Awesome to be next to cool people!

2010

Awesome to be next to cool people!

2010

Awesome to be next to cool people!


2010 - GrTr Conf. on Stat. Mech. and Dyn. Sys. [Turunc - Rhodos]
Anyway, everybody super cool here!


INSIDE
34


INSIDE
34


34

Statistical Mechanics for Complexity

The 80th BD of Prof. Constantino Tsallis

Rio de Janeiro, Nov 6-10, 2023

Network-Oriented Dynamics and Data Science

nodds

KADİR HAS ÜNİVERSİTESİ 1997

Deniz Eroglu

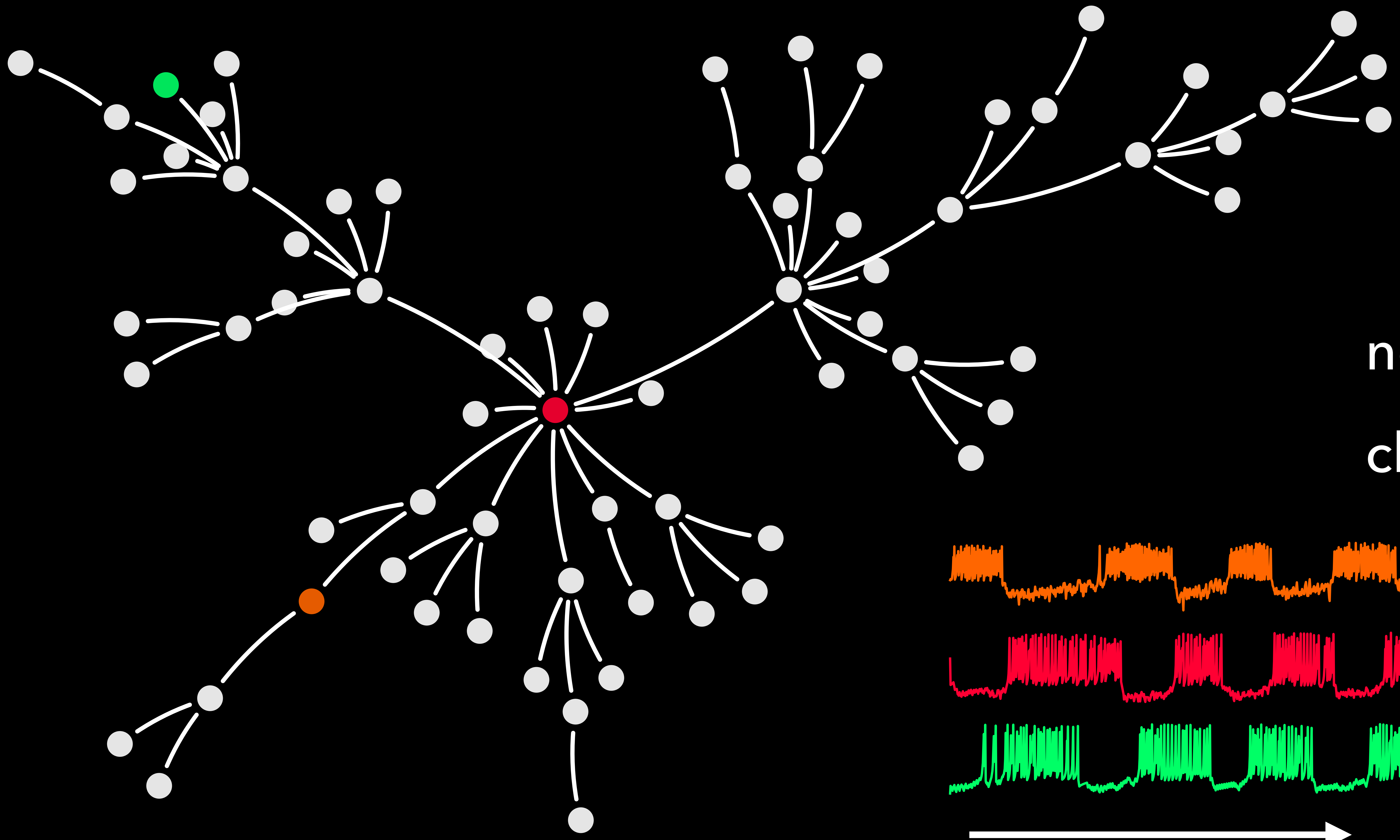
DATA-DRIVEN NETWORK DYNAMICS RECONSTRUCTION AND PREDICTION

EMERGENT HIGHER-ORDER INTERACTIONS AND CRITICAL PHENOMENA


neuron networks
climate networks etc.
time

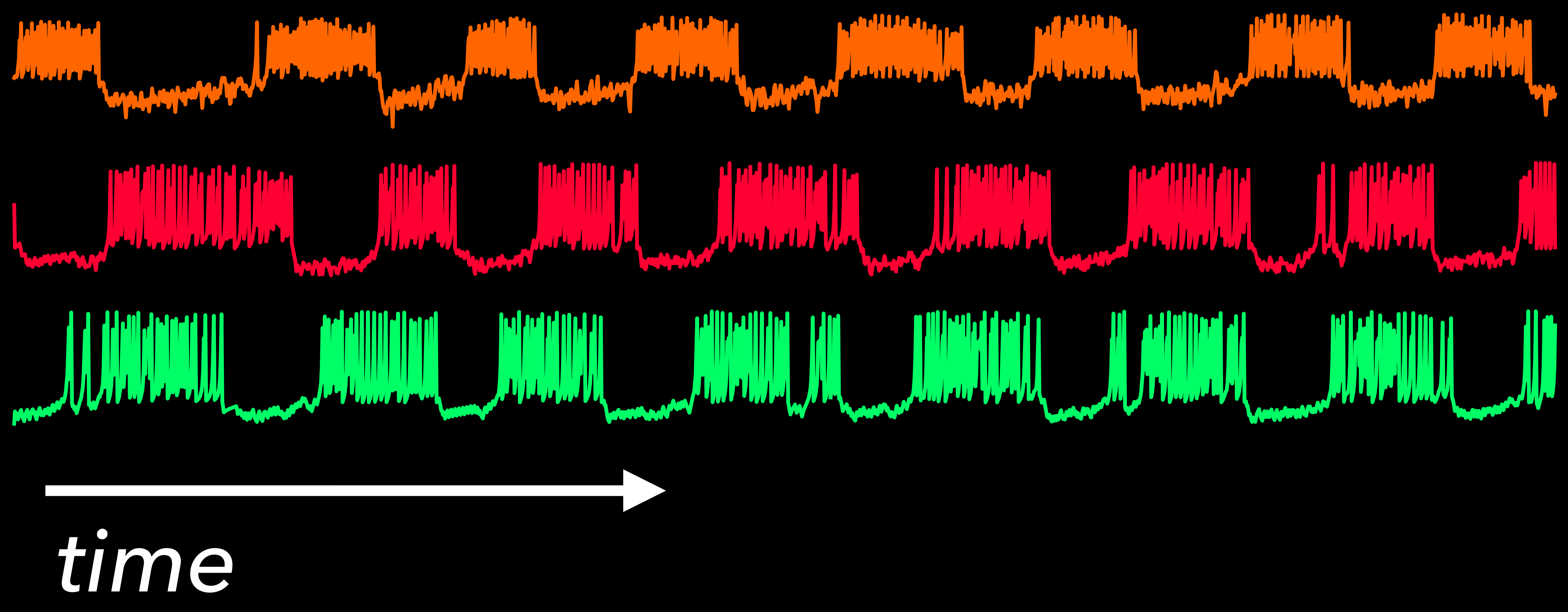
INVERSE PROBLEM

create an oscillator network with (G,f,H) to construct the data



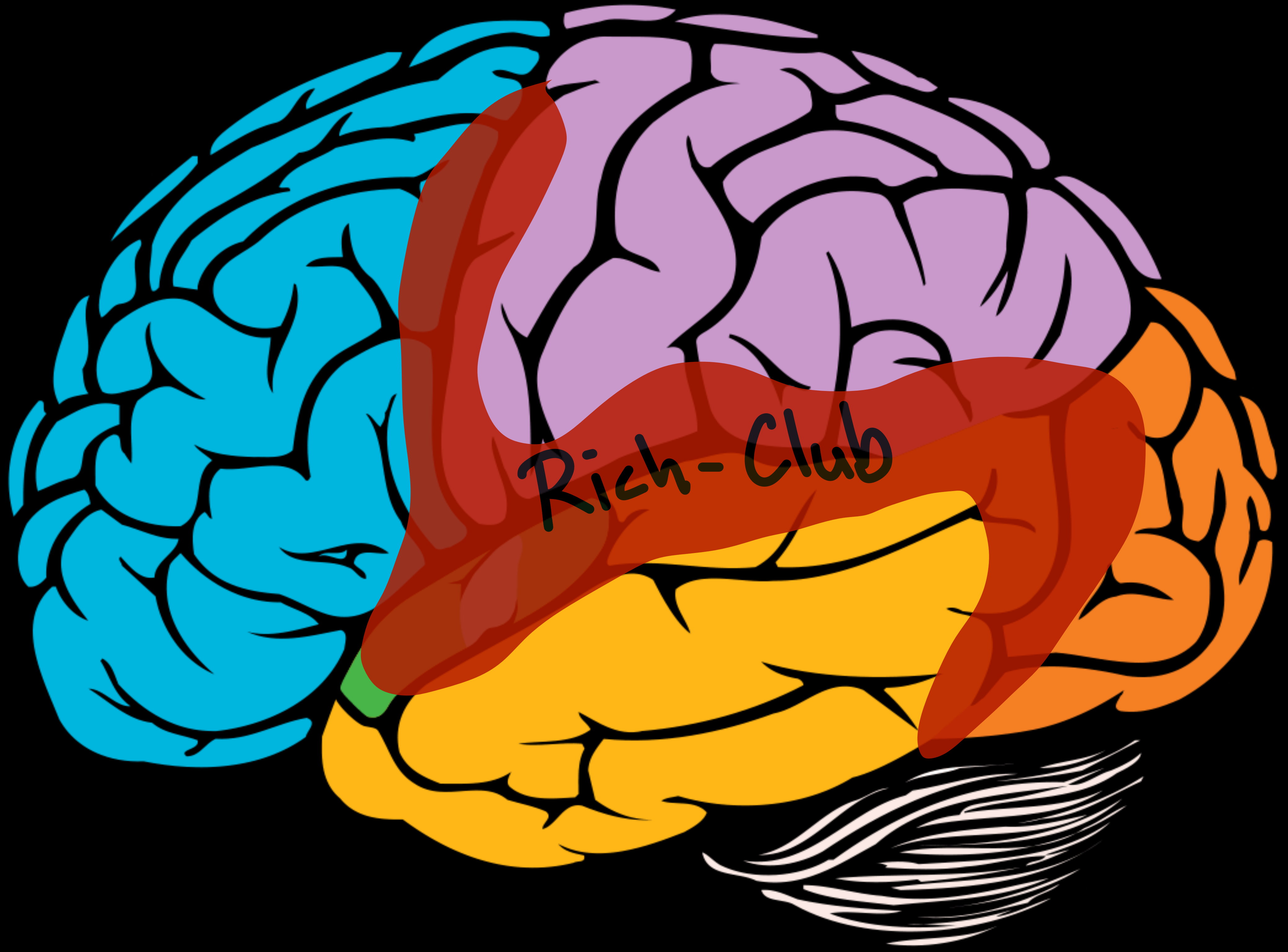
neuron networks

climate networks etc.



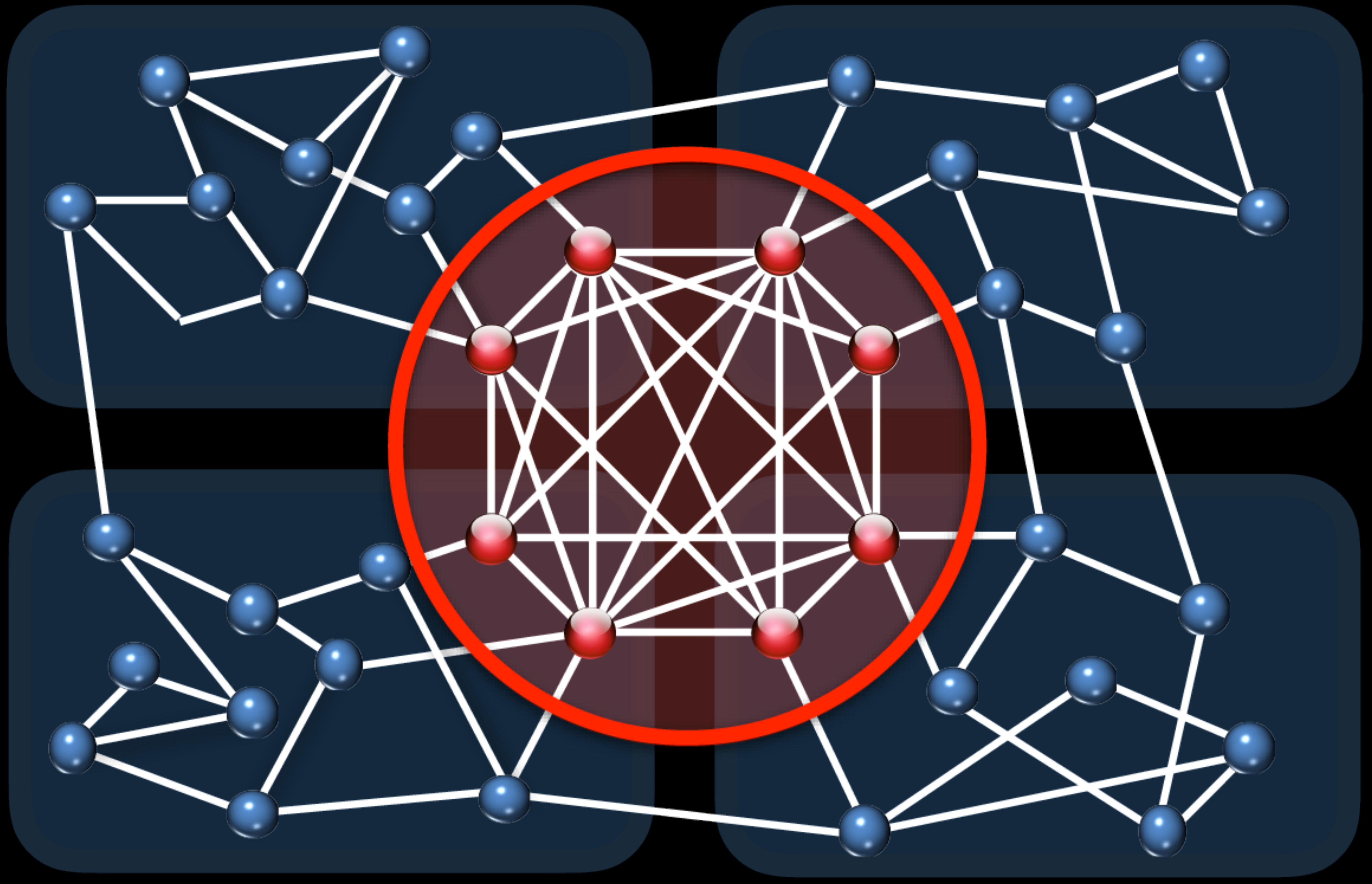
BRAIN

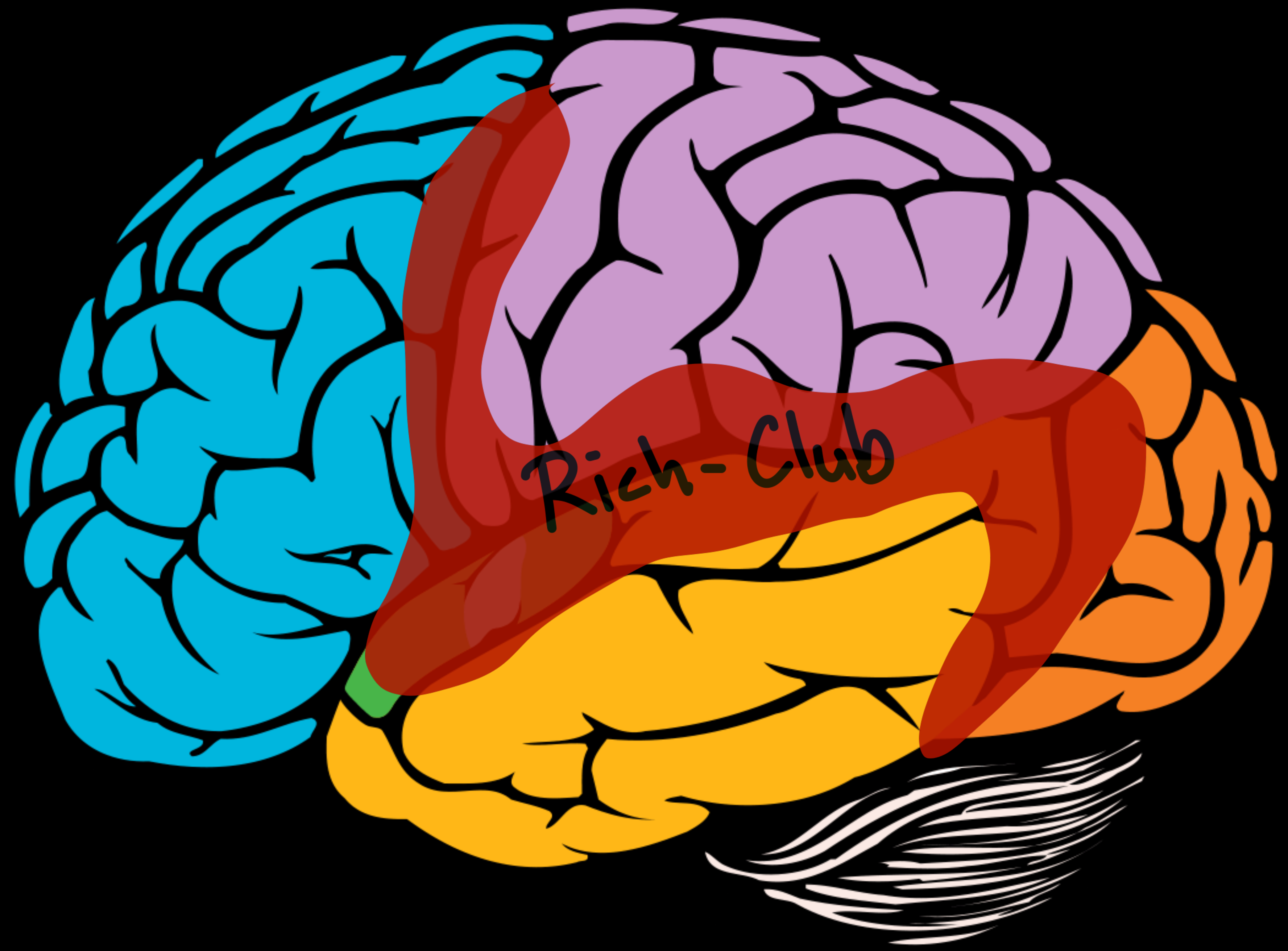
clusters



WHAT WE WANT TO SOLVE

rich-club structures



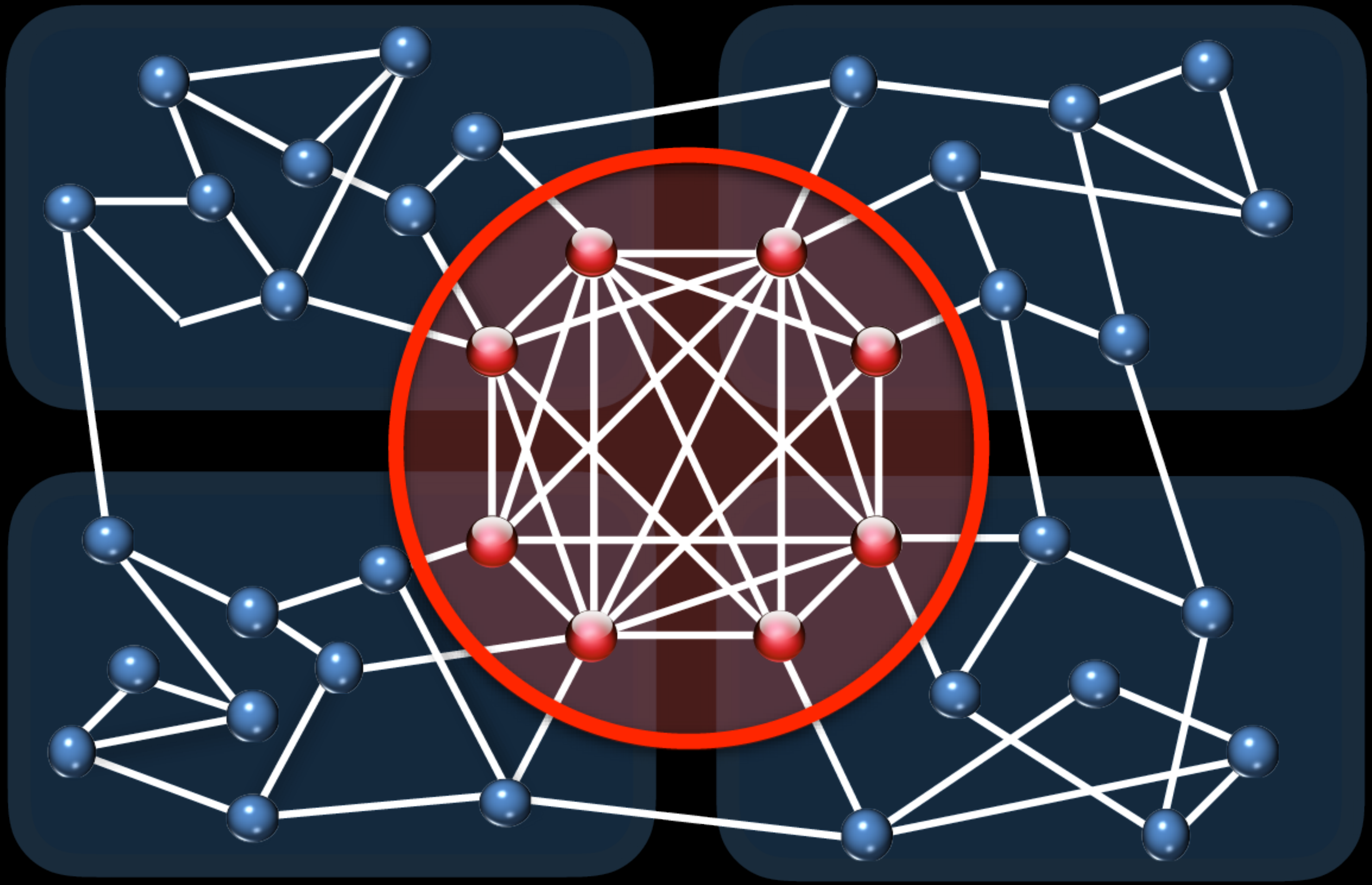


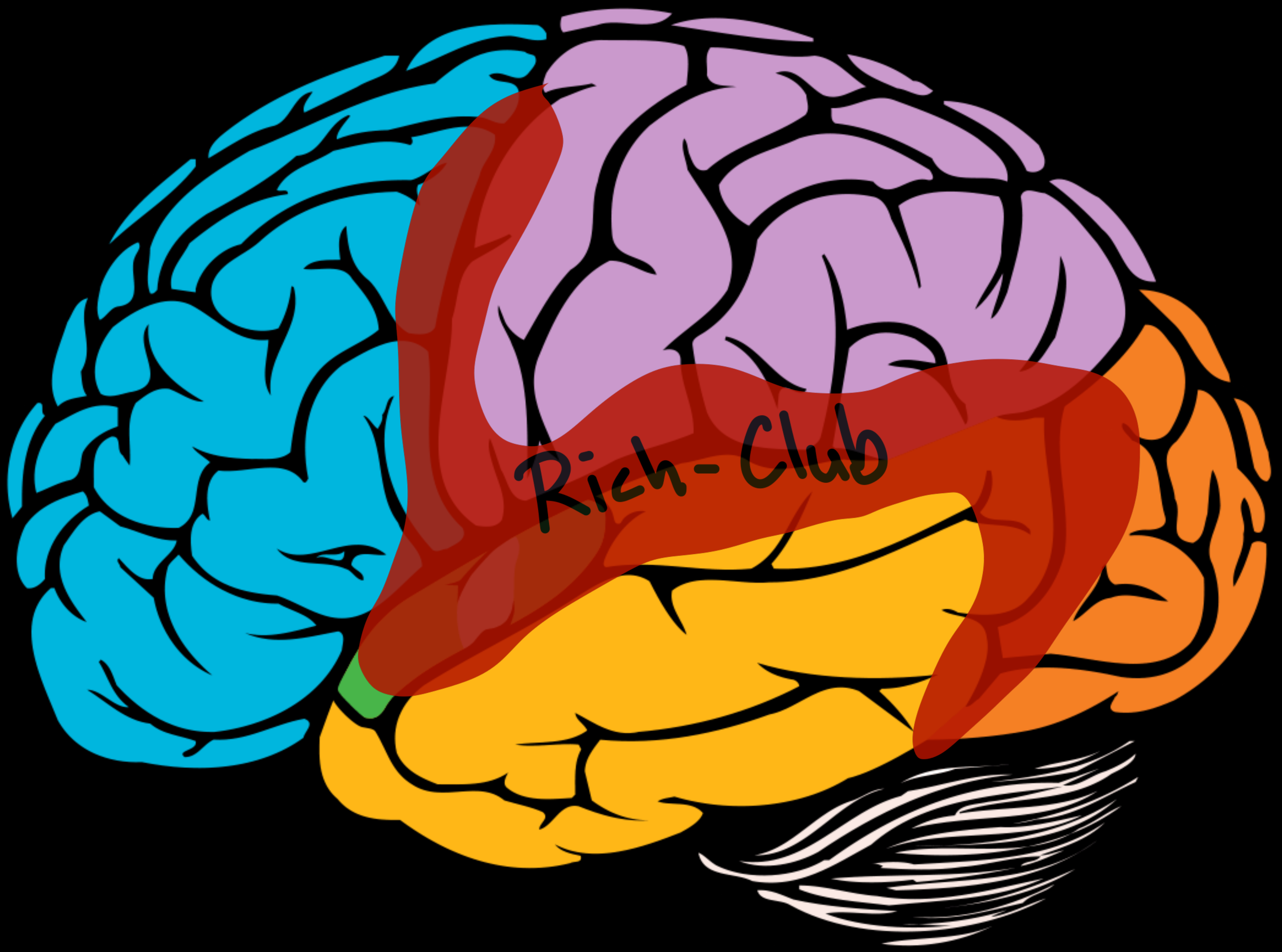
Scannell, J. W. & Young, M. P. *Curr.*(1993)
Van Den Heuvel, M. P., & Sporns, O. (2011)

