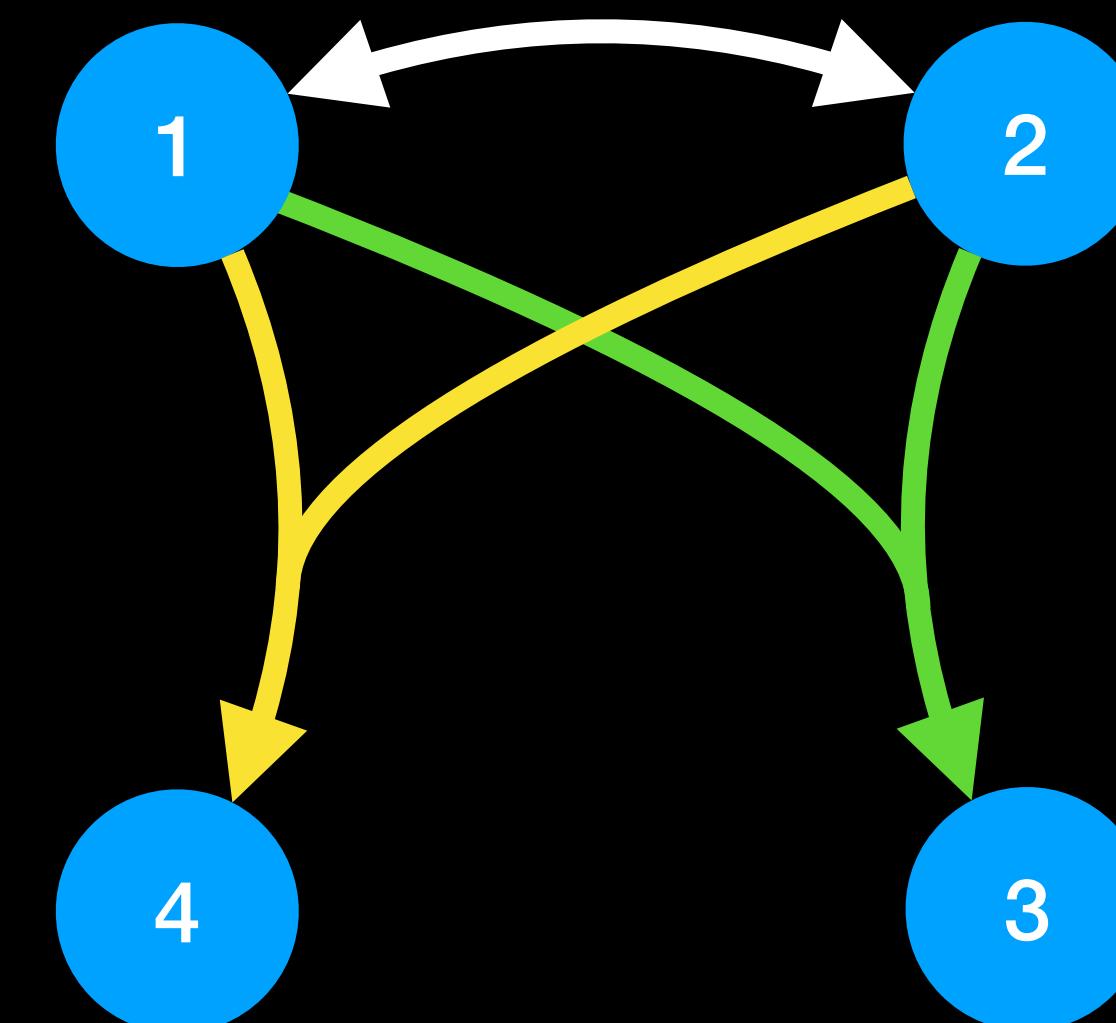
NOT a diffusive coupling!