WHAT WE WANT TO SOLVE

rich-club structures

time series = local dynamics + coupling





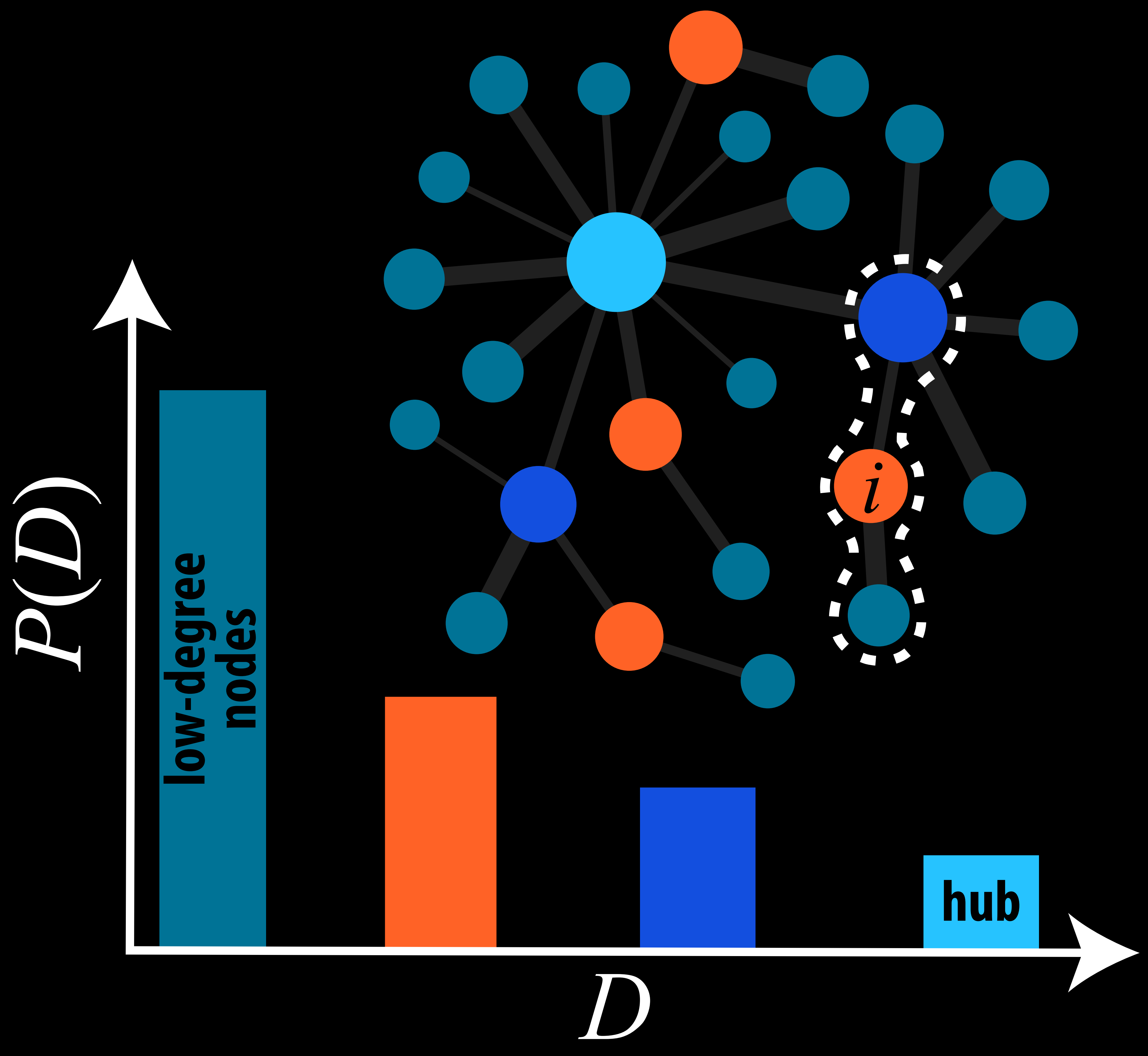
Scannell, J. W. & Young, M. P. *Curr.*(1993)

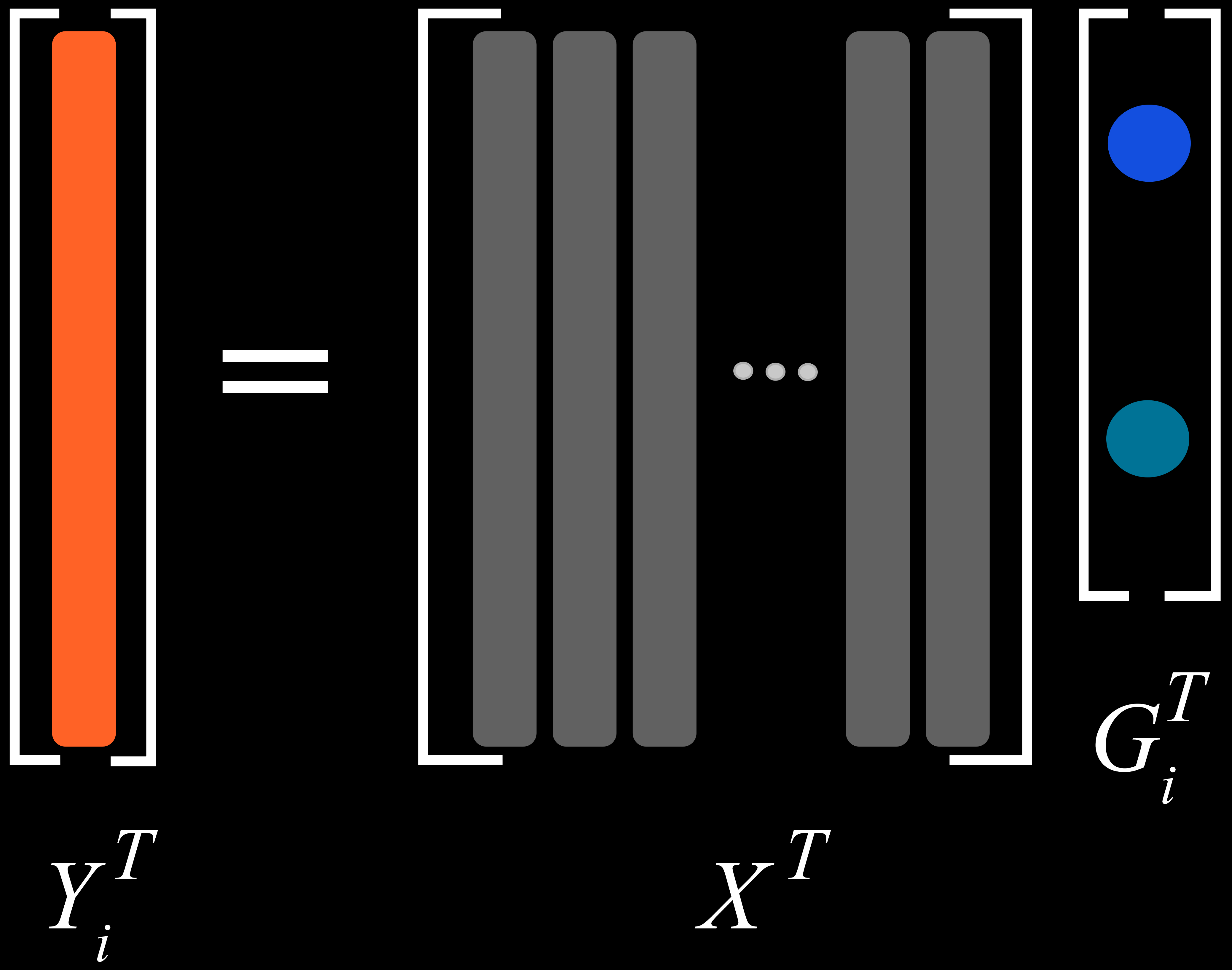
Van Den Heuvel, M. P., & Sporns, O. (2011)

MICROSCOPIC INVESTIGATION

reduction theorem and sparse regression

time series = mean field + fluctuations



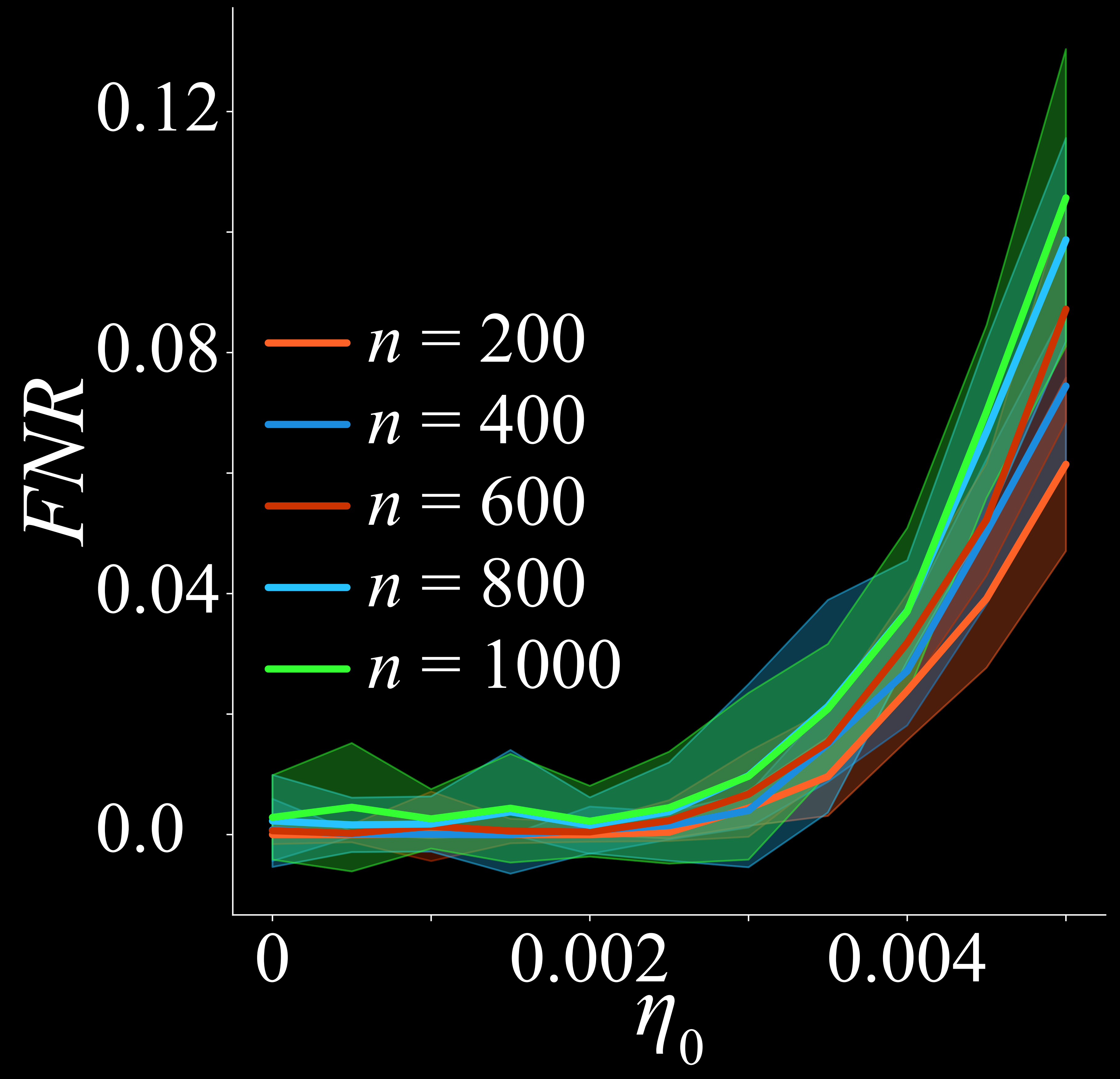


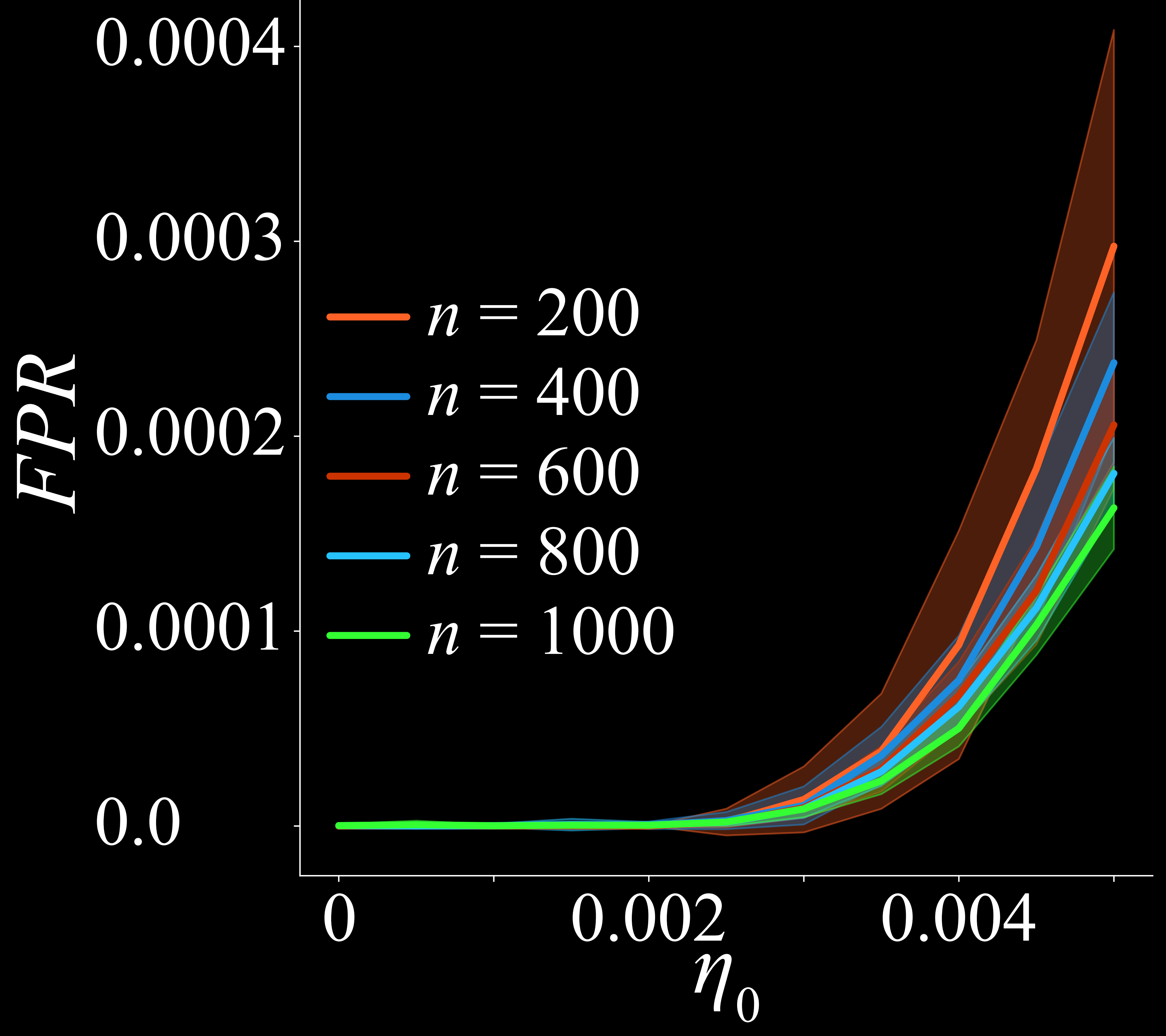
Eroglu, D., Tanzi, M., van Strien, S., Pereira, T. *Physical Review X* 10 (2020)
Candes, E J., Justin K. R, and Terence T., Comm. Pure Appl. Math 59 (2006)

ROBUST AGAINST NOISE

better reconstruction, better prediction!

time series = mean field + fluctuations + noise

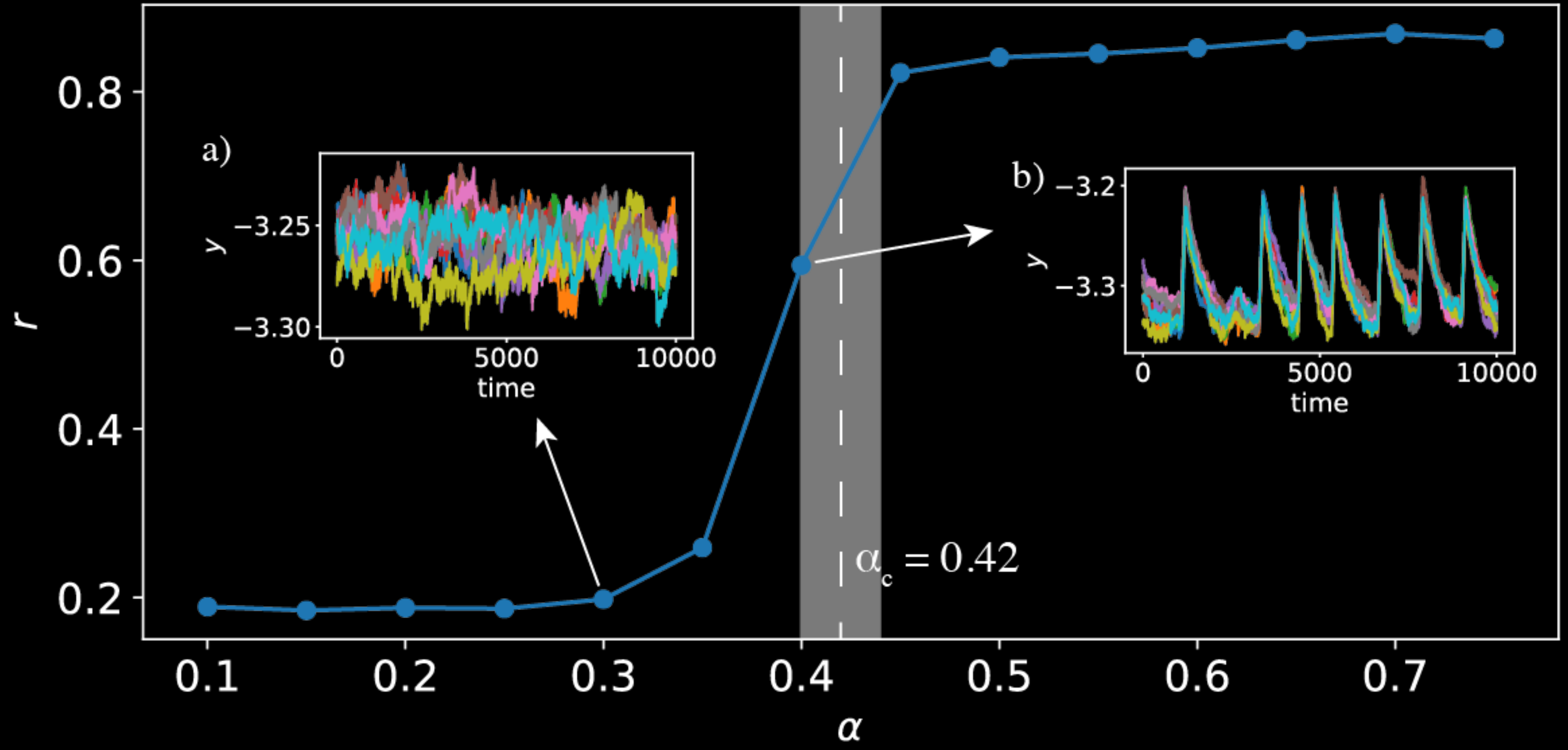




Eroglu, D., Tanzi, M., van Strien, S., Pereira, T. *Physical Review X* 10 (2020)
Topal, I and Eroglu, D. *Physical Review Letters* 130 (2023)

PREDICTION

critical transitions



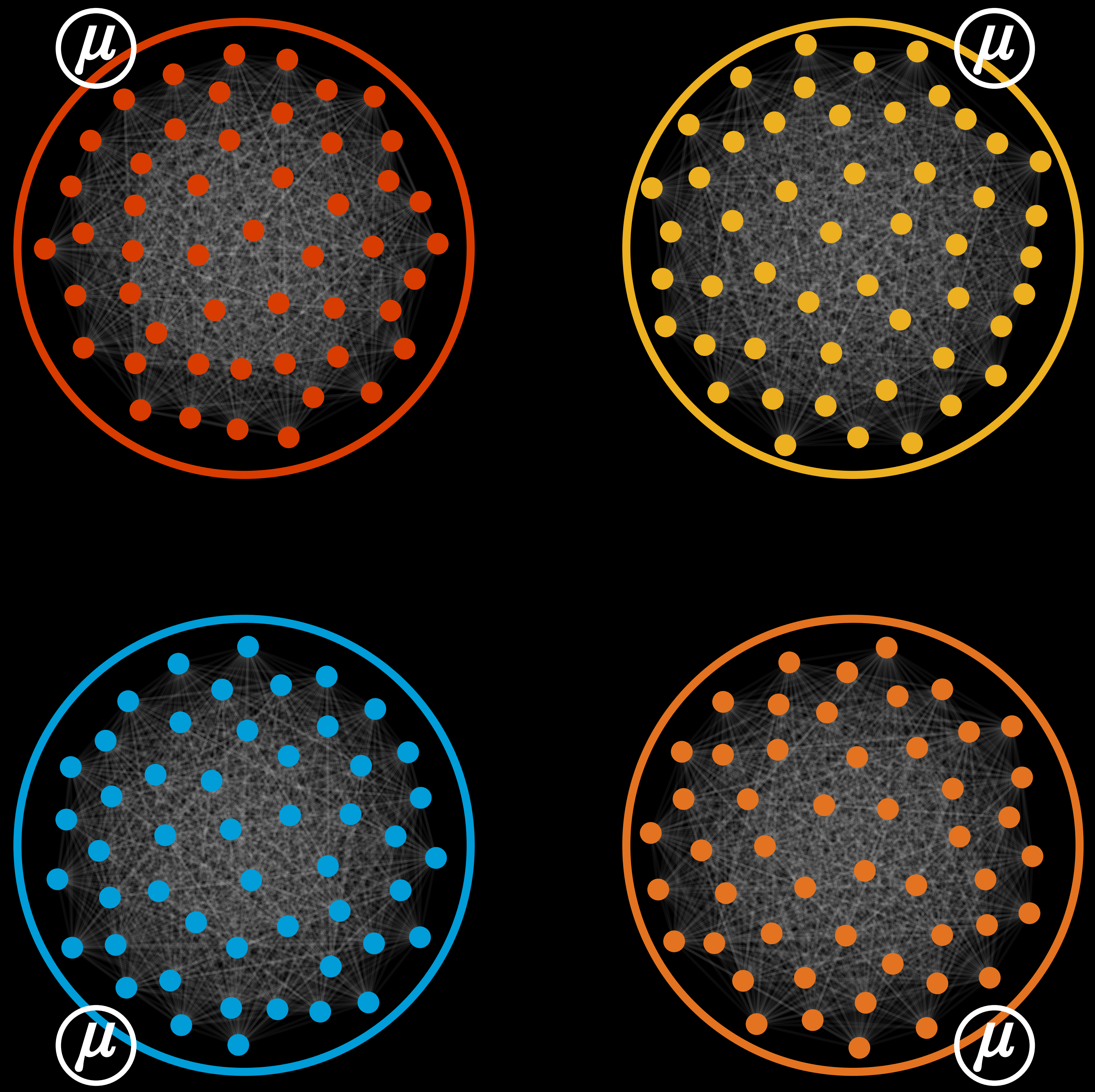
Eroglu, D., Lamb J., Pereira T. *Contemporary Physics* **58** 207 (2017)

Pereira, T., Eroglu, D., Bagci, GB., Tirnakli, U., Jensen, HJ.,. *Physical Review Letters* 110 (2013)

Duan, C., Nishikawa, T., Eroglu, D., Motter., AE. *Science Advances* 8 (2022)