# SURPRISING PREDICTIONS!

*ring topology to driven system*



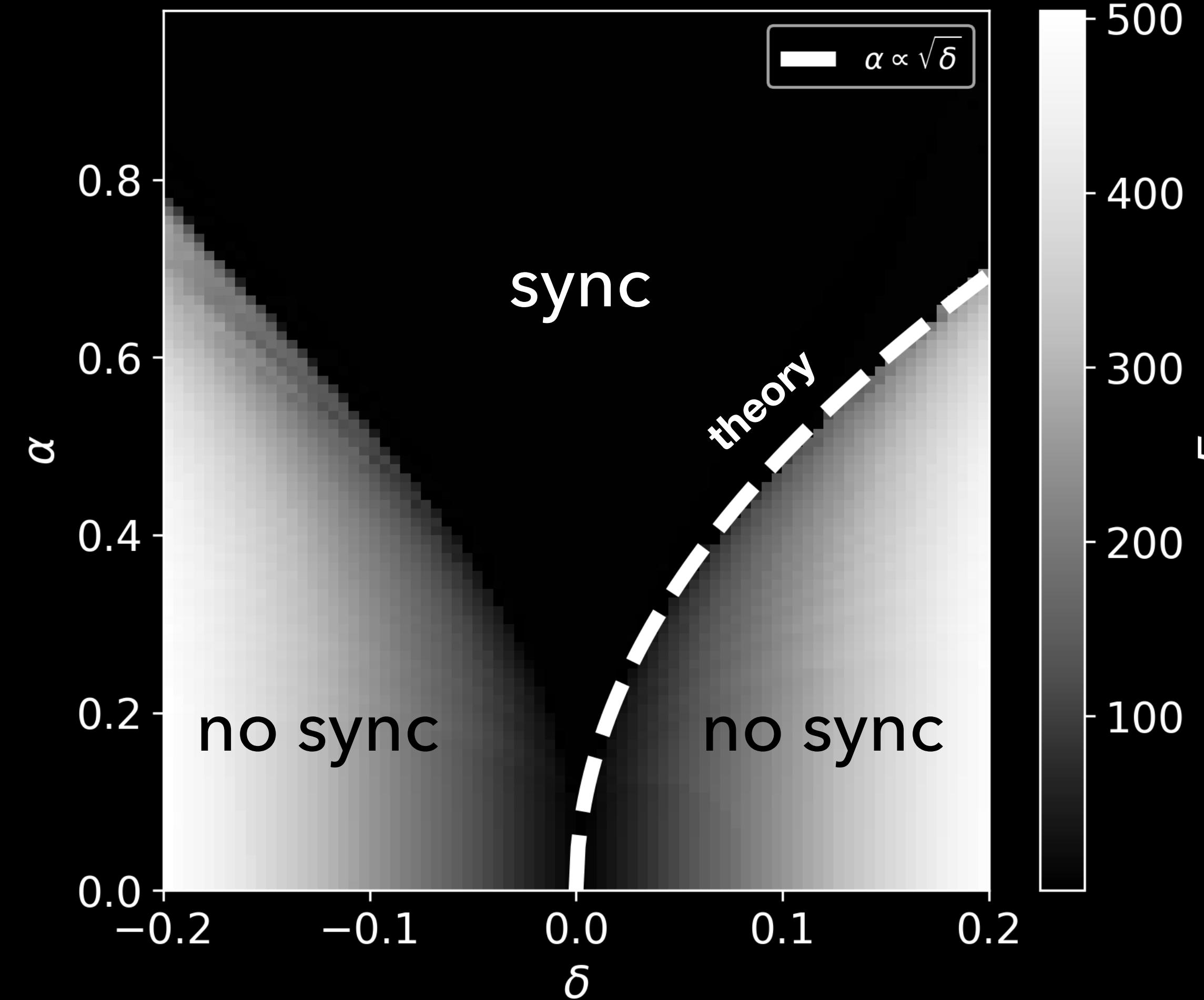
original network



reconstructed hypernetwork

# ANOMALOUS SYNC

*synchronization tongue*

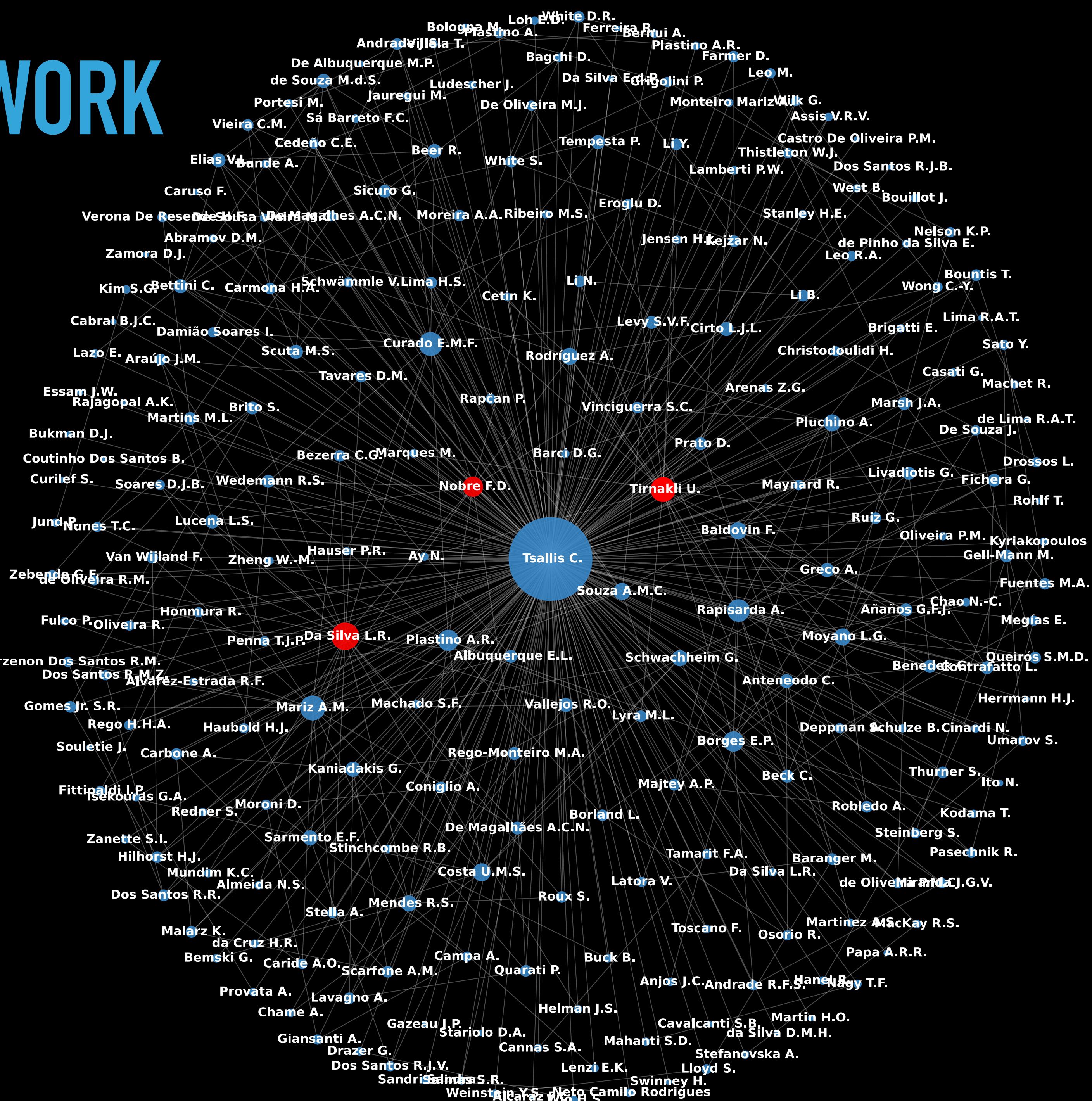