PROBING DATA

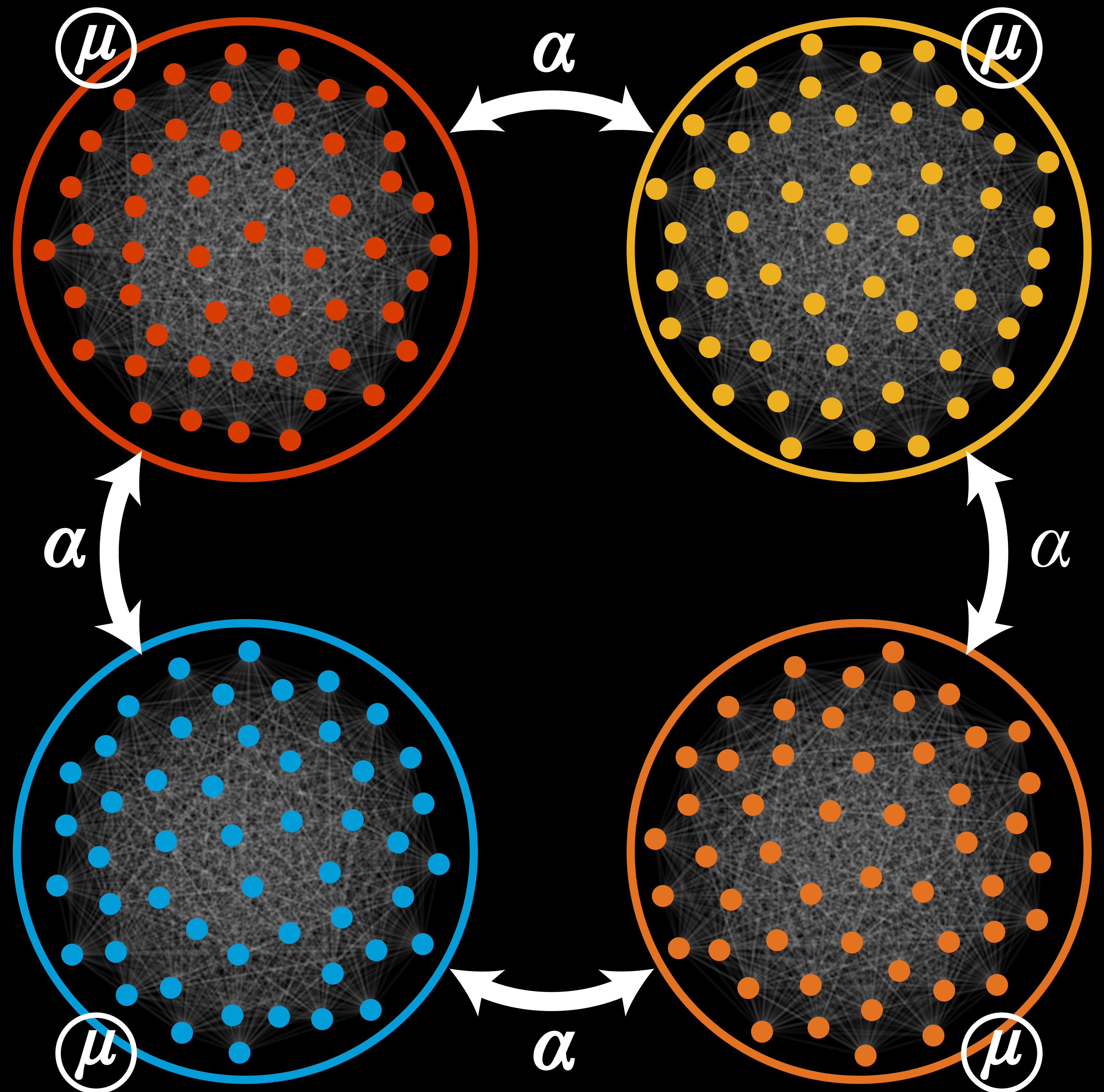
mean-field measurements



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N}\sum_{n=1}^{N}\sin(\psi_{kn} - \psi_{km})$$

PROBING DATA

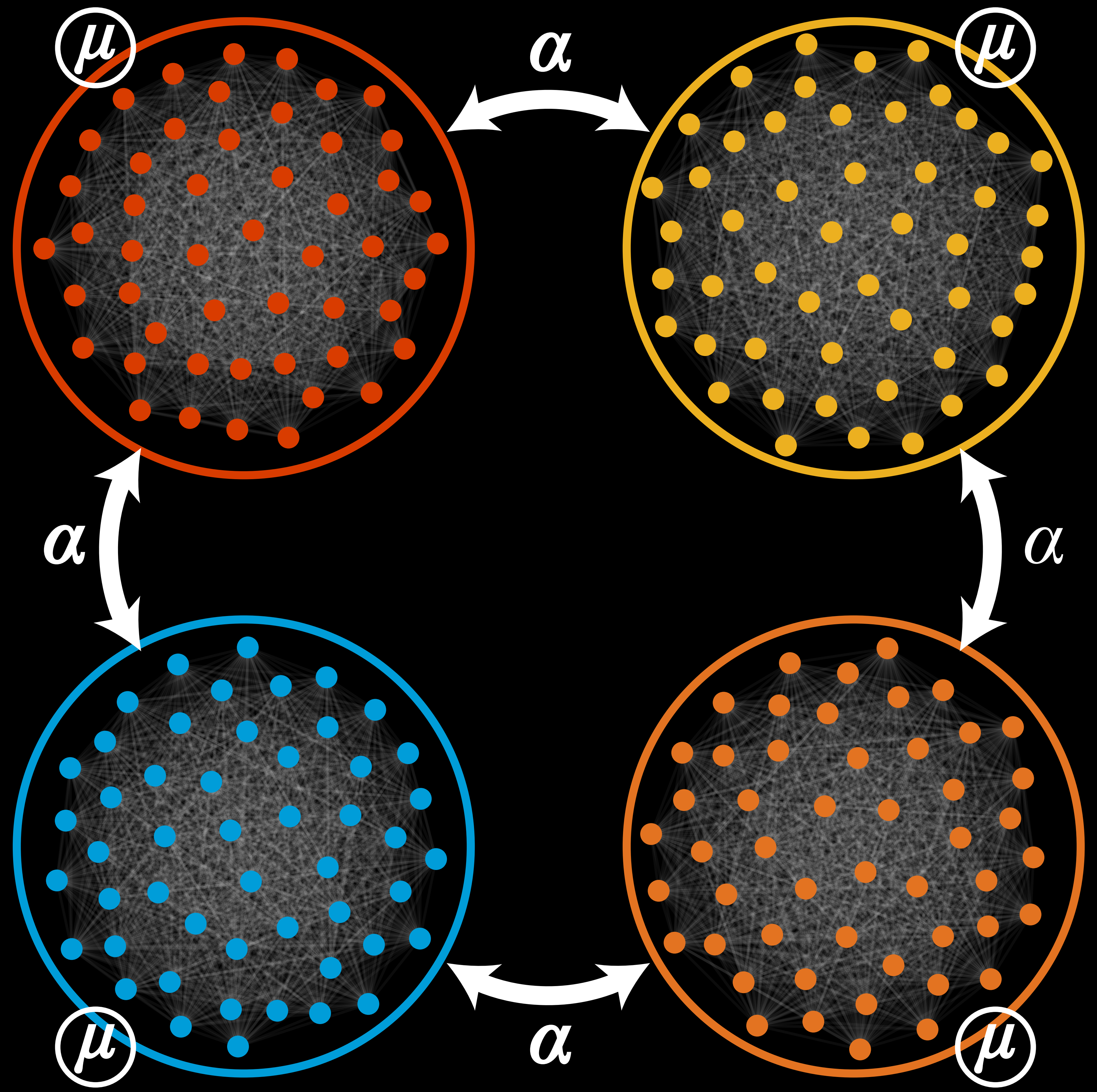
mean-field measurements



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N}\sum_{n=1}^{N}\sin(\psi_{kn} - \psi_{km}) + \sum_{\ell=1}^{4} A_{k\ell}\left(\frac{\alpha}{N}\sum_{n=1}^{N}\sin(\psi_{\ell n} - \psi_{km})\right)$$

PROBING DATA

mean-field measurements



$$\dot{\psi}_{km} = \omega_{km} + \frac{\mu}{N}\sum_{n=1}^{N}\sin(\psi_{kn}-\psi_{km}) + \sum_{\ell=1}^{4} A_{k\ell}\left(\frac{\alpha}{N}\sum_{n=1}^{N}\sin(\psi_{\ell n}-\psi_{km})\right)$$

In terms of mean-fields

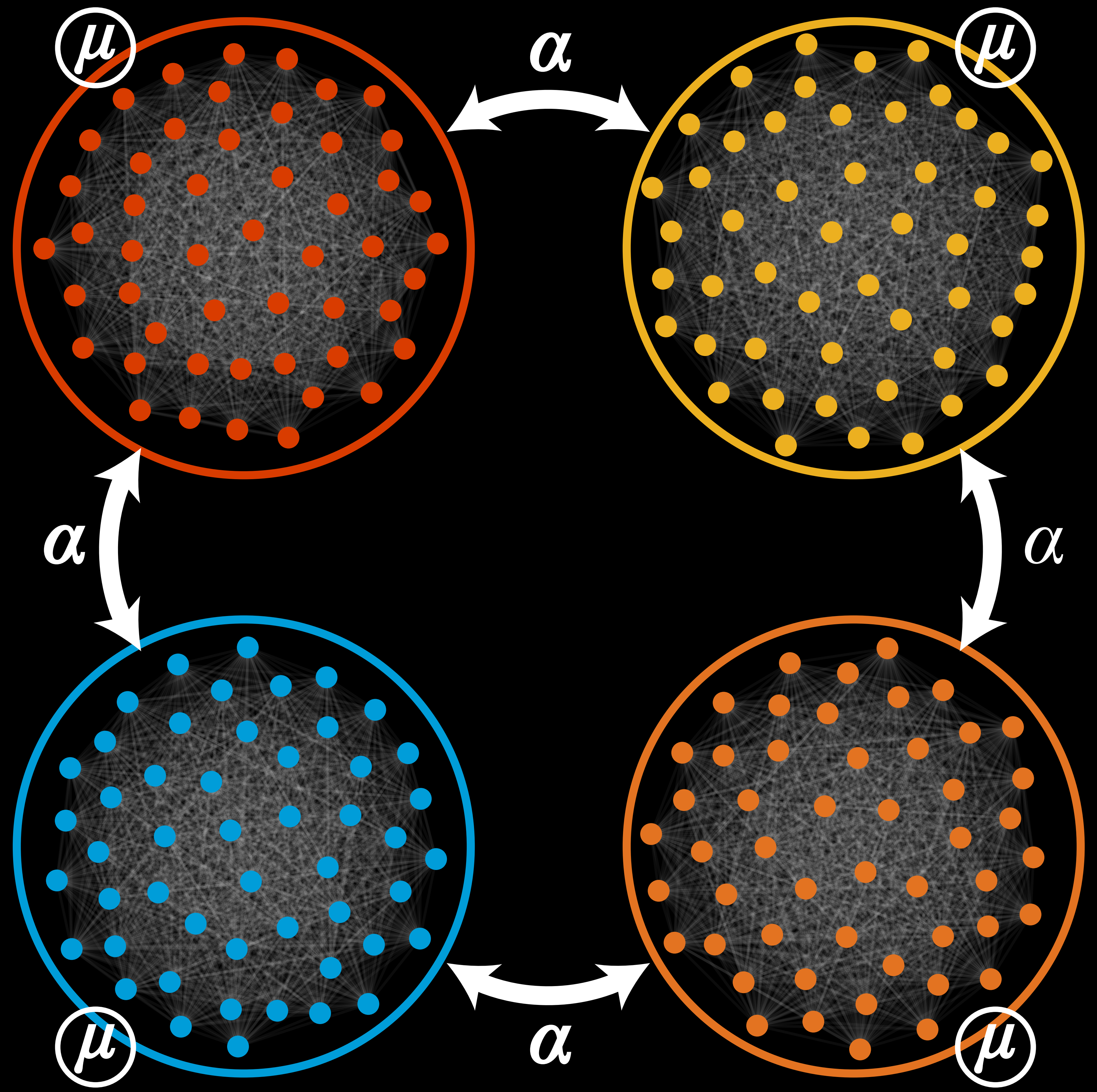
$$\dot{\psi}_{km} = \omega_{km} + \mathrm{Im}\left(\mu z_k + \alpha \sum A_{kl} z_\ell\right) e^{-i\psi_{km}}$$

where

$$z_k = \frac{1}{N}\sum_{m=1}^{N} e^{i\psi_{km}}$$

PROBING DATA

assume infinitely many neurons

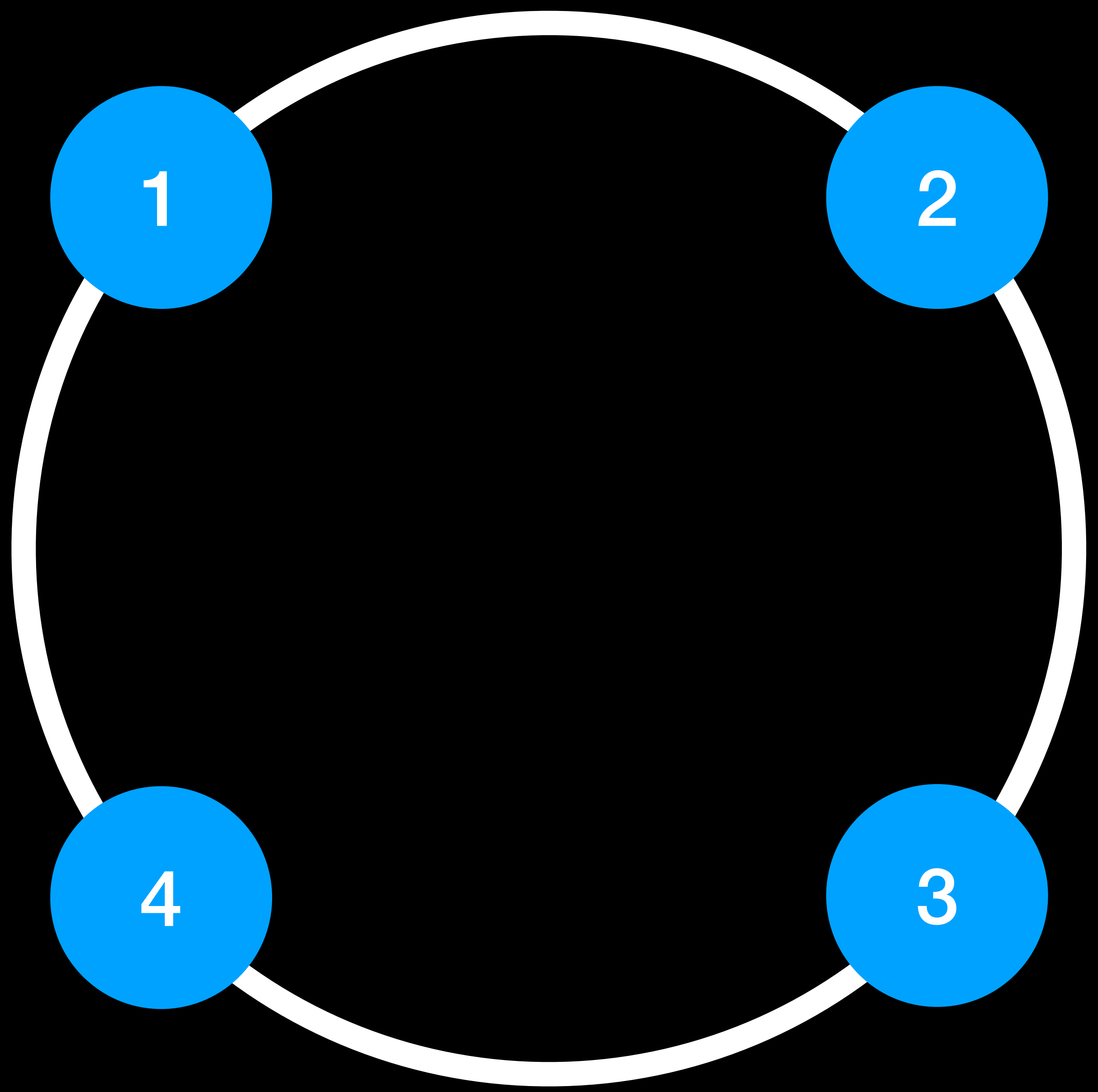


Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^{4} A_{k\ell} h(z_k, z_\ell)$$

RING GRAPH

undirected and cubic polynomial interaction



Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^{4} A_{k\ell} h(z_k, z_\ell)$$

where

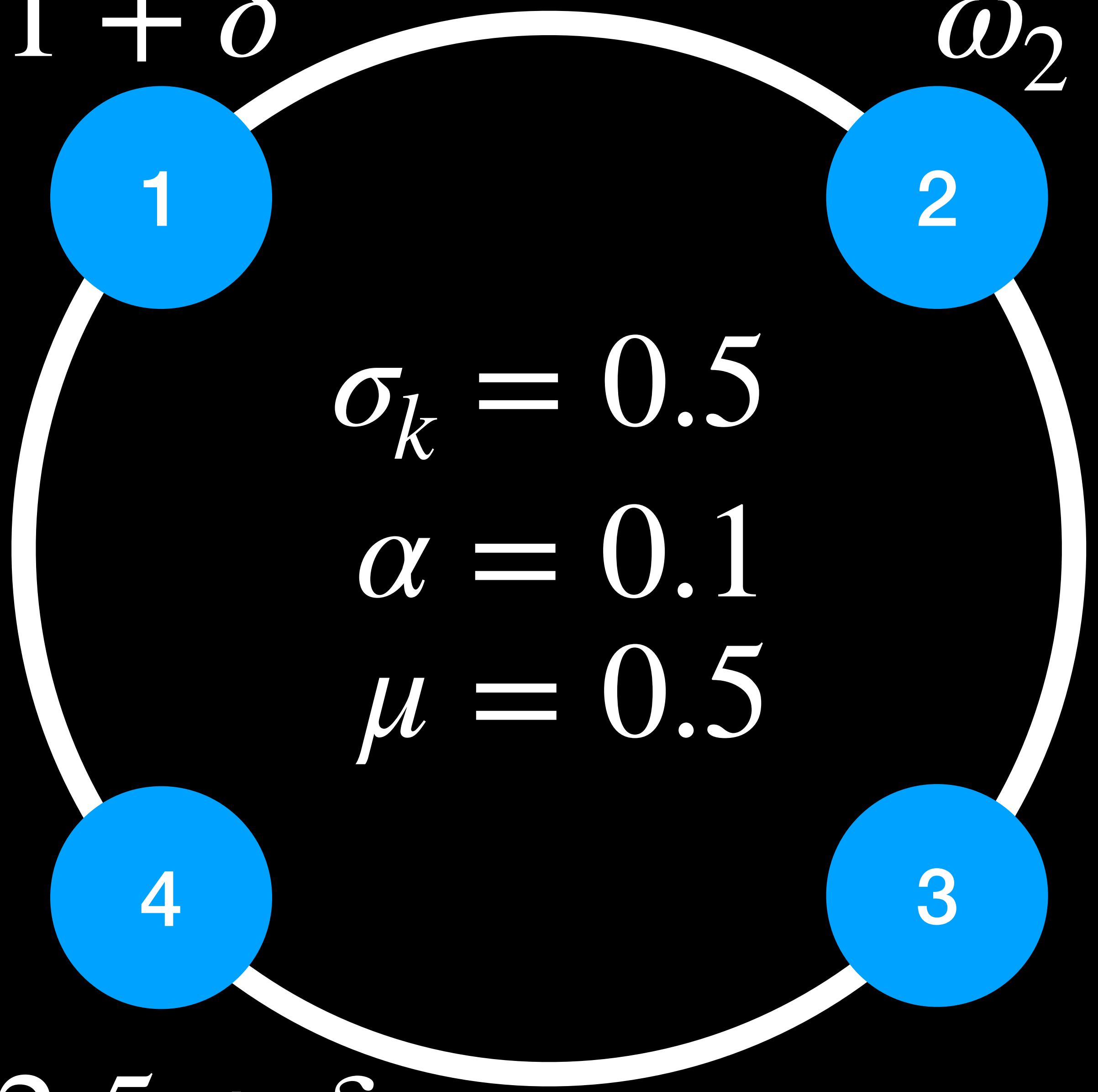
$$f_k(z_k) = \gamma_k z_k + \beta_k z_k \;\; z_k \;{}^2; \qquad \gamma_k = (i\Omega_k + \mu - \sigma_k)$$

$$\beta_k = -\mu; \qquad h(z_k, z_\ell) = \alpha z_\ell + \alpha \bar{z}_\ell z_k^2$$

RING GRAPH

undirected and cubic polynomial interaction

$\omega_1 = 1 + \delta$ $\qquad$ $\omega_2 = 2.5$



$\sigma_k = 0.5$

$\alpha = 0.1$

$\mu = 0.5$

$\omega_4 = 2.5 + \delta$ $\qquad$ $\omega_3 = 1.5$

Applying the Ott-Antonsen ansatz

$$\dot{z}_k = f_k(z_k) + \sum_{\ell=1}^{4} A_{k\ell} h(z_k, z_\ell)$$

where

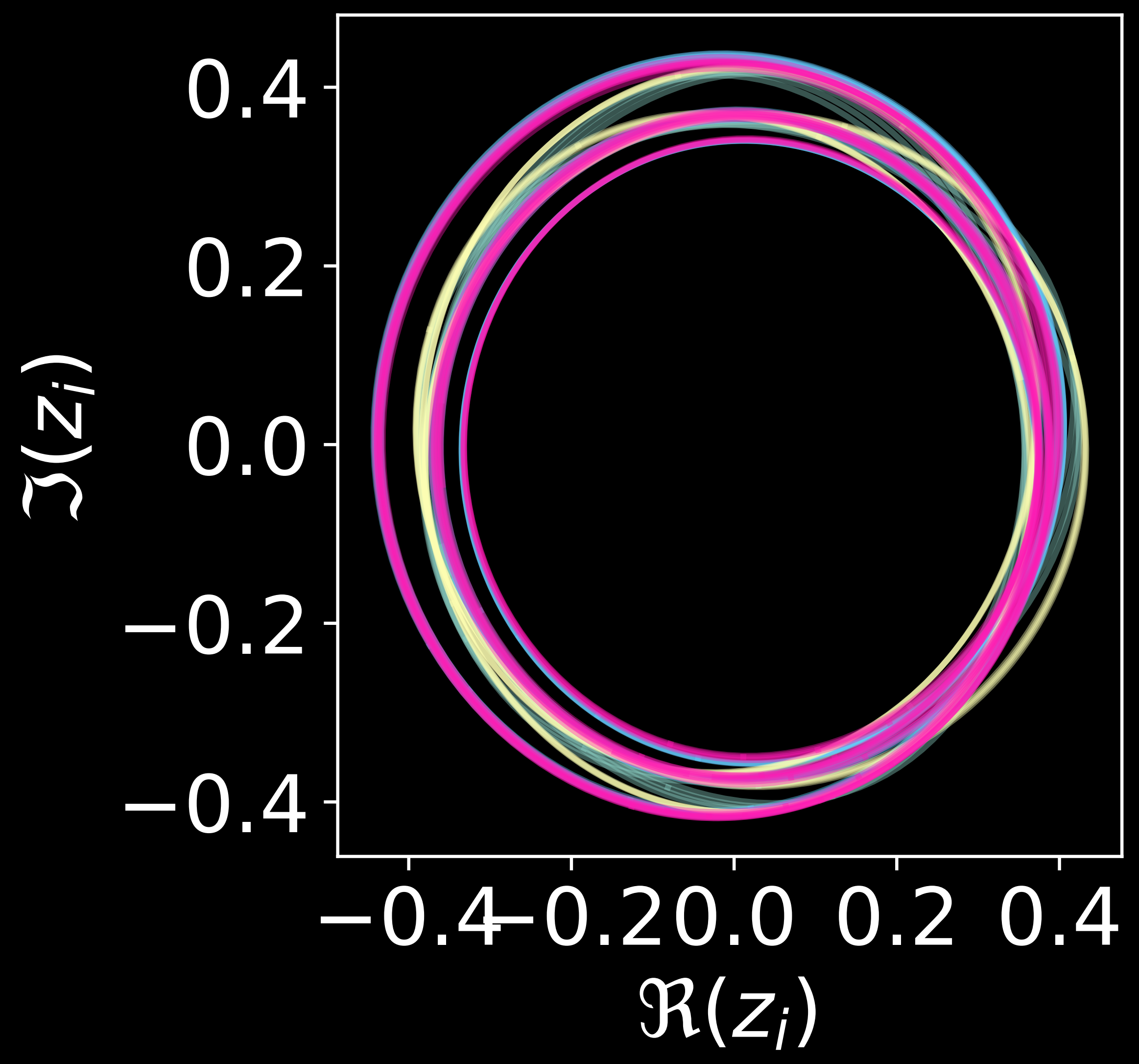
$$f_k(z_k) = \gamma_k z_k + \beta_k z_k \;\; z_k \;{}^2; \qquad \gamma_k = (i\Omega_k + \mu - \sigma_k)$$

$$\beta_k = -\mu; \qquad h(z_k, z_\ell) = \alpha z_\ell + \alpha \bar{z}_\ell z_k^2$$