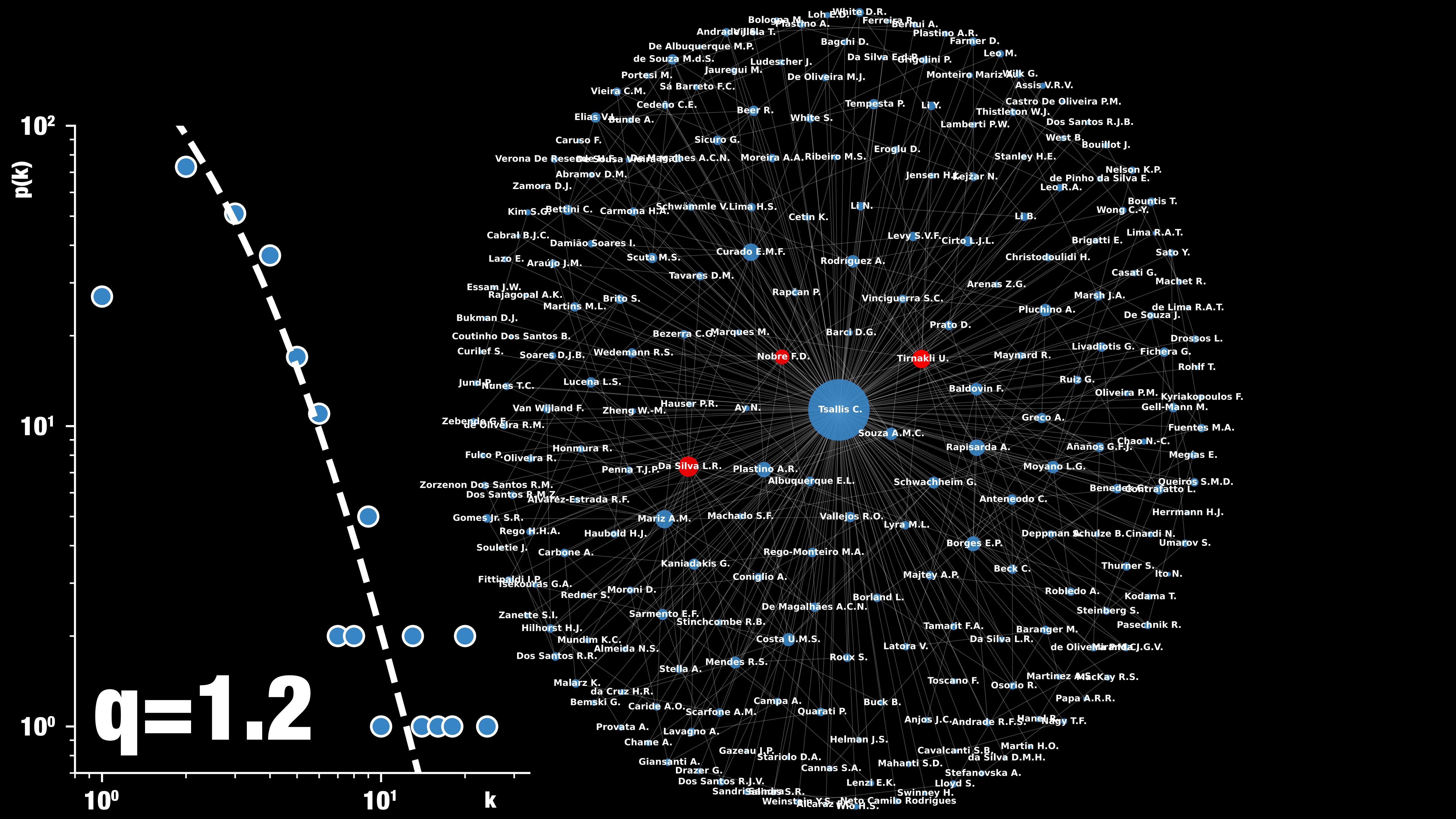
$$E = \frac{1}{T} \sum_{t=1}^T \phi(t)$$

I AM NOT ONLY RECONSTRUCTING BRAIN!