Resonance satisfying condition: $\omega_2 = \omega_1 + \omega_3 \quad \& \quad \omega_4 = \omega_1 + \omega_3$

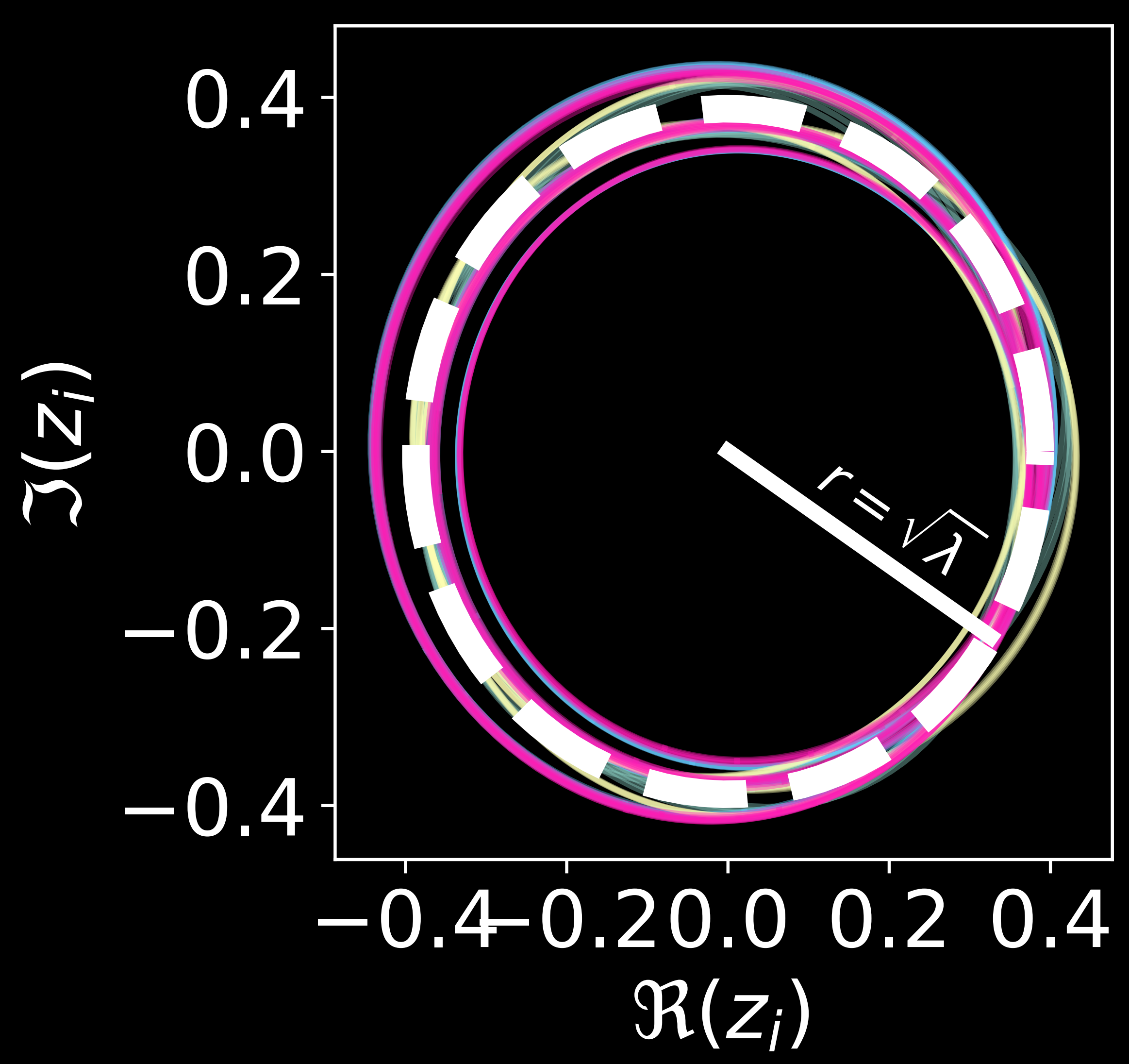
RING GRAPH

simulation



RING GRAPH

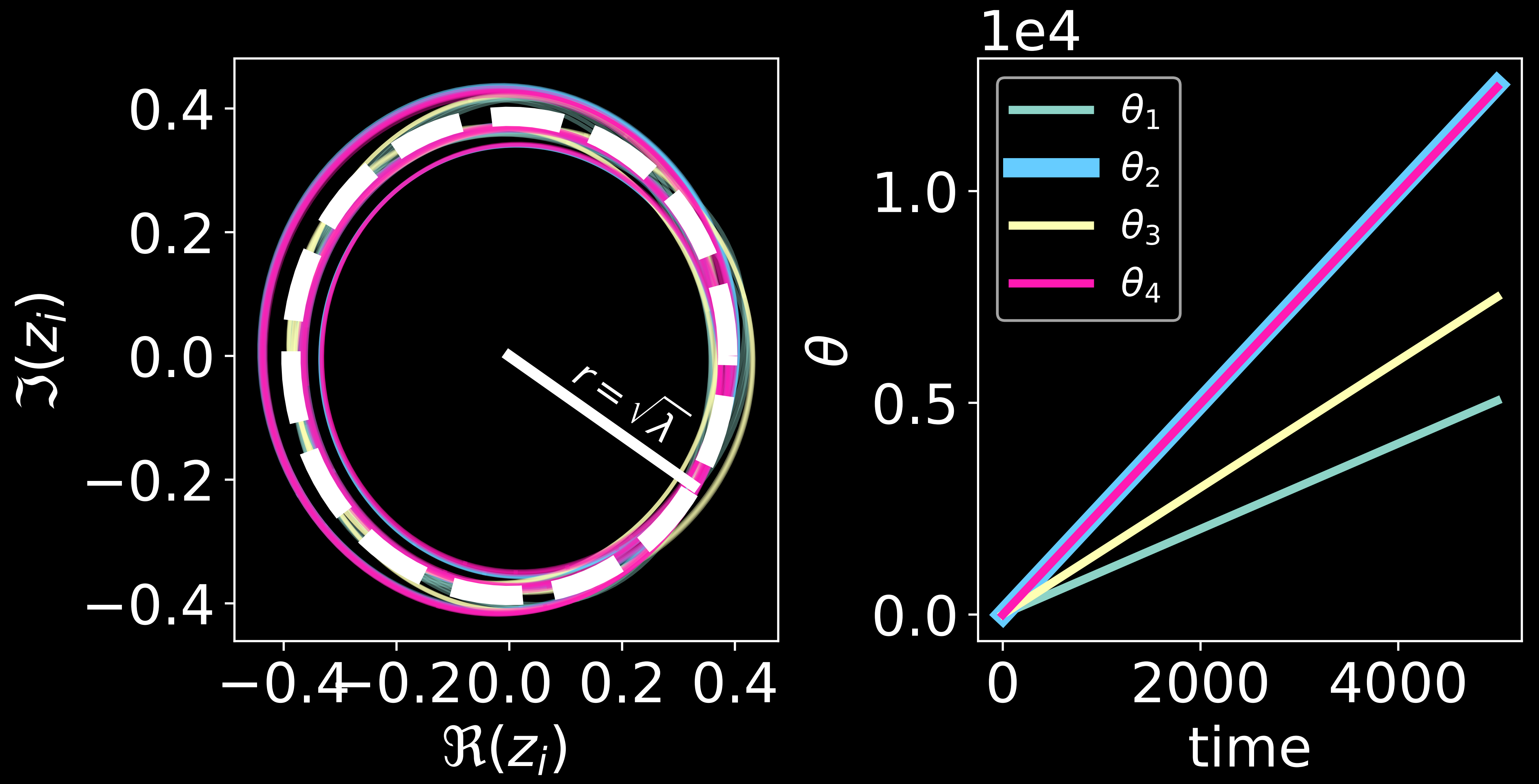
simulation



$$\lambda_k = \frac{\mu - \sigma_k}{\mu}$$

RING GRAPH

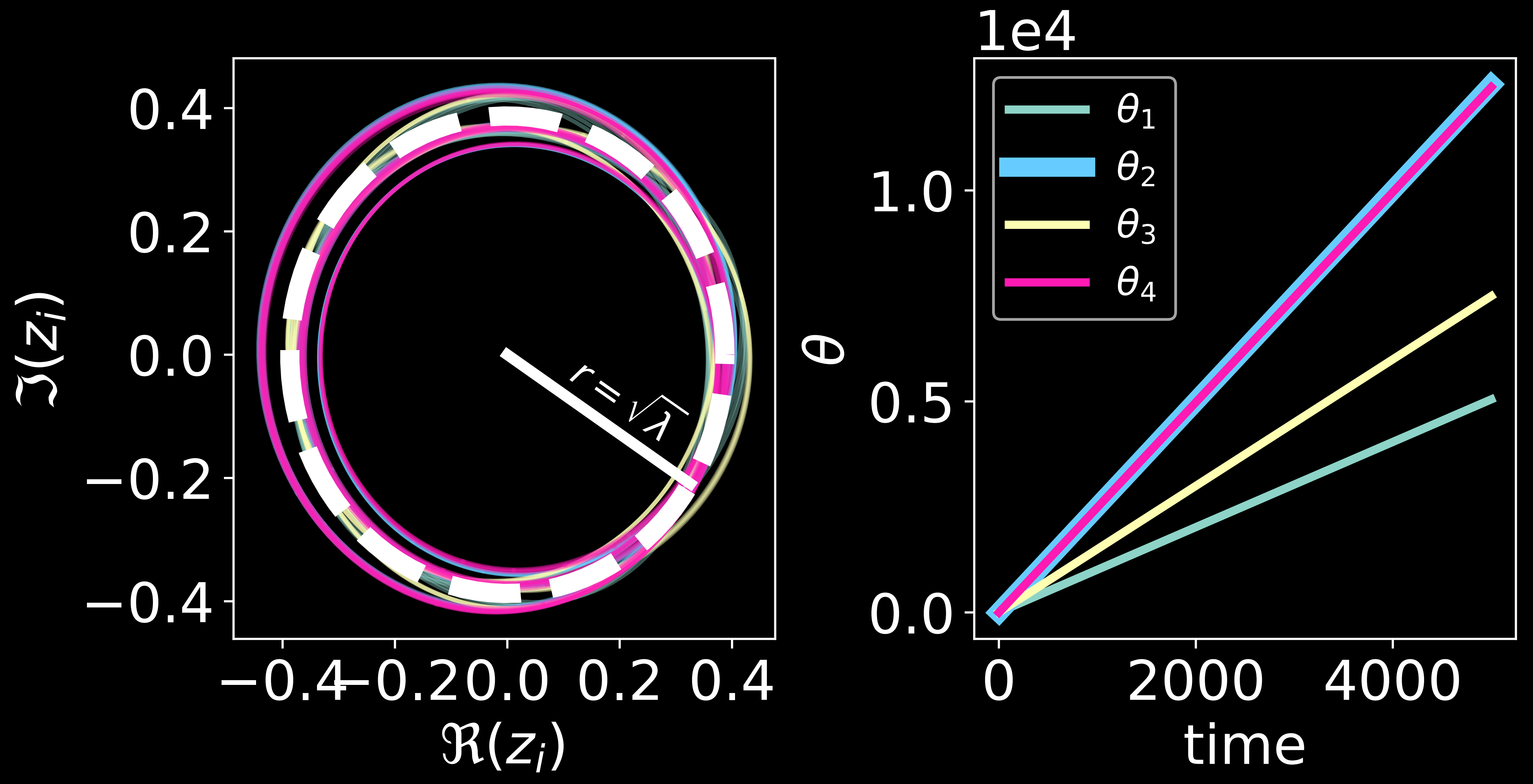
defining phases



$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$

RING GRAPH

defining slow variables



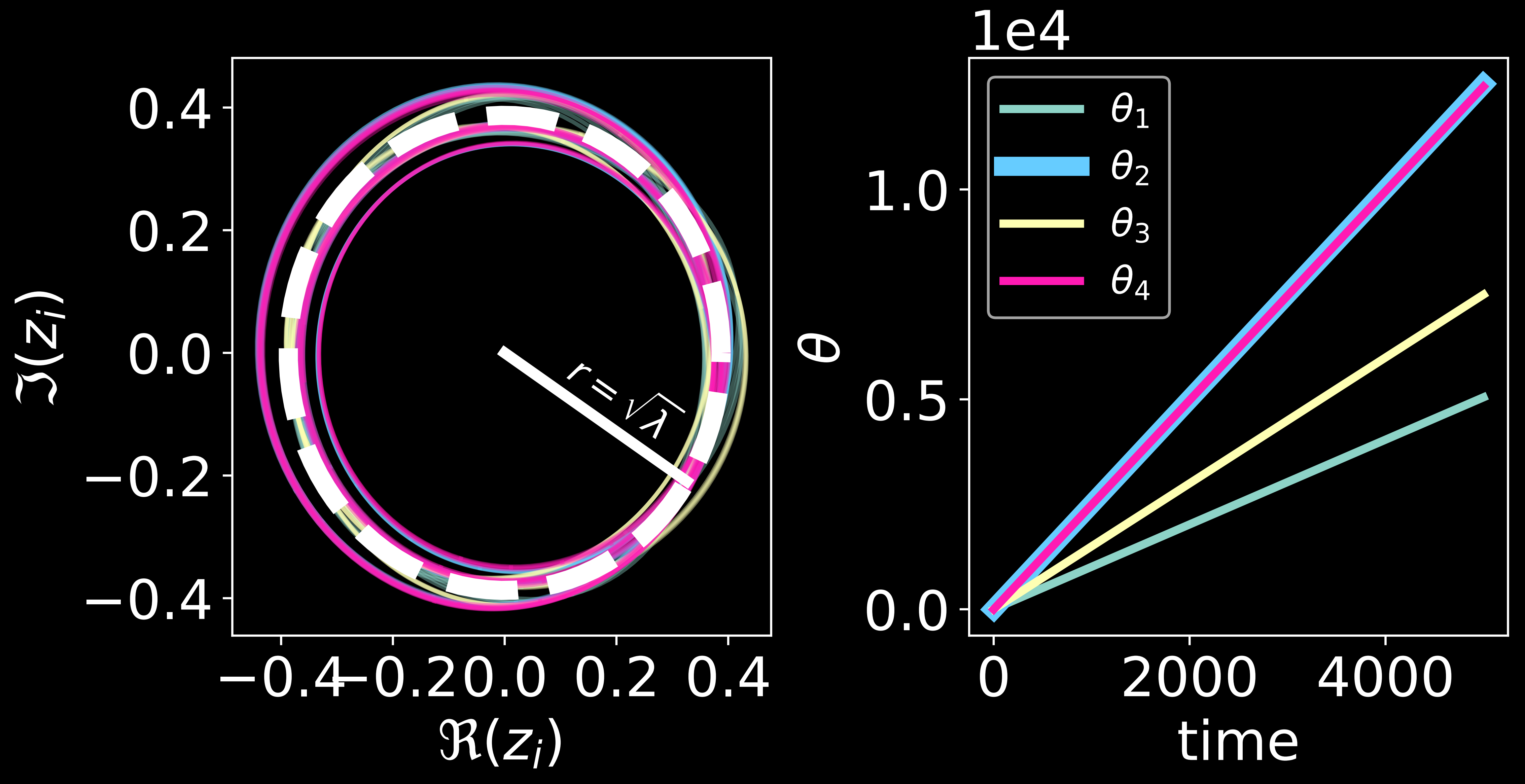
$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$

$$\vartheta_k(t) = \theta_k(t) - \Omega_k t$$

Then we would like to reconstruct ϑ_i from data

RING GRAPH

defining slow variables



$$z_k(t) = r_k(t)e^{i\theta_k(t)}$$

$$\vartheta_k(t) = \theta_k(t) - \Omega_k t$$

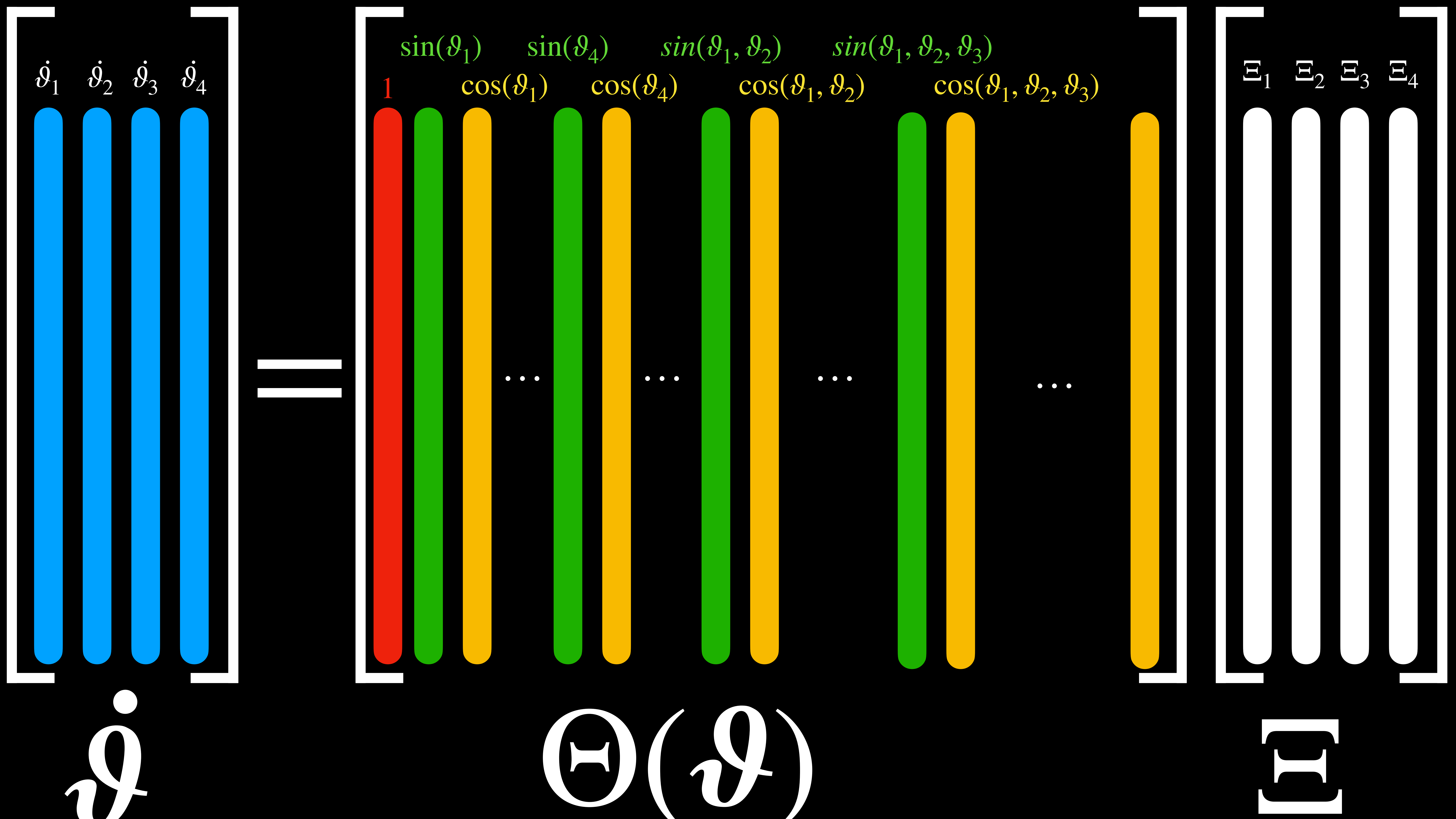
Then we would like to reconstruct ϑ_i from data

$$\dot{\boldsymbol{\vartheta}} = \Theta(\boldsymbol{\vartheta})\Xi$$

COMPRESSED SENSING

Candes, E J., Justin K. R, and Terence T. , 2006

model reconstruction from data



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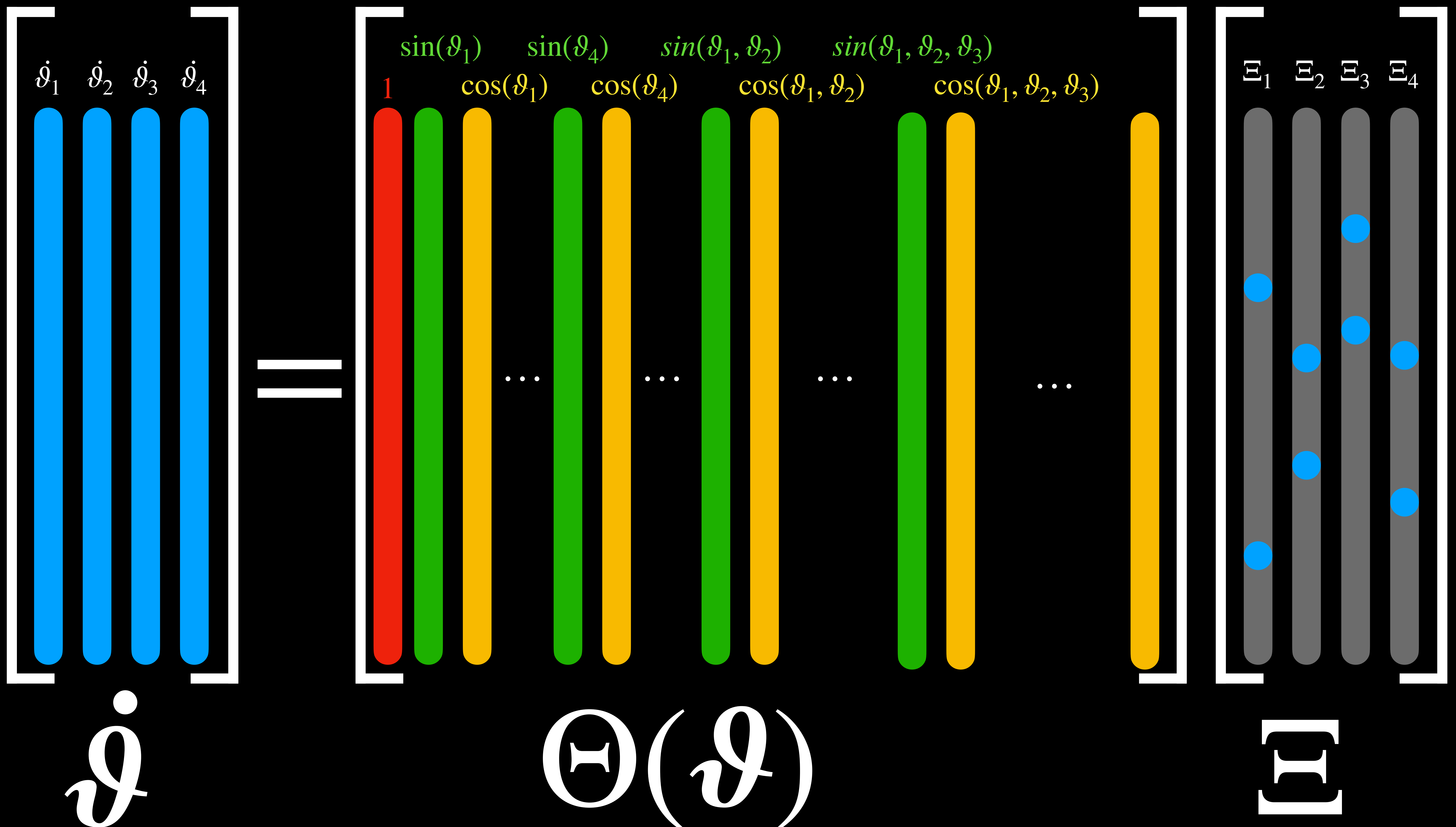
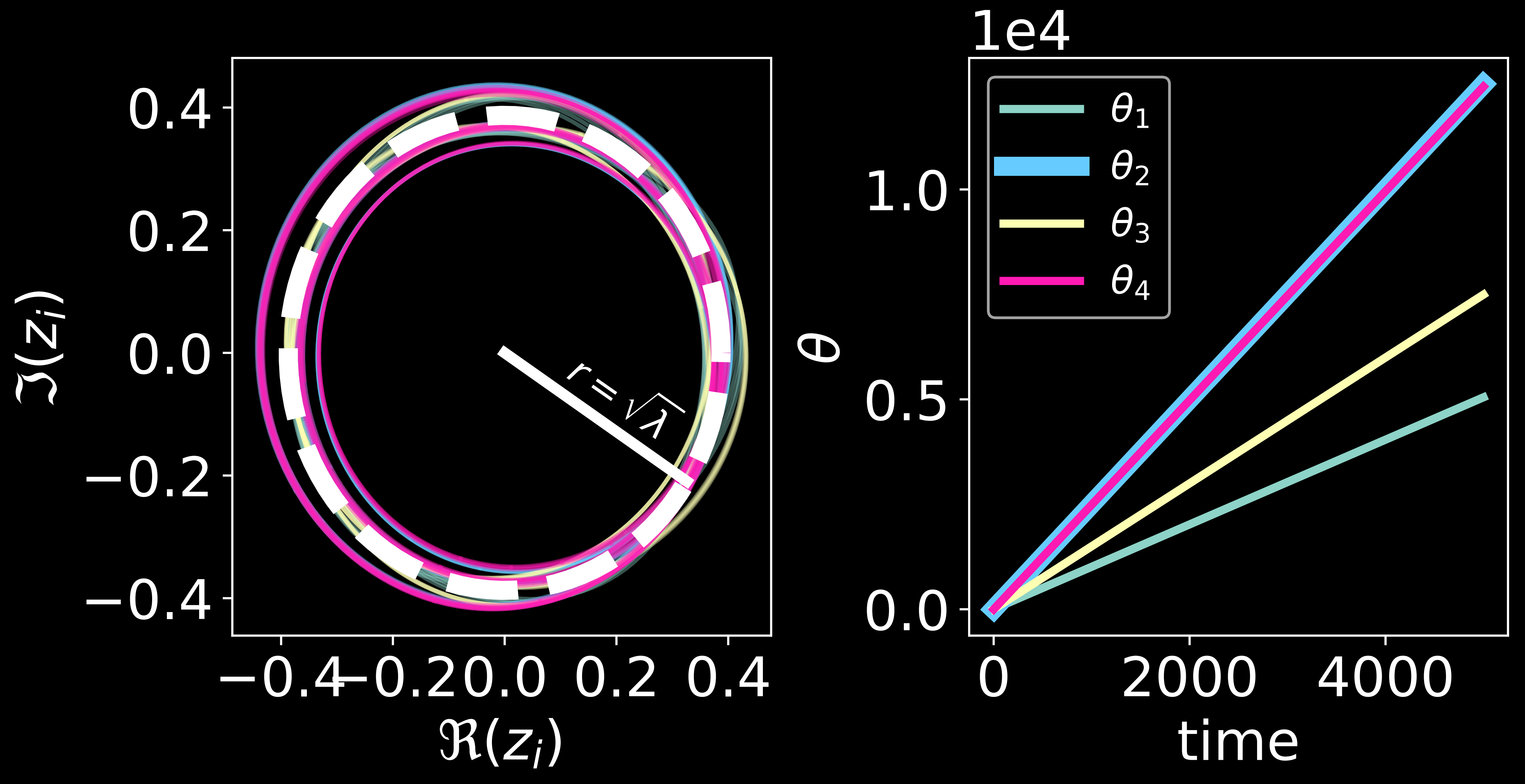