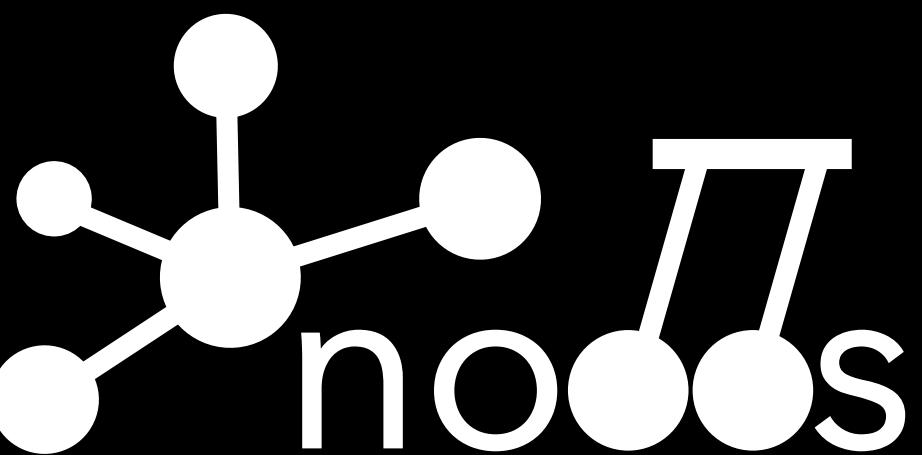
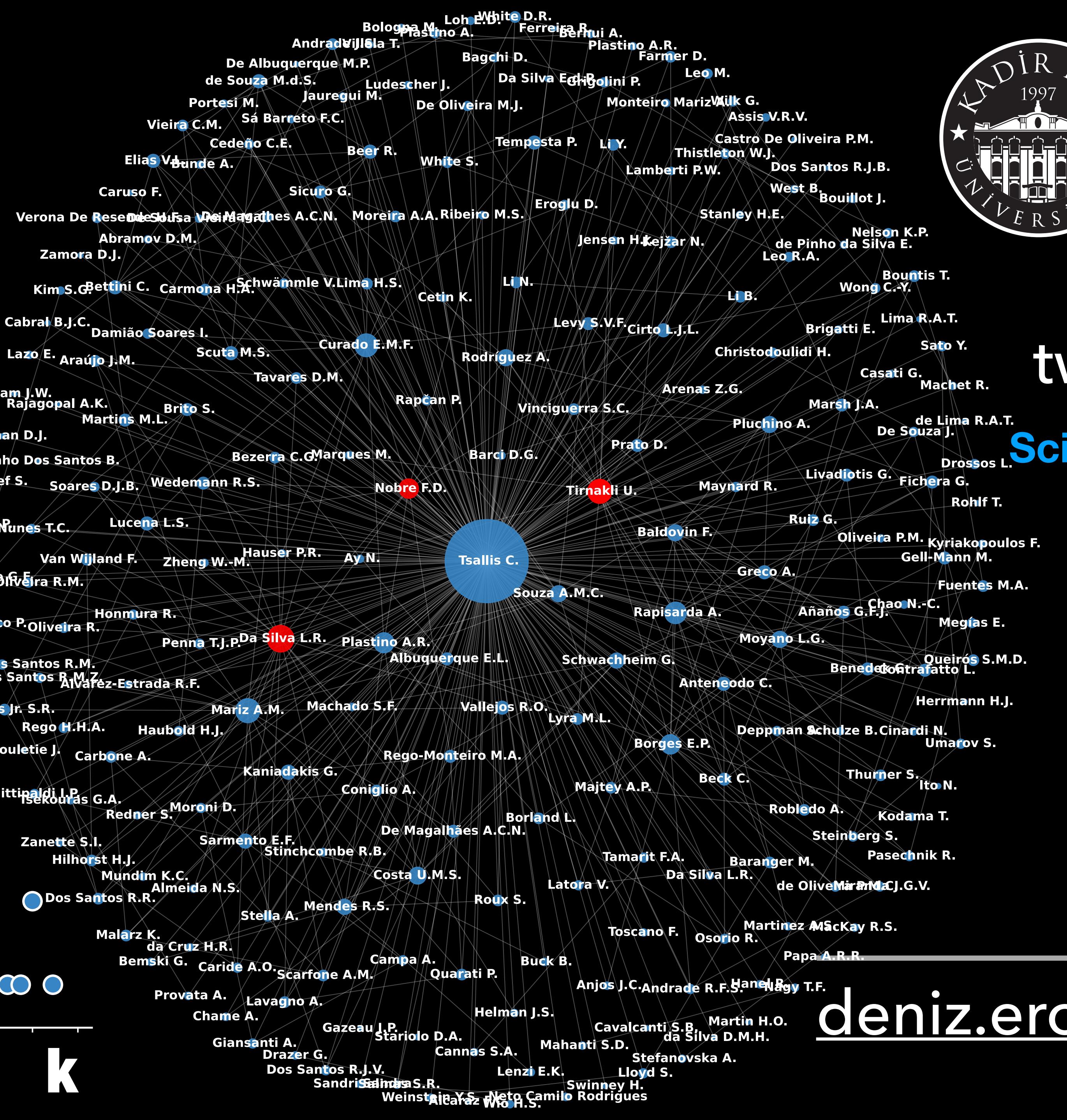
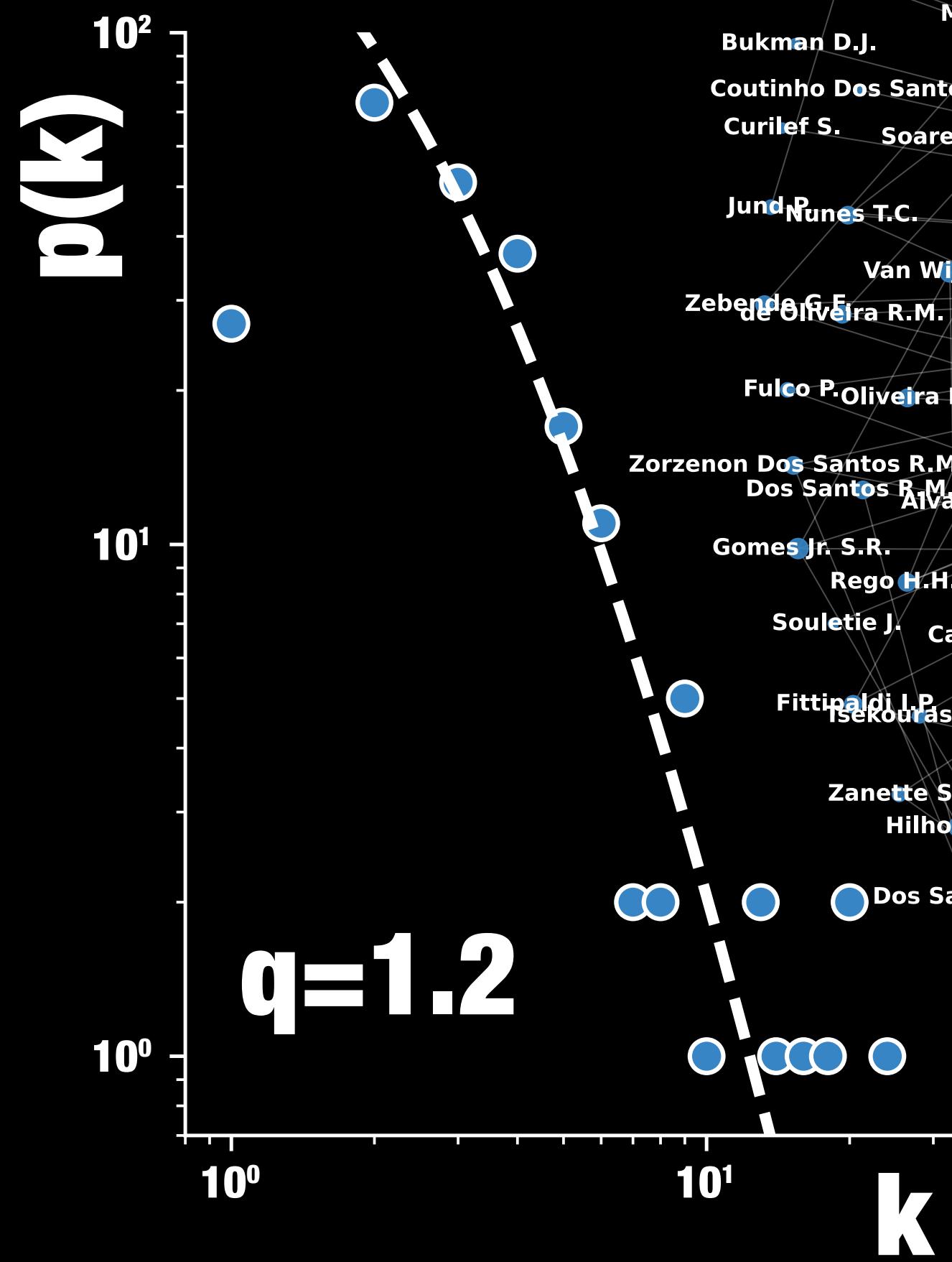
ALSO?

# TSALLIS NETWORK





# THANKS!



# **Network-Oriented Dynamics and Data Science**

**twitter: noddslab**

# Science Academy, TUBITAK ERC, EU MSCA CNPq, FAPESP NSF

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**FINALLY!!**