RING GRAPH

reconstruction of slow phases



$$\dot{\theta}_1 = 1.010 + 0.001\cos(\theta_1 - \theta_2 + \theta_3) - 0.001\cos(\theta_1 - \theta_4 + \theta_3)$$

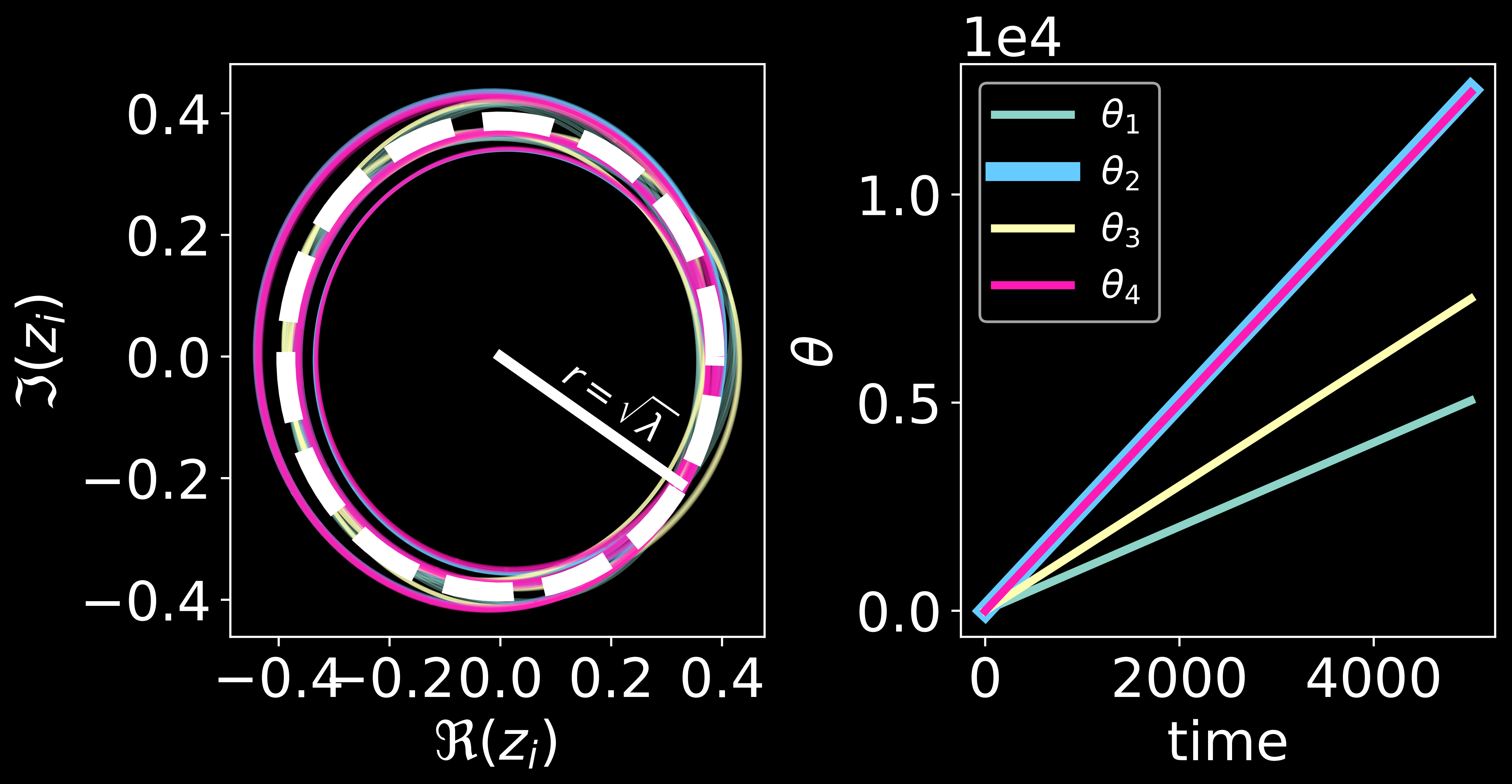
$$\dot{\theta}_2 = 2.489 - 0.005\cos(\theta_1 - \theta_2 + \theta_3)$$

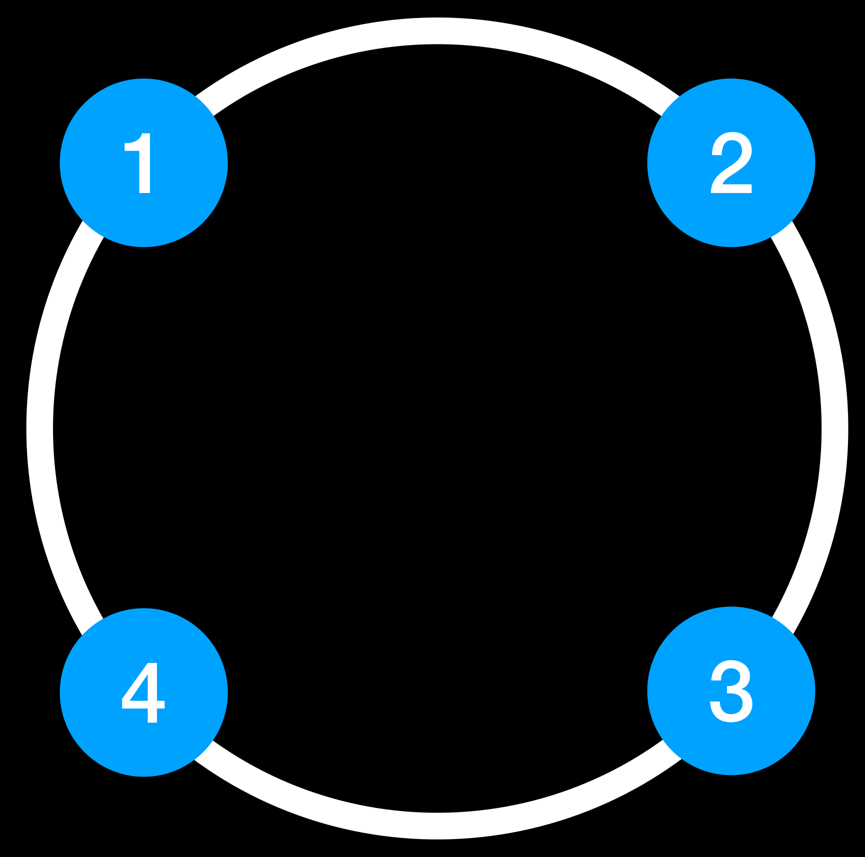
$$\dot{\theta}_3 = 1.499 + 0.001\cos(\theta_1 - \theta_2 + \theta_3) - 0.001\cos(\theta_1 - \theta_4 + \theta_3)$$

$$\dot{\theta}_4 = 2.508 + 0.005\cos(\theta_1 - \theta_4 + \theta_3)$$

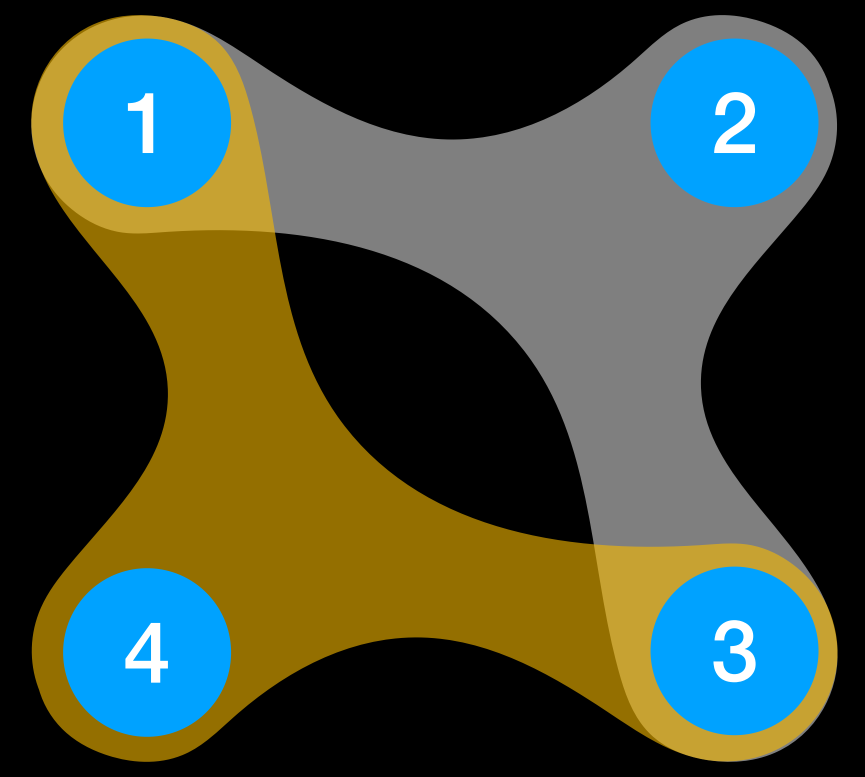
RING GRAPH

emergent hypergraphs





original network



reconstructed hypernetwork

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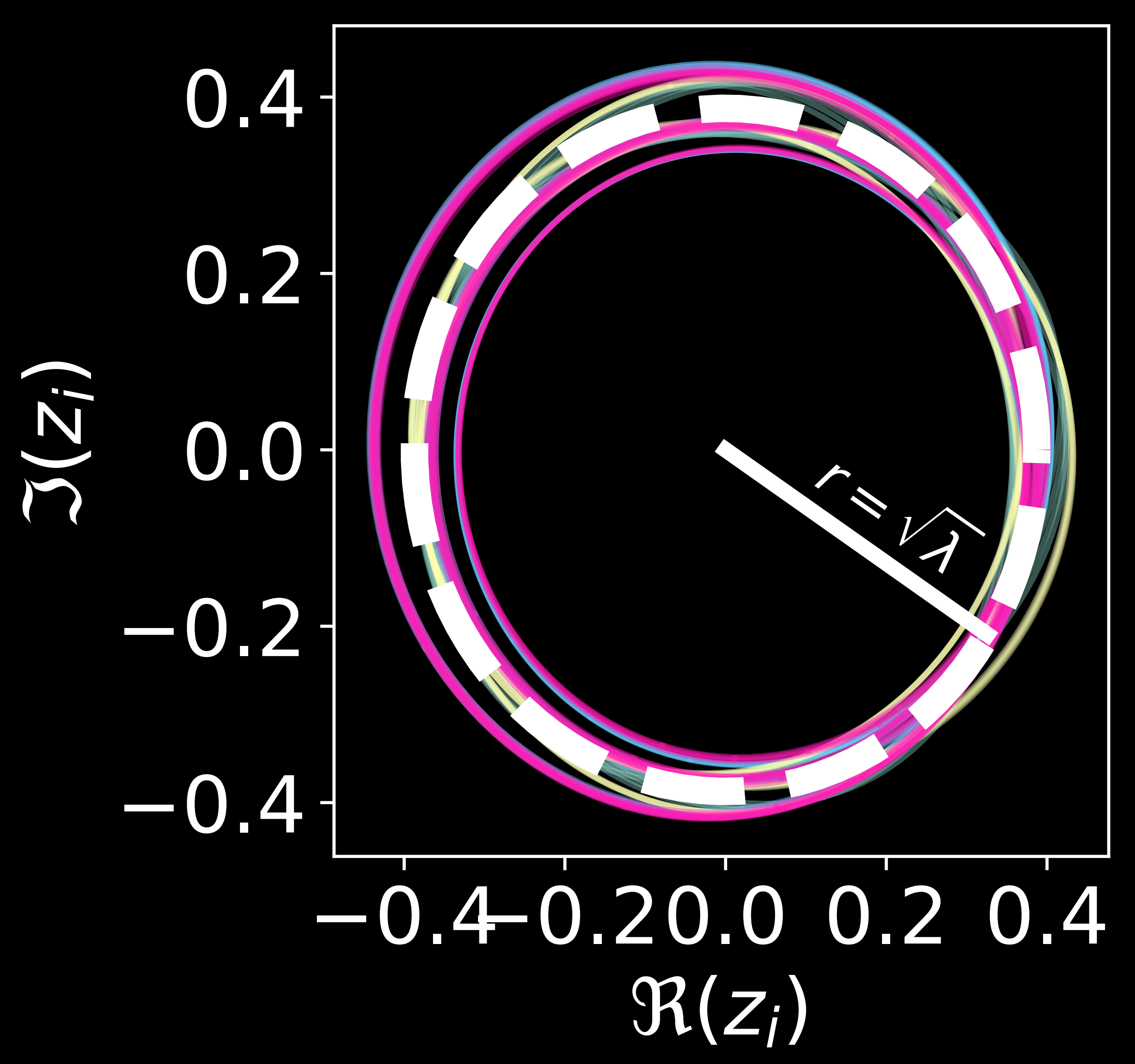
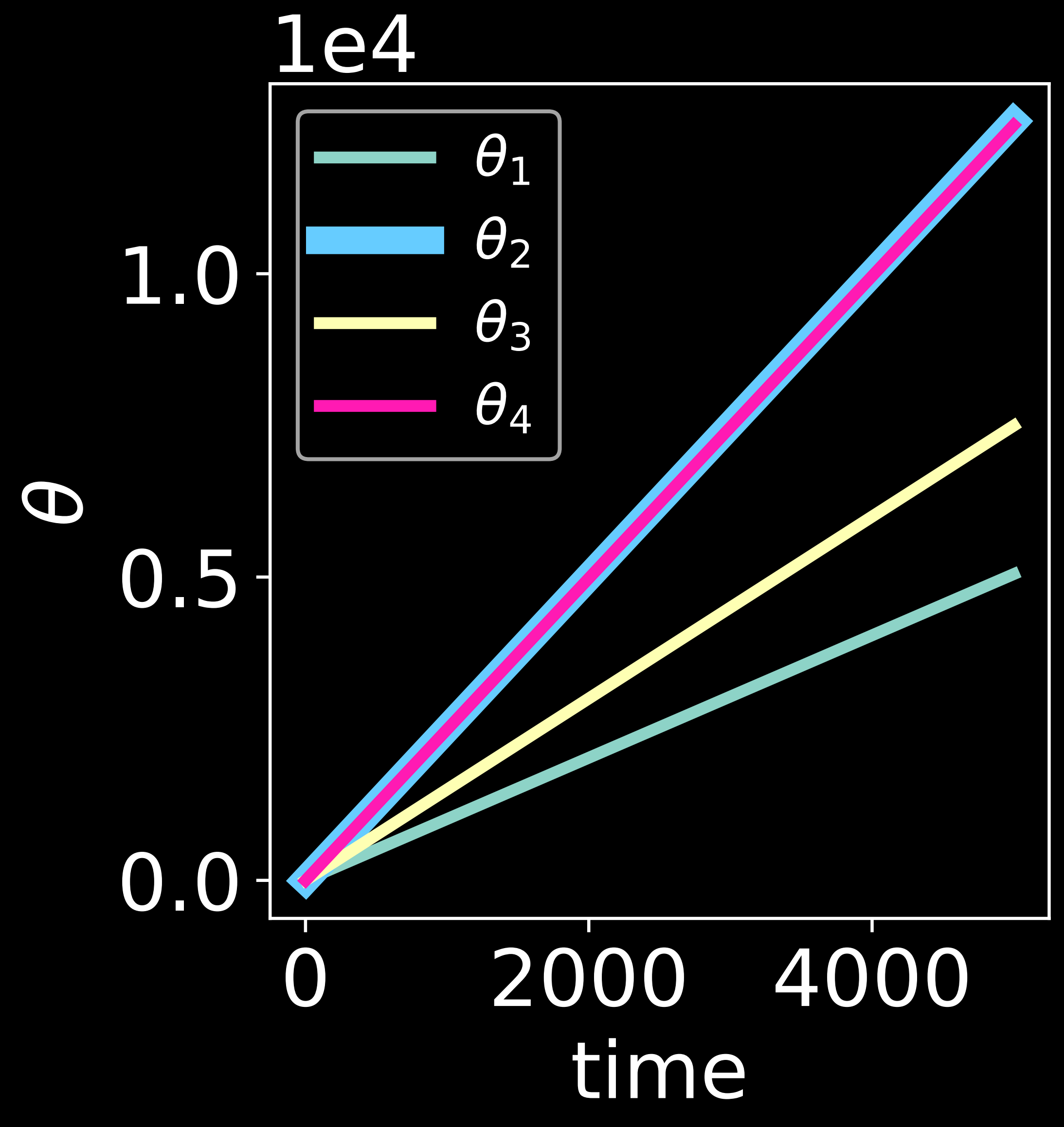
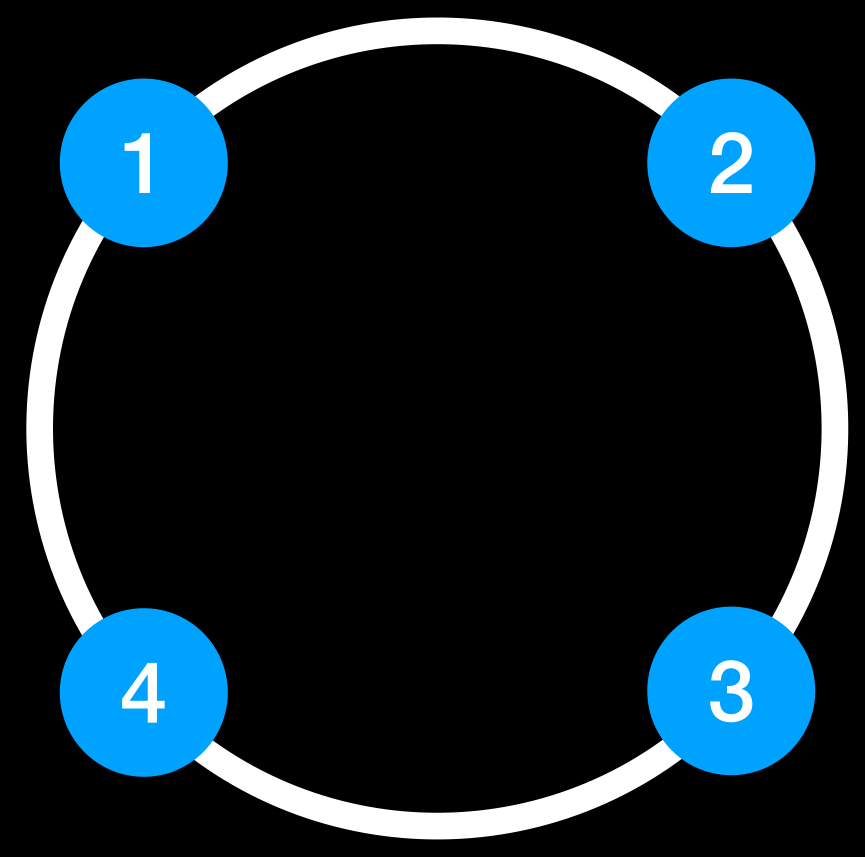
RING GRAPH

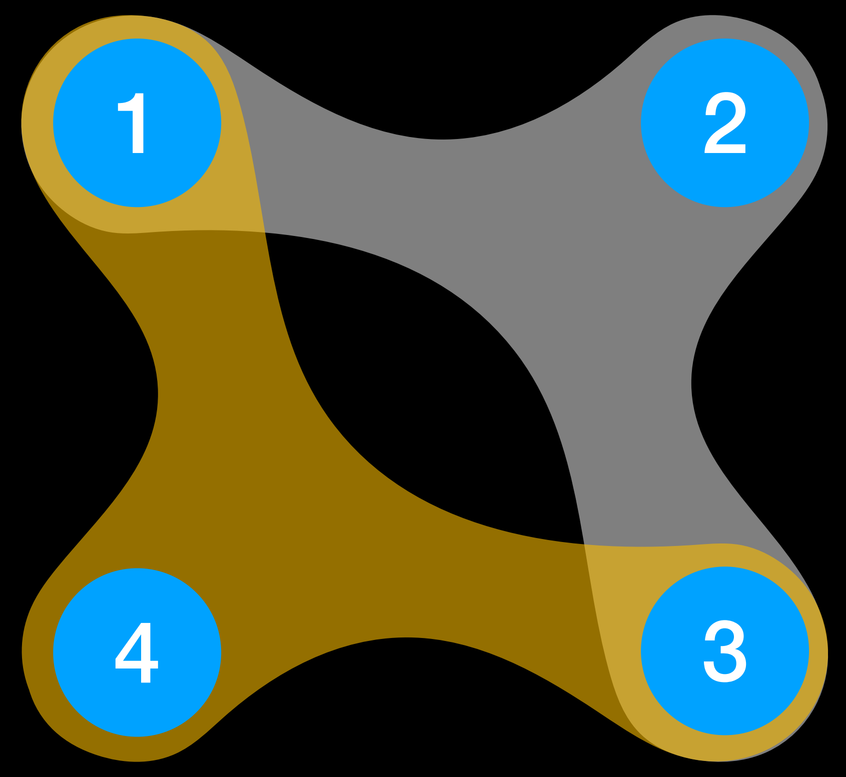
emergent hypergraphs







original network



reconstructed hypernetwork

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WHY?

NORMAL FORM THEORY

looking for the sparsest solution

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

NORMAL FORM THEORY

looking for the sparsest solution

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

a coordinate transformation of the form $w_k = z_k - \alpha P_k(z)$

for some polynomials $P_k(z) = \sum_{\ell=1}^{n} A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$

NORMAL FORM THEORY

looking for the sparsest solution

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

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for some polynomials $P_k(z) = \sum_{\ell=1}^{n} A_{k\ell} \tilde{h}_{k\ell}(z_k, z_\ell)$

NORMAL FORM THEORY

looking for the sparsest solution

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

transformation generates additional undesired terms from the isolated dynamics

NORMAL FORM THEORY

looking for the sparsest solution

$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

second coordinate transformation of the form $u_k = w_k - \alpha Q_k(w)$

NORMAL FORM THEORY

looking for the sparsest solution

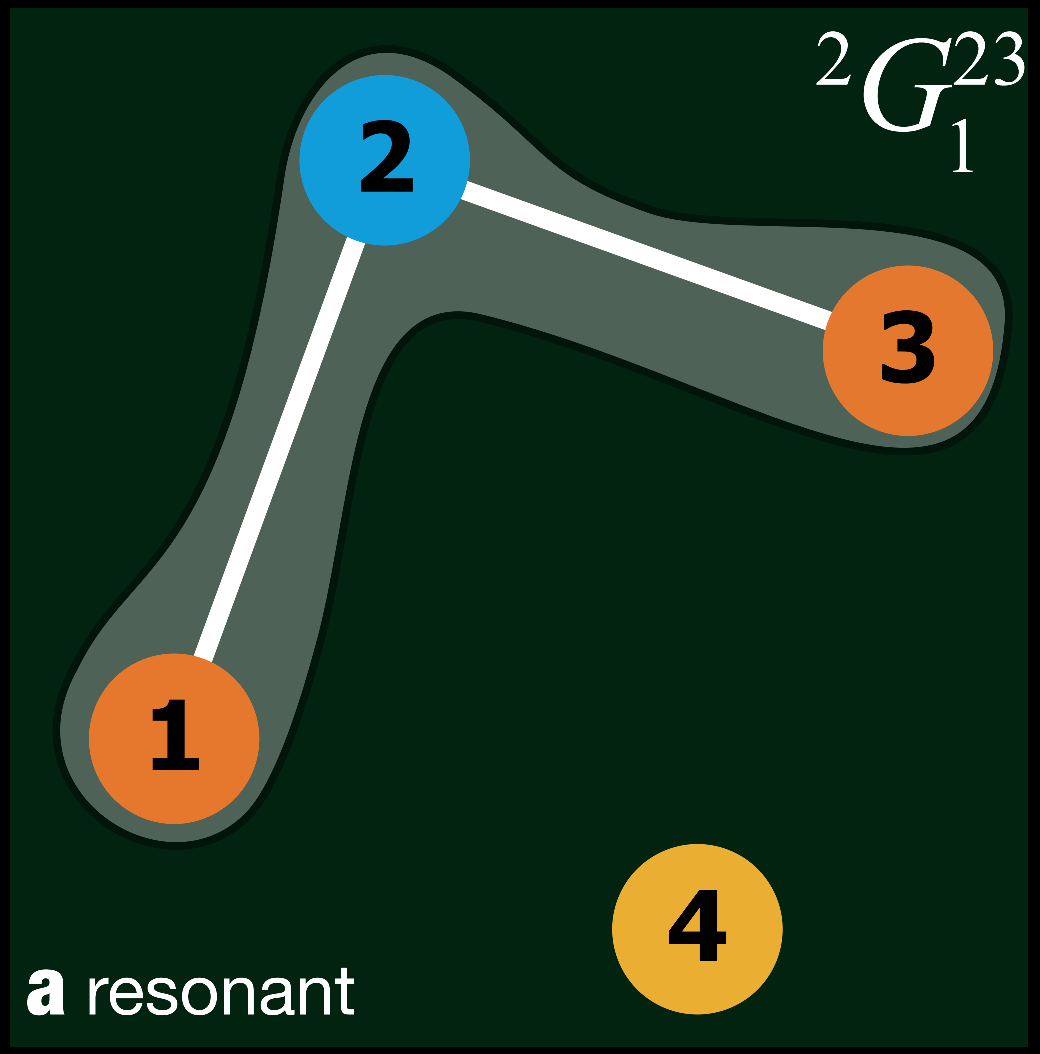
$$\dot{z}_k = f_k(z_k) + \alpha \sum_{\ell=1}^{n} A_{k\ell} h_k(z_k, z_\ell)$$

second coordinate transformation of the form $u_k = w_k - \alpha Q_k(w)$

nontrivial combinatorial problem tackled by introducing a special bracket [●||●] on the space of polynomials.

NORMAL FORM THEORY

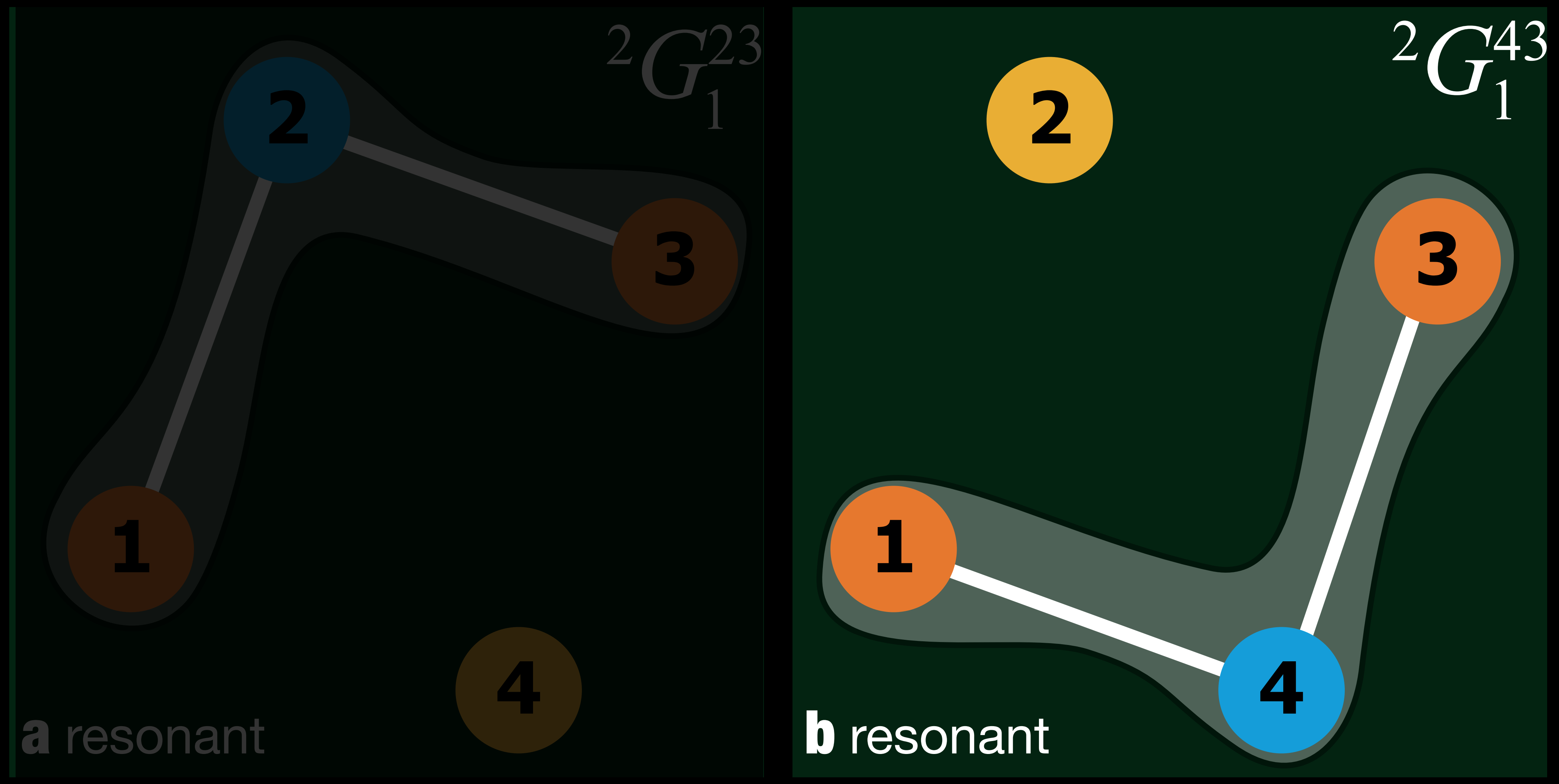
rules for resonant and nonresonant terms



Checking only for node-1

NORMAL FORM THEORY

rules for resonant and nonresonant terms



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NORMAL FORM THEORY

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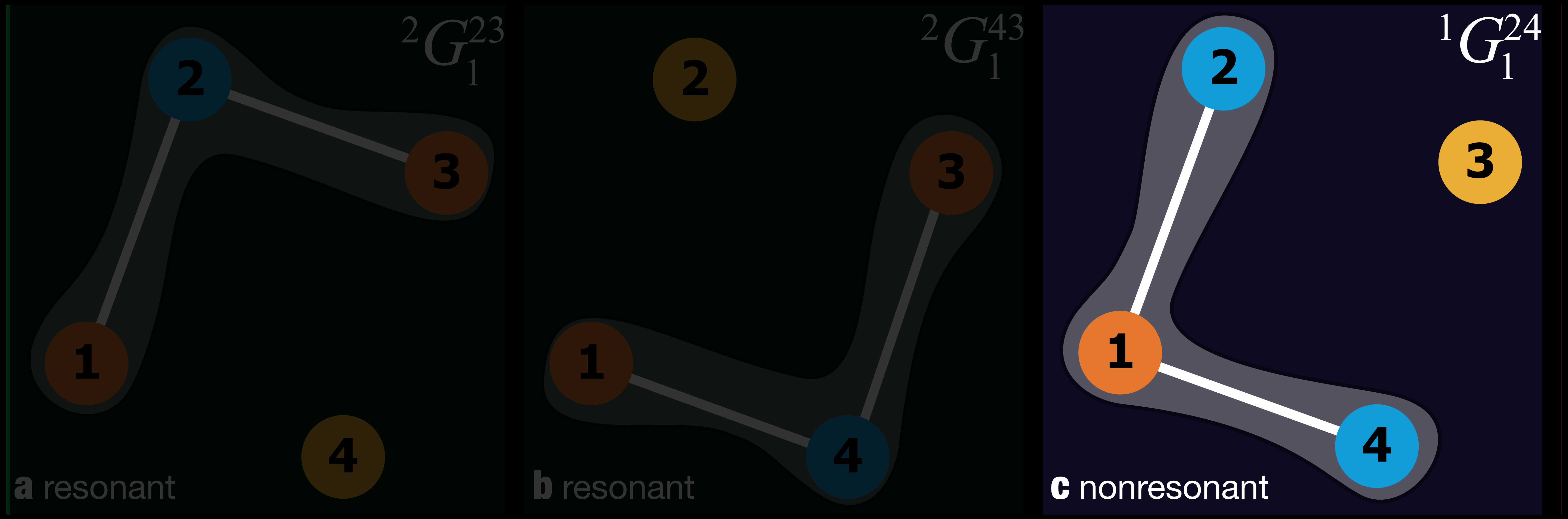

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NORMAL FORM THEORY

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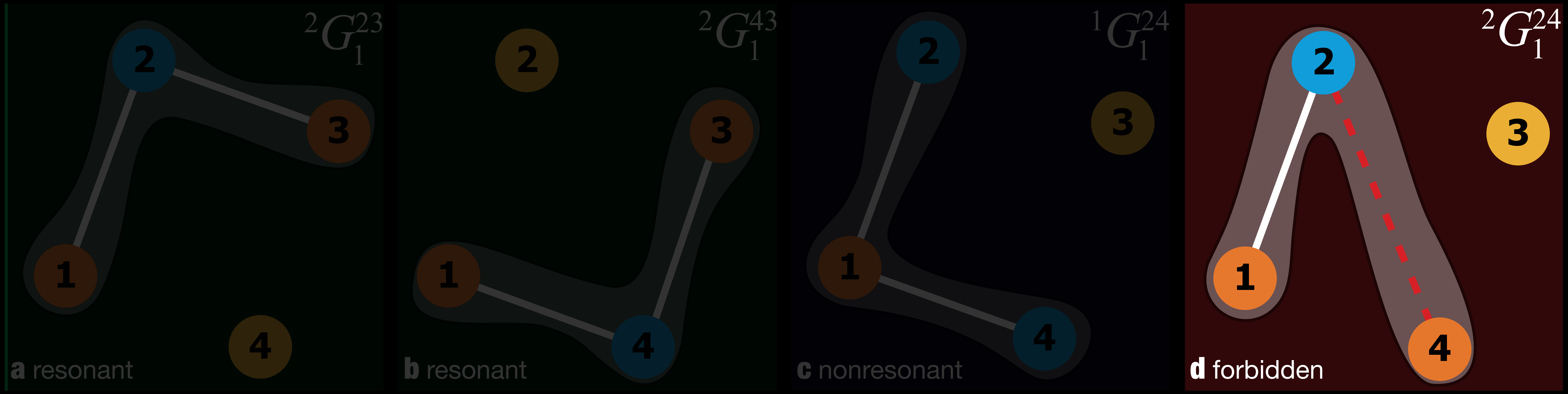
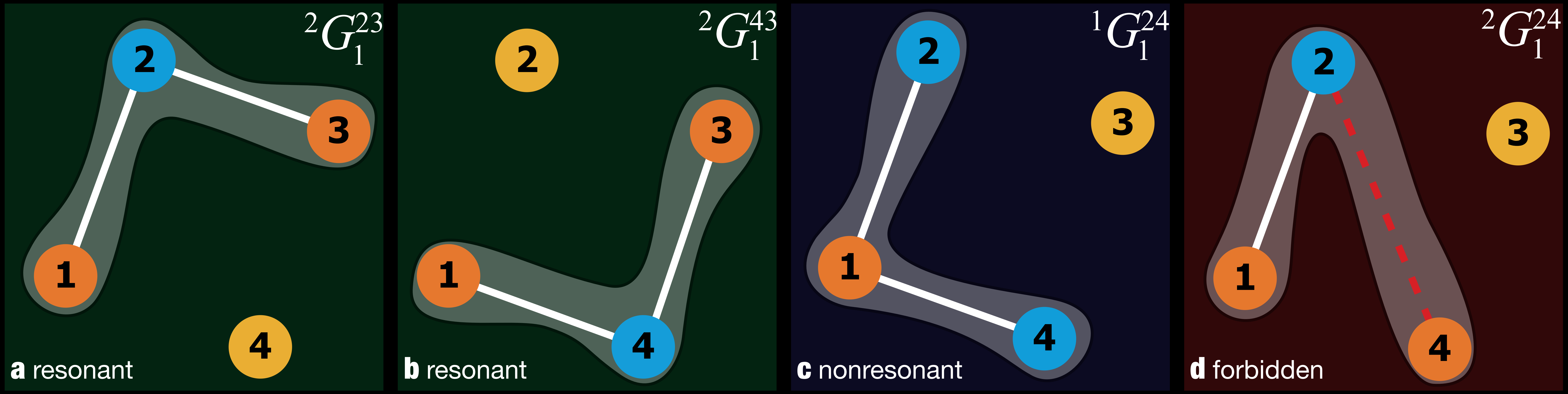


Checking only for node-1

NORMAL FORM THEORY

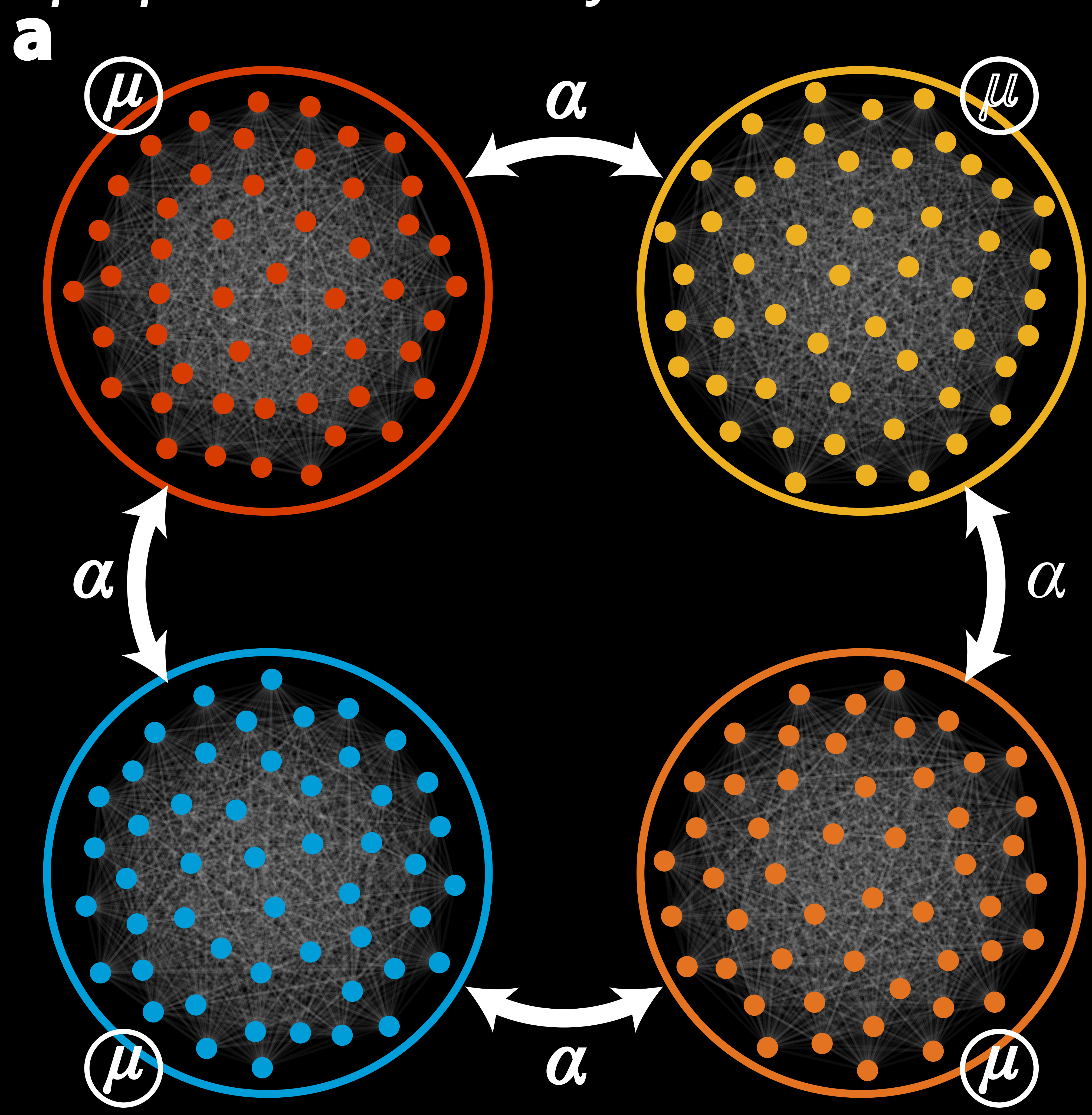
rules for resonant and nonresonant terms



Checking only for node-1

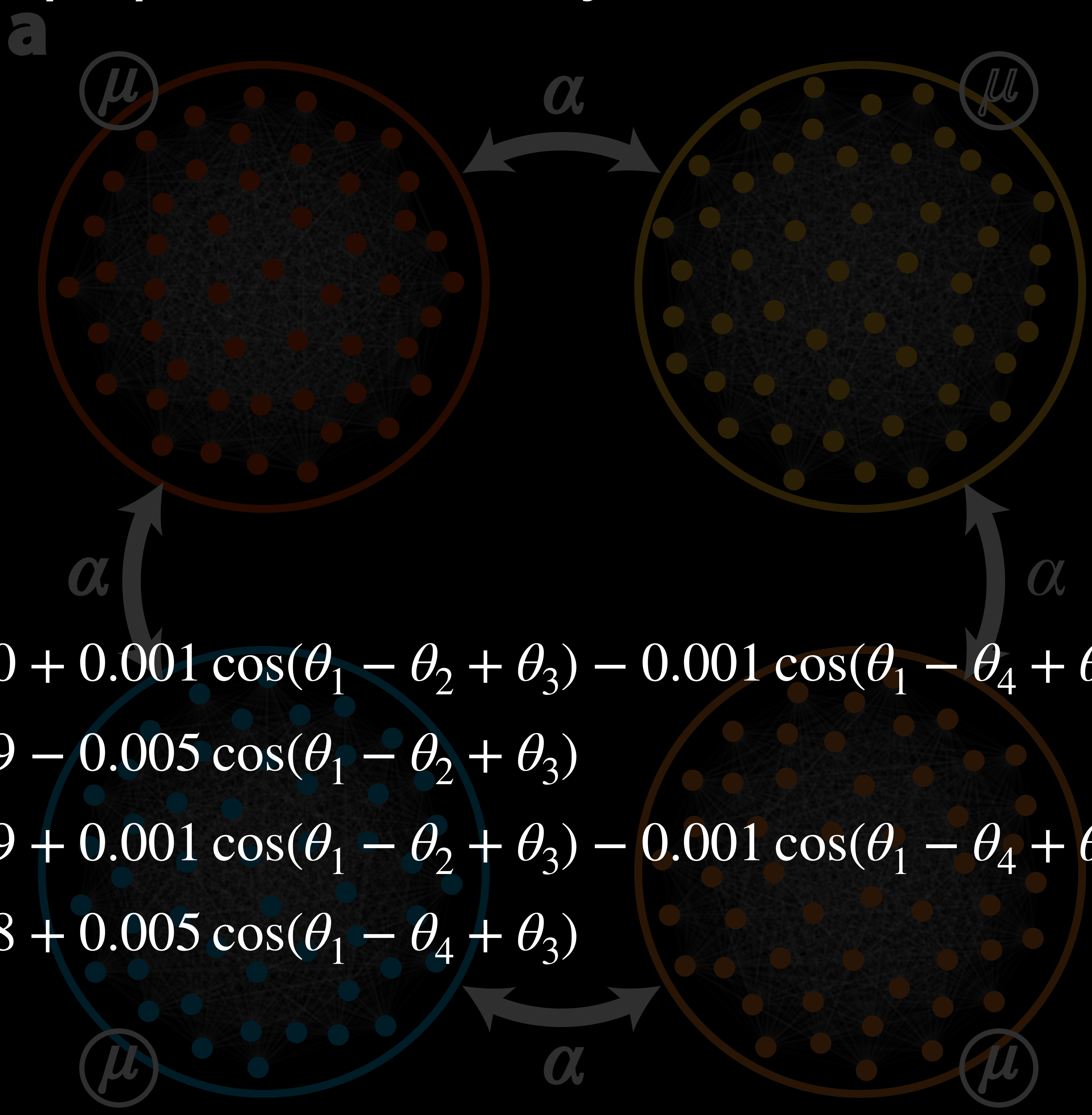
EMERGENT HYPERNETWORK

mean-field pops nonlinearity



EMERGENT HYPERNETWORK

mean-field pops nonlinearity



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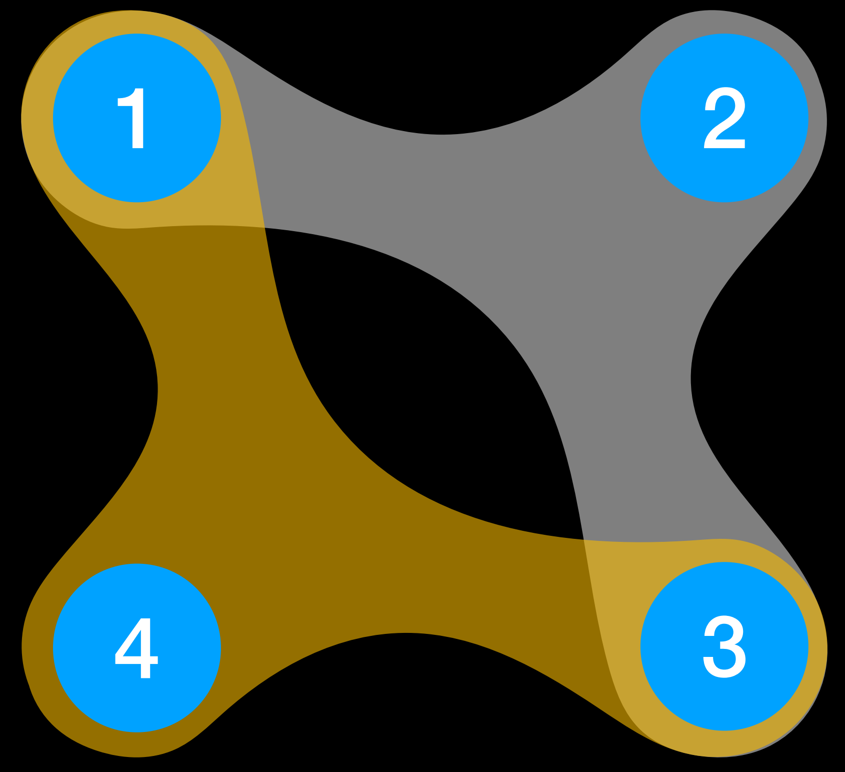
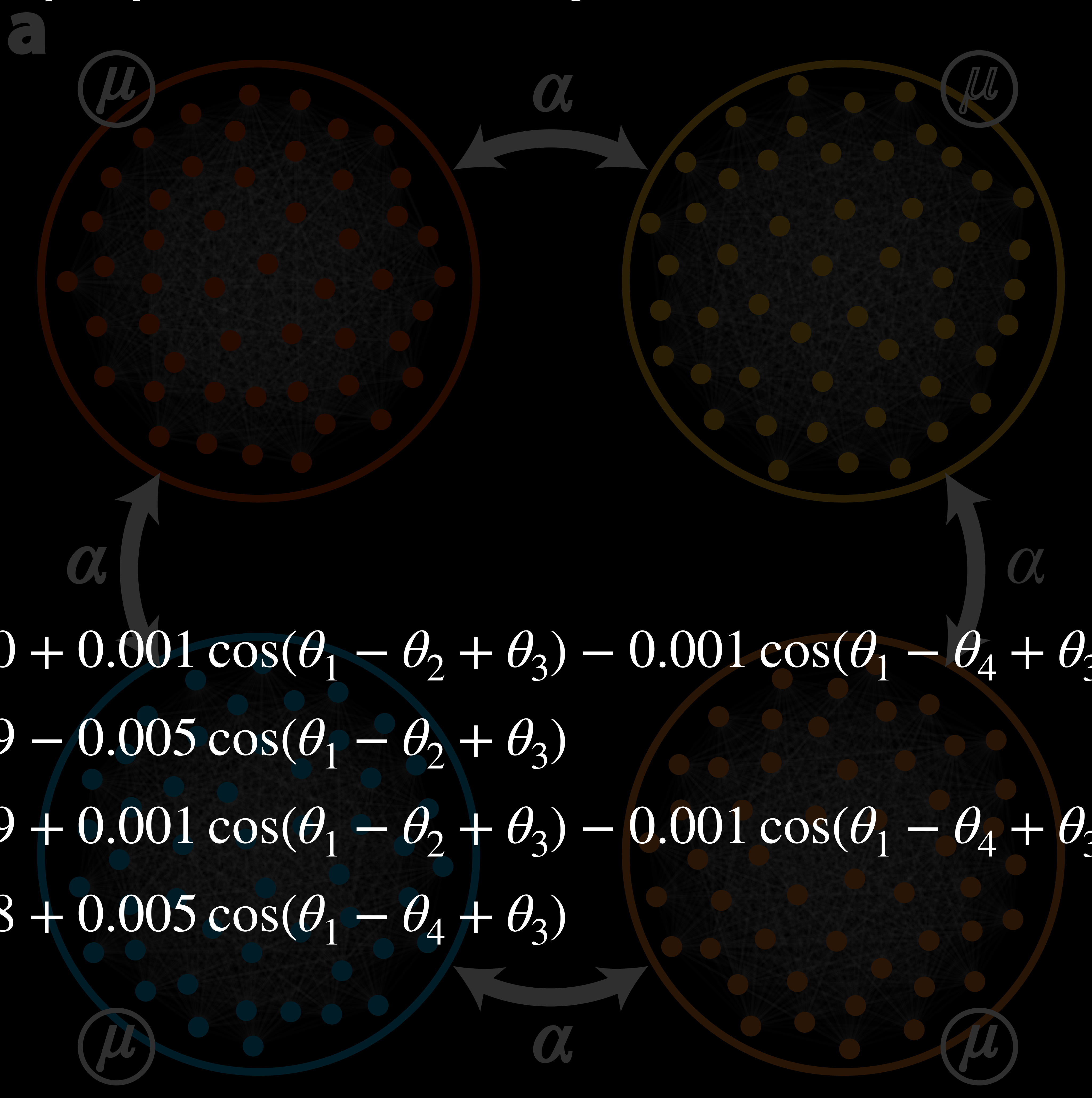


$$\varphi_1 = \theta_1 - \theta_2 + \theta_3$$
$$\varphi_2 = \theta_2 - \theta_4 + \theta_4$$

EMERGENT HYPERNETWORK

mean-field pops nonlinearity



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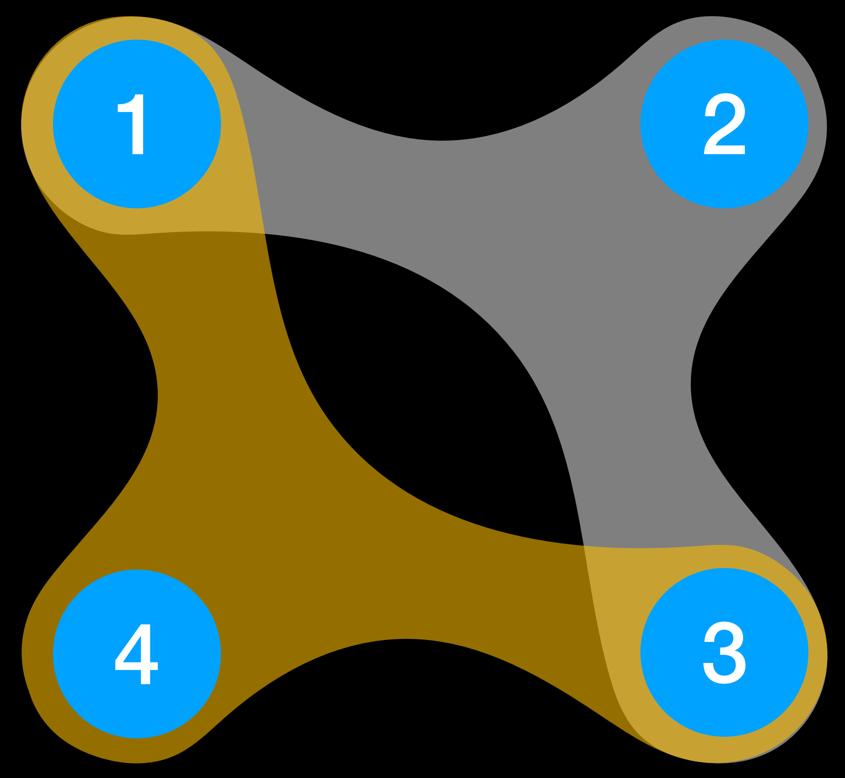
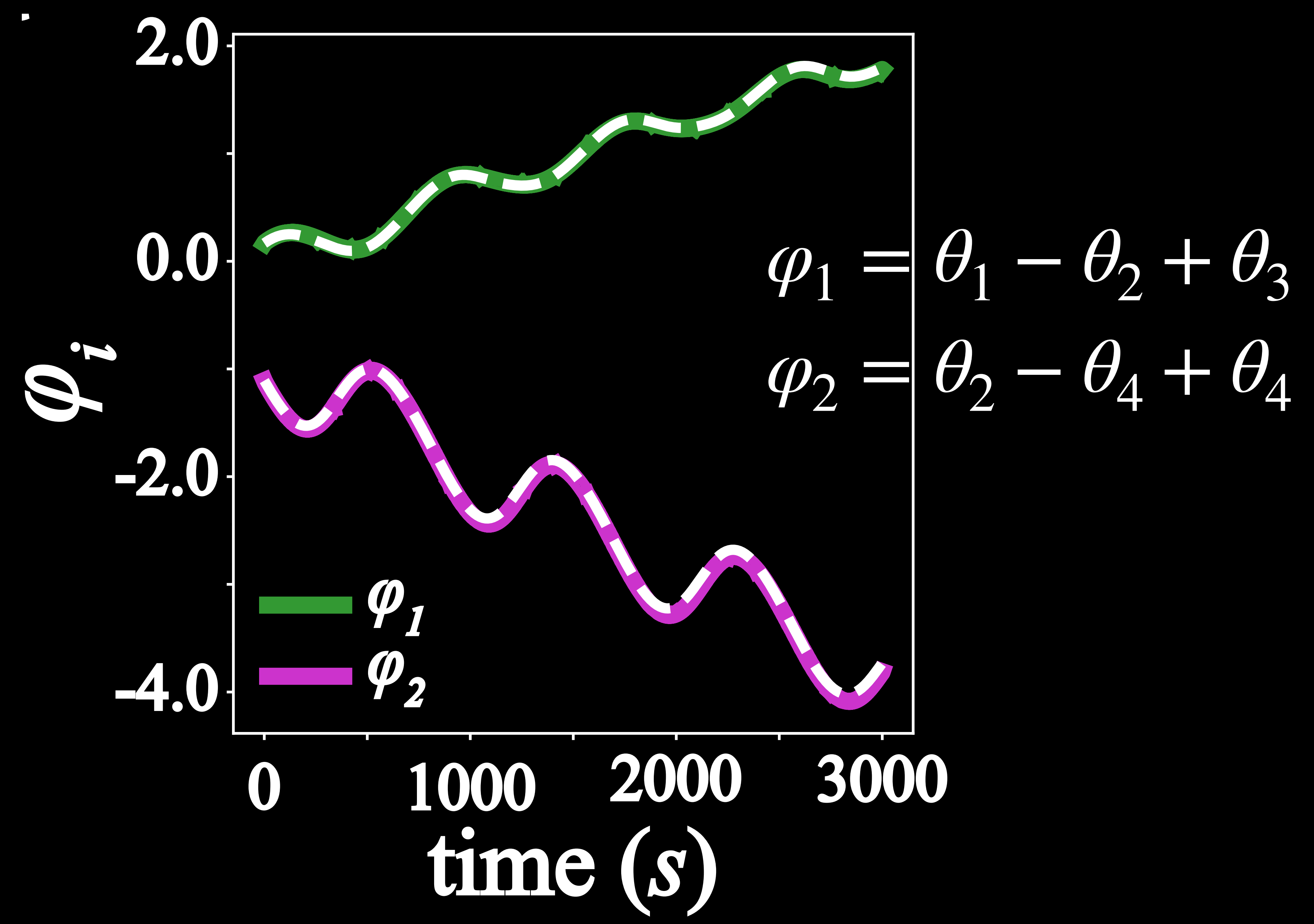
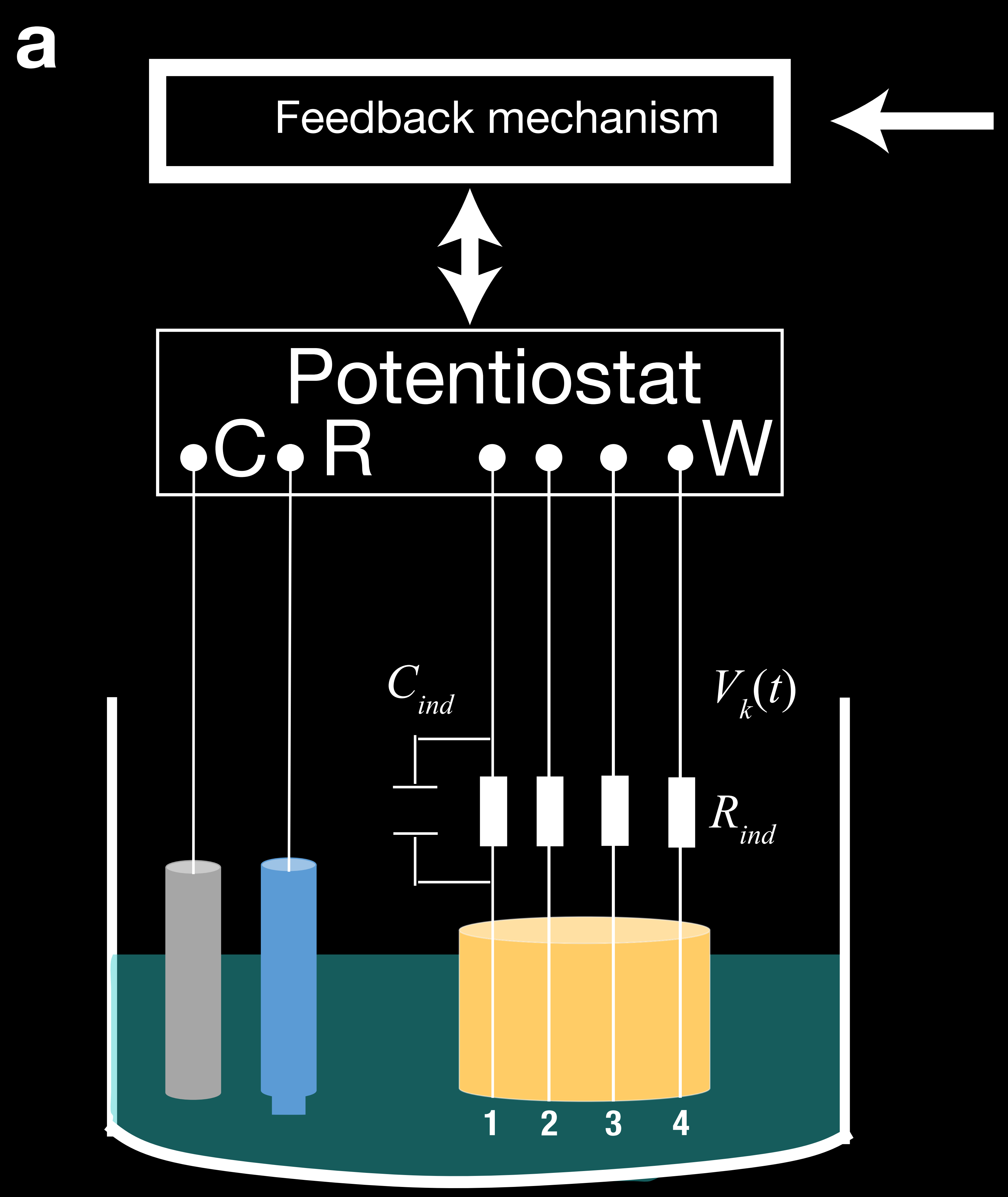
$$\dot{\theta}_4 = 2.508 + 0.005\cos(\theta_1 - \theta_4 + \theta_3)$$





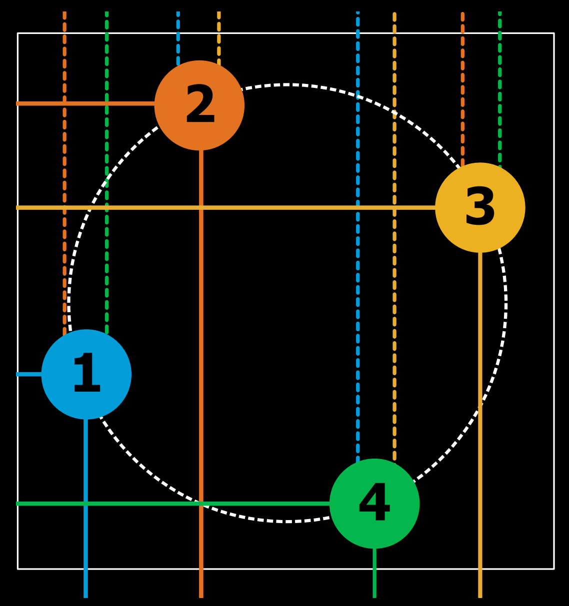
REAL-WORLD EXPERIMENT

electrochemical oscillators



REAL-WORLD EXPERIMENT

electrochemical oscillators



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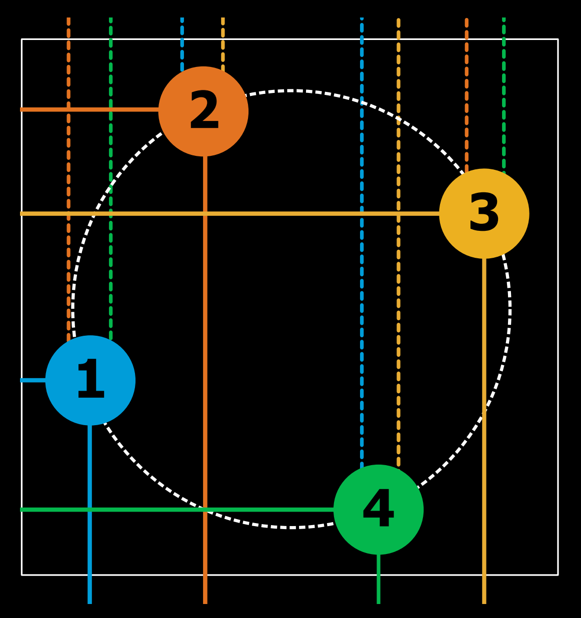
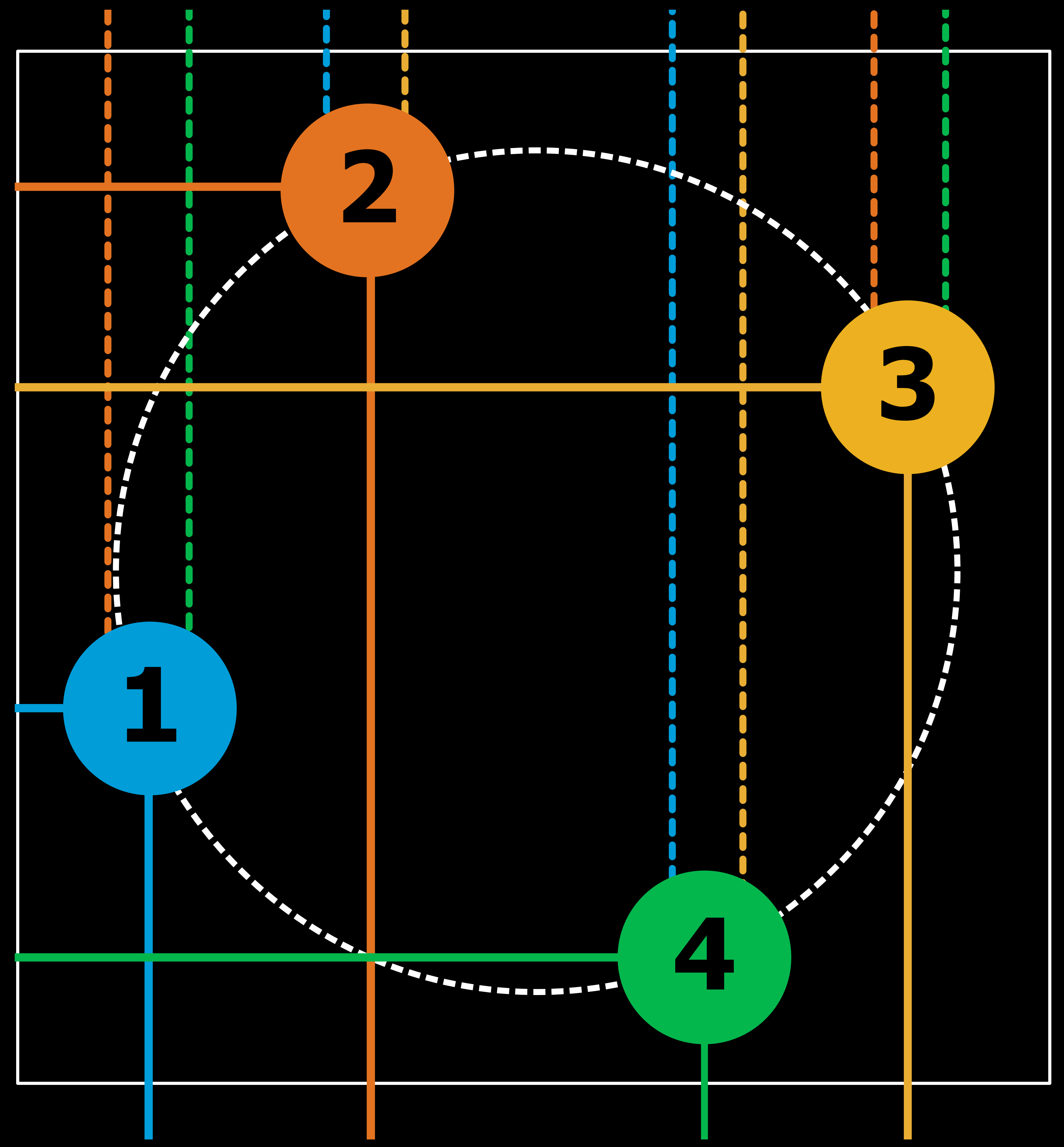
electrochemical oscillators



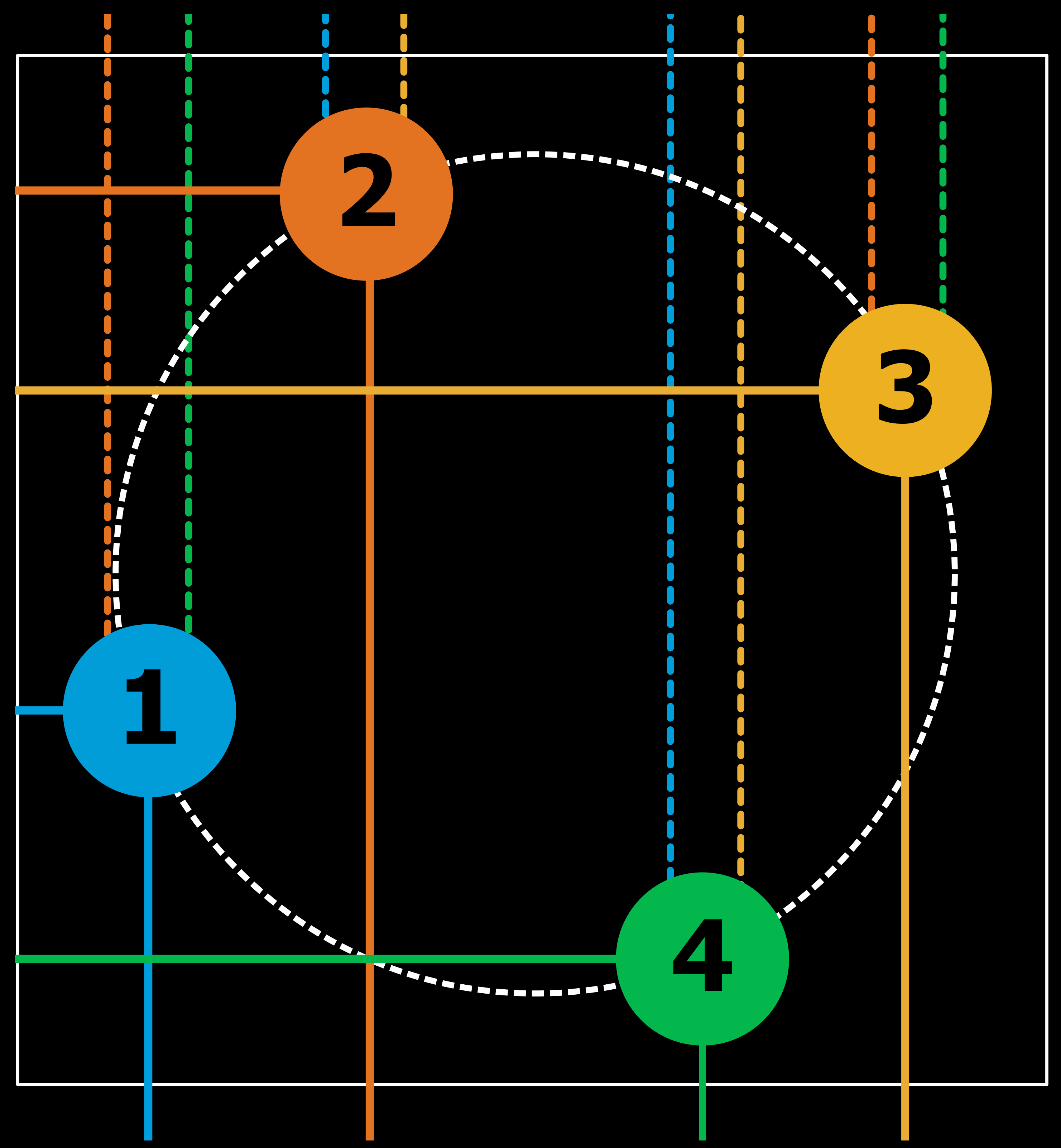
REAL-WORLD EXPERIMENT

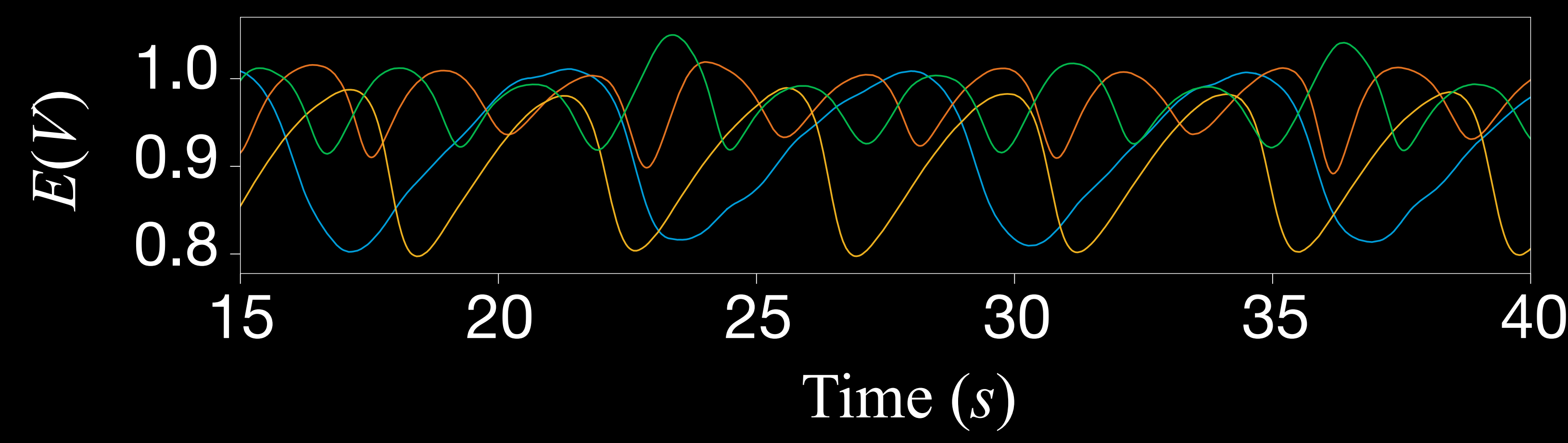
electrochemical oscillators



REAL-WORLD EXPERIMENT

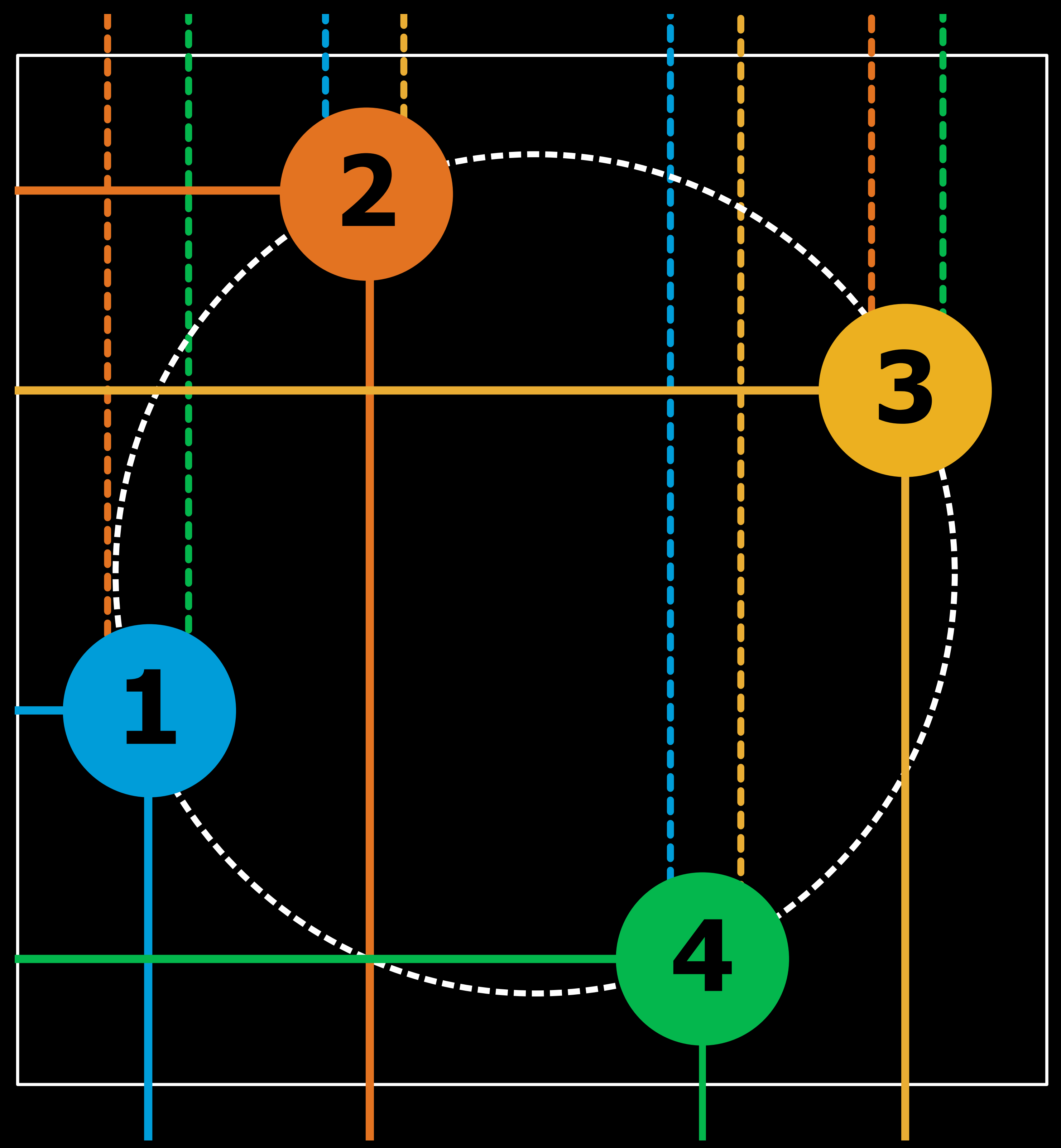
electrochemical oscillators

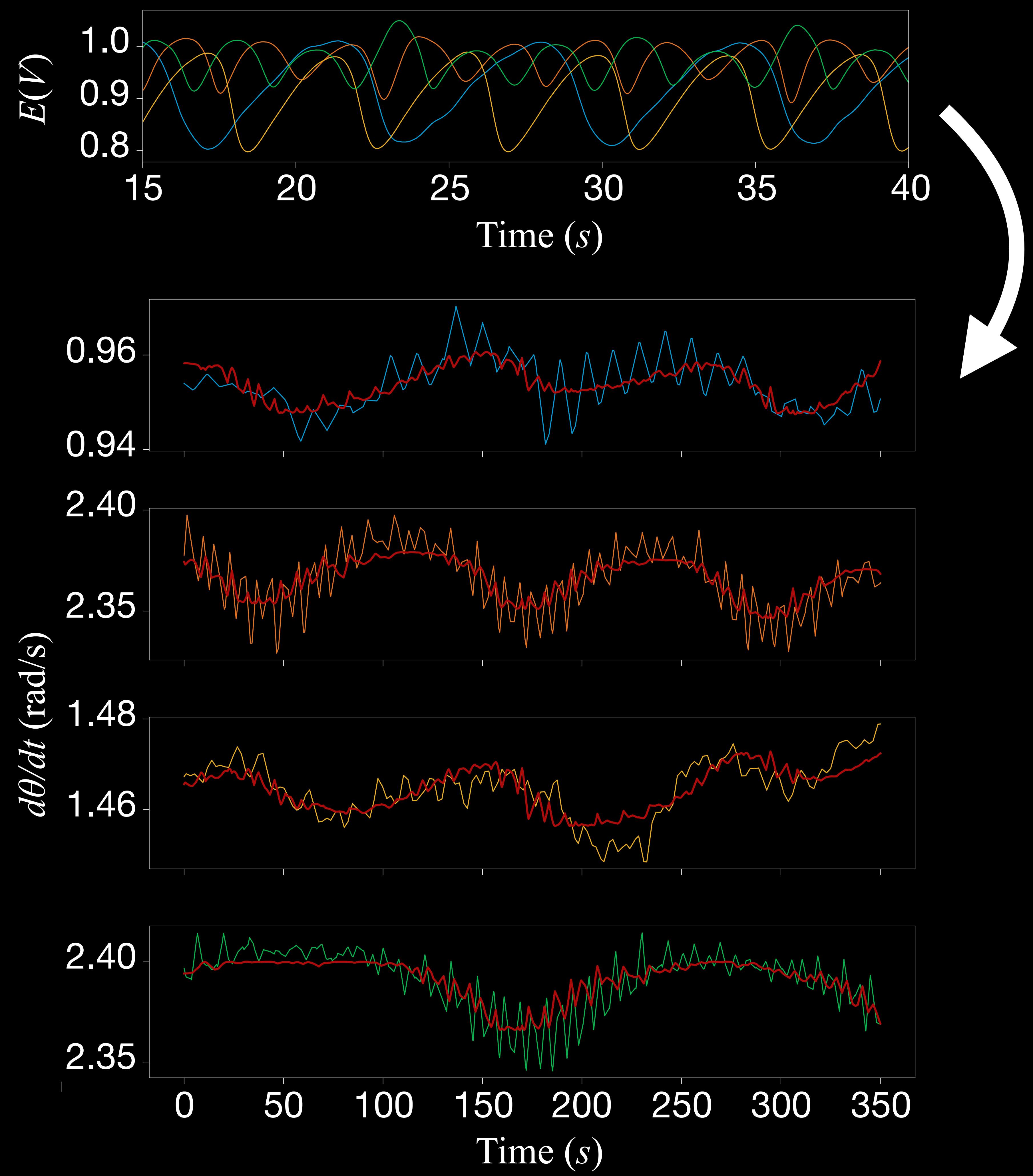




REAL-WORLD EXPERIMENT

electrochemical oscillators





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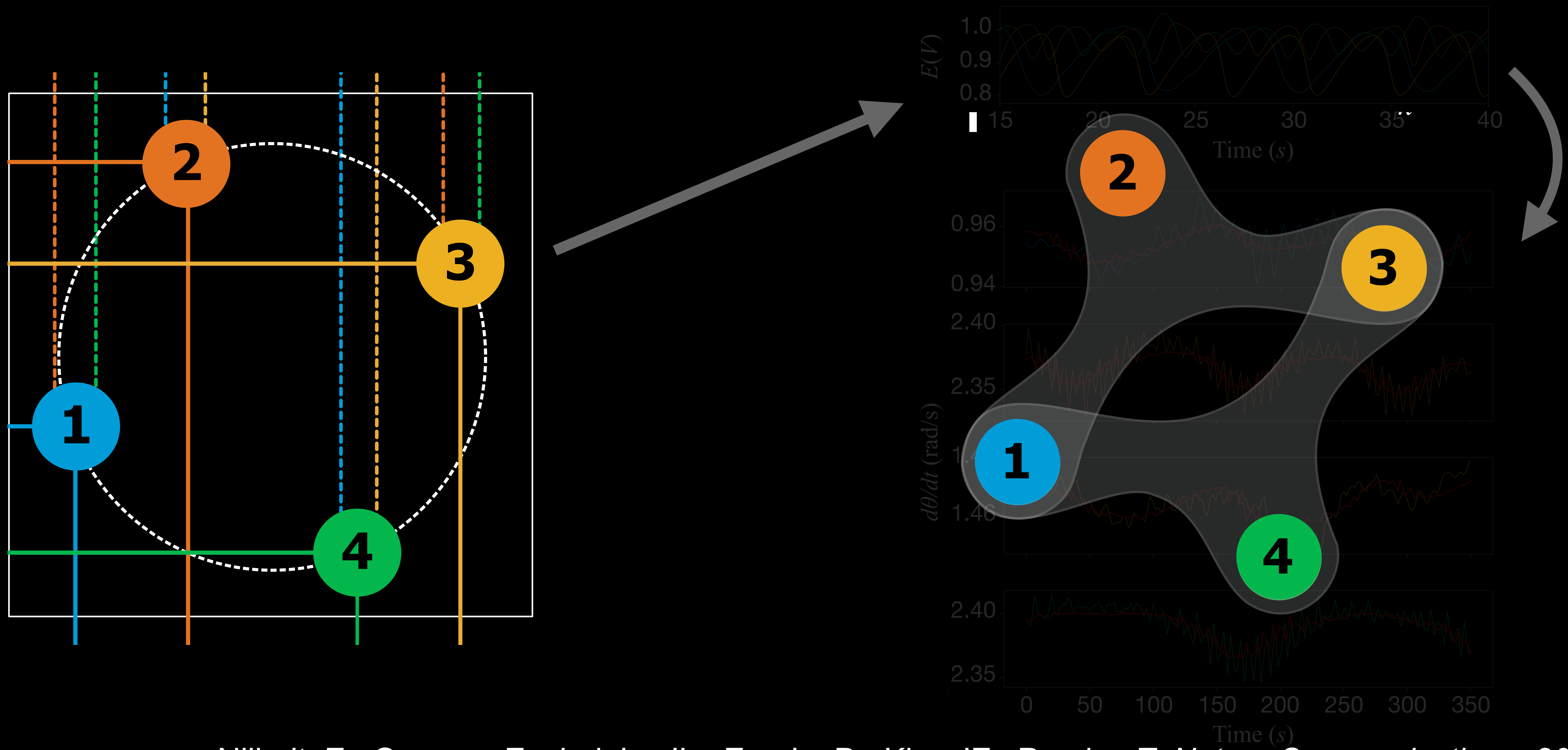
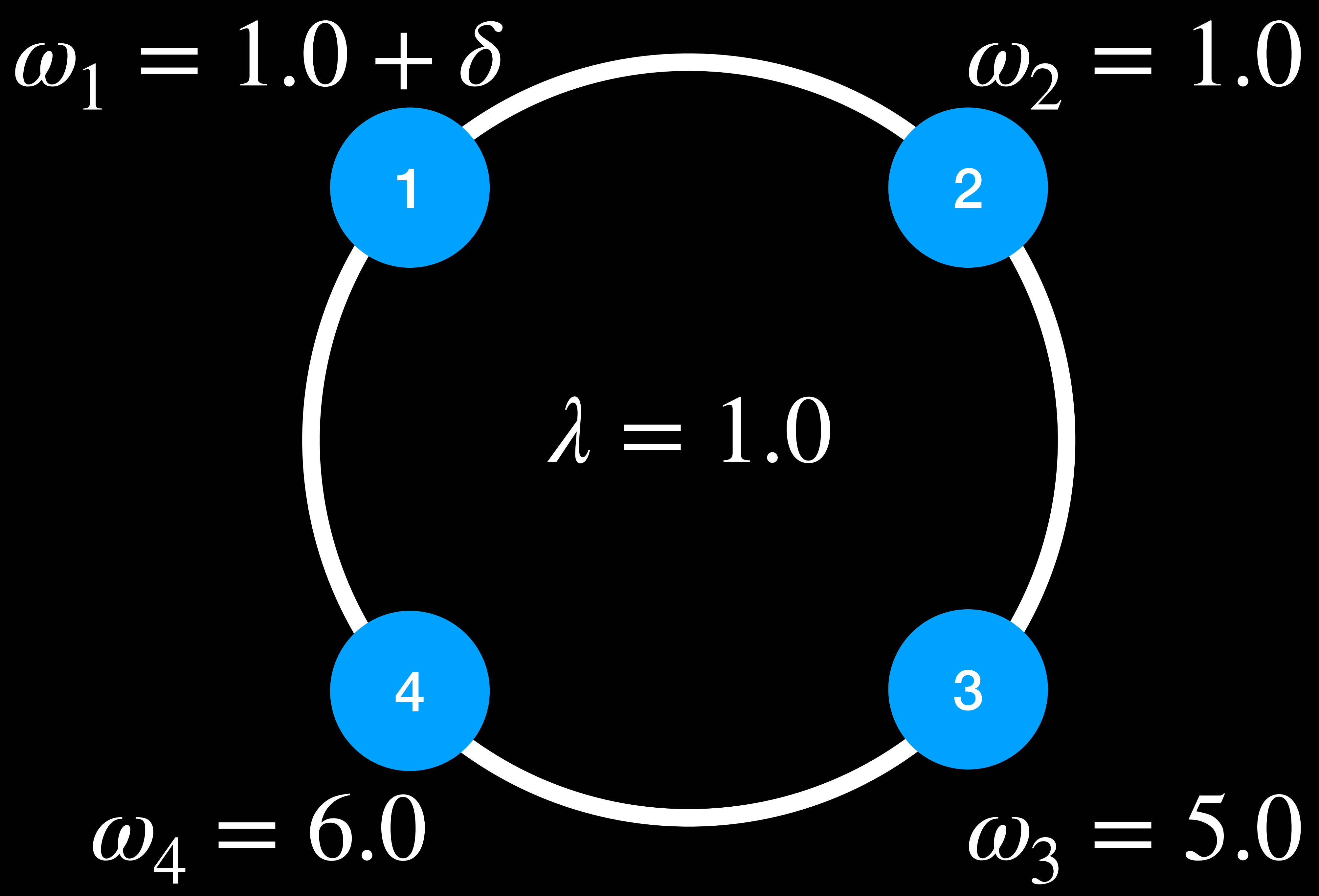
electrochemical oscillators



Nijholt, E., Ocampo-Espindola, JL., Eroglu, D., Kiss, IZ., Pereira, T. *Nature Communications, 2022;* 13:4849

SURPRISING PREDICTIONS!

ring topology with quadratic coupling



$$h(z, w) = z\bar{w}$$

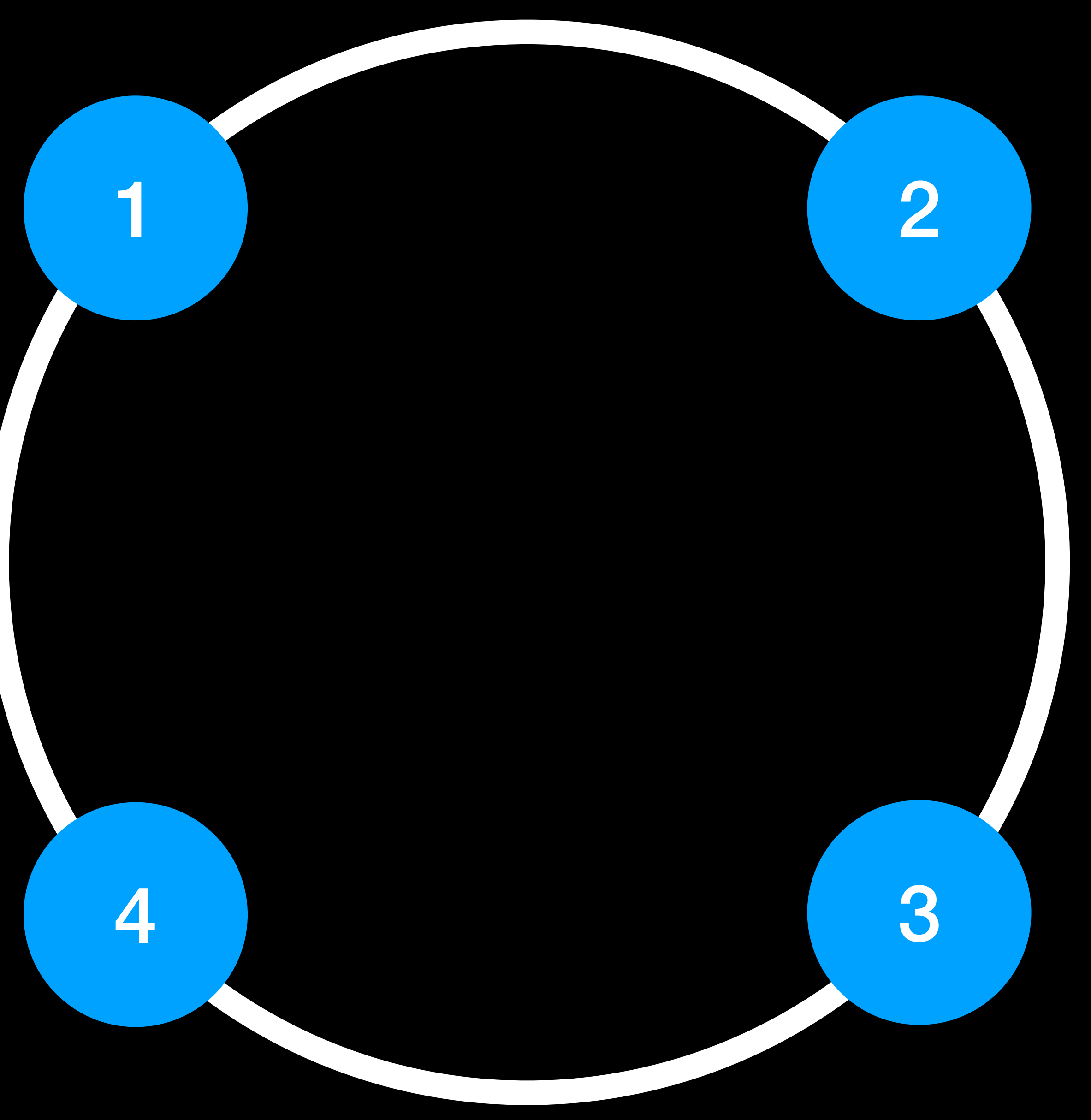
pairwise coupling function

Jacobian vanishes at the origin

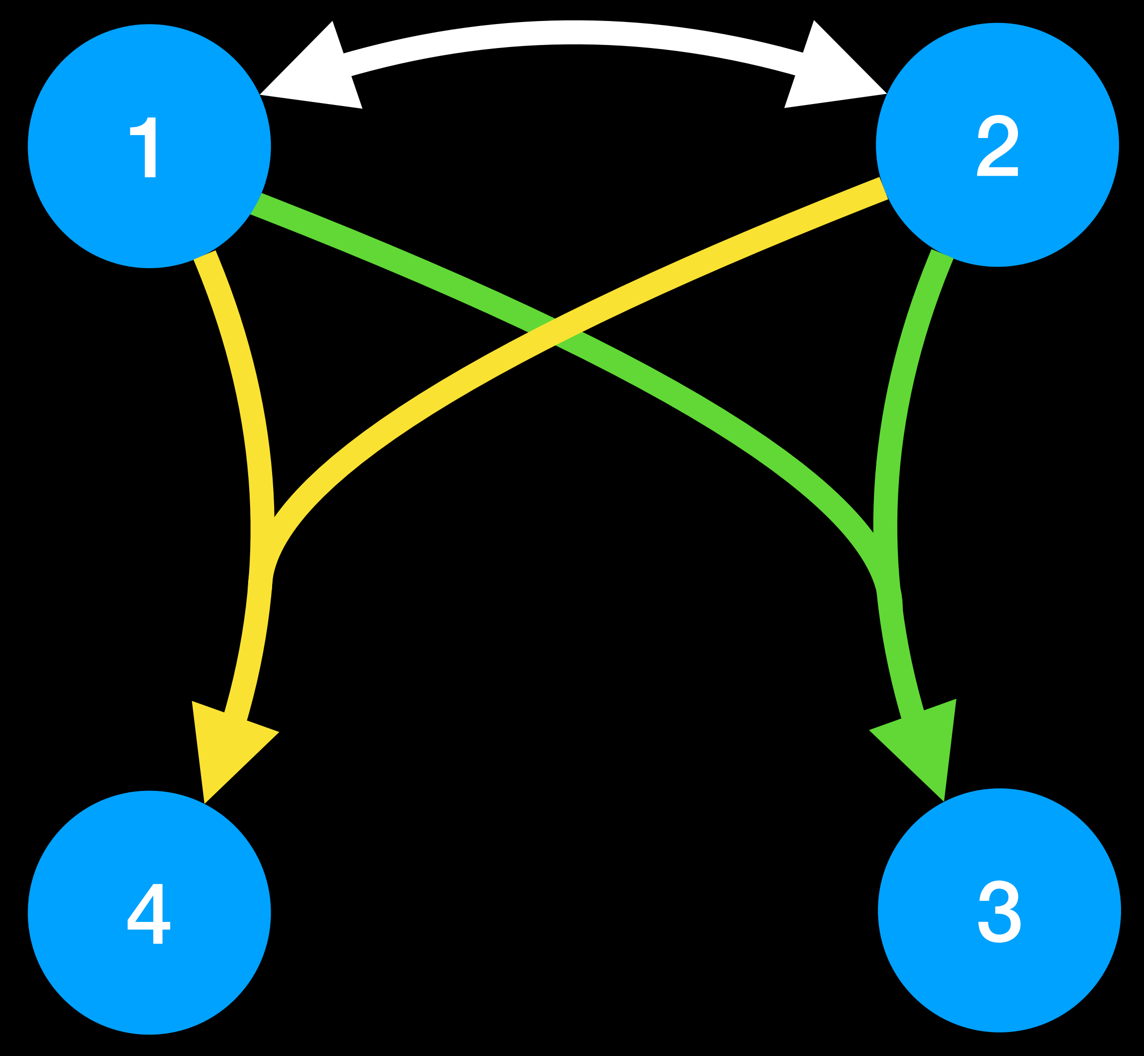
NOT a diffusive coupling!

SURPRISING PREDICTIONS!

ring topology to driven system



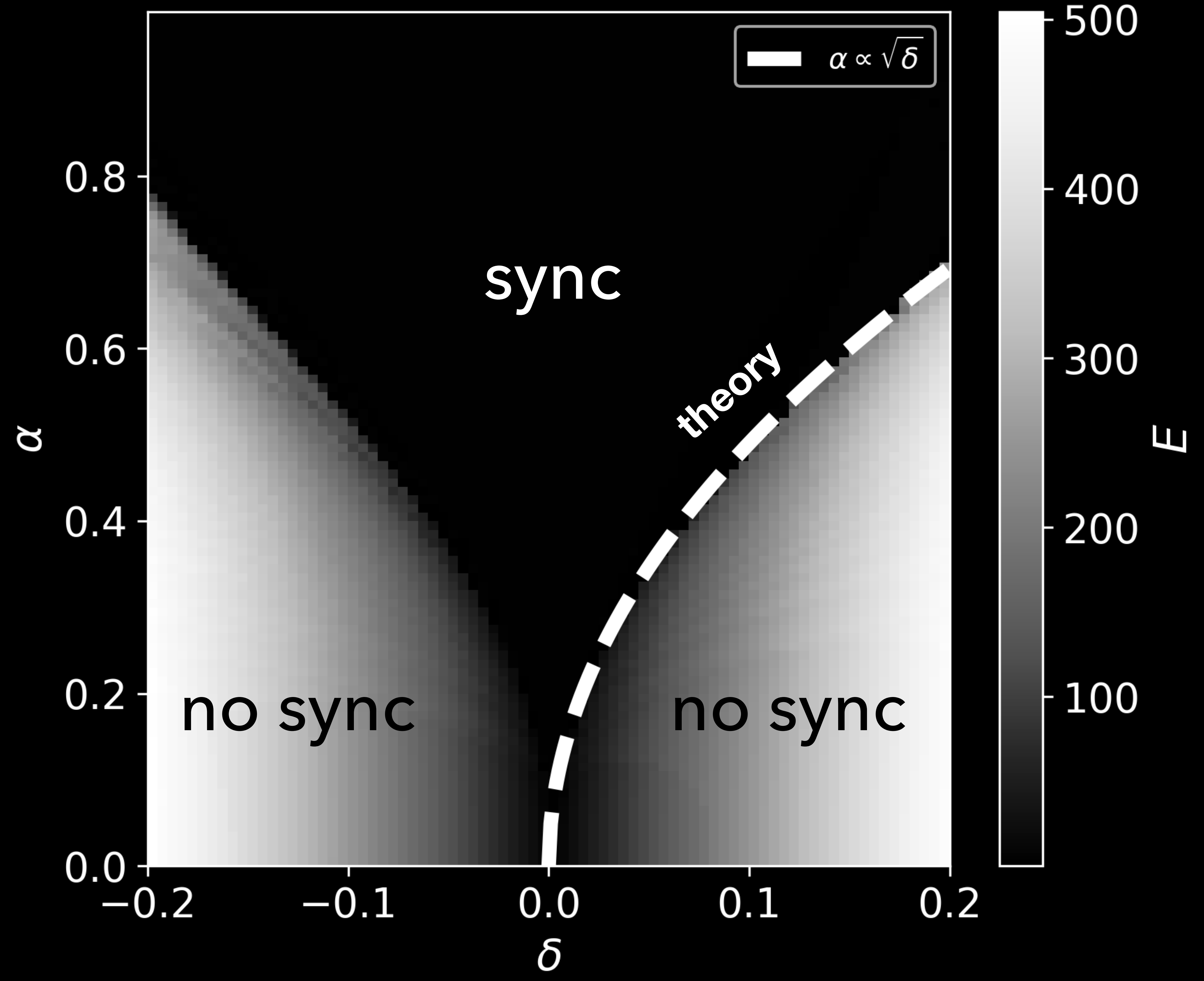
original network



reconstructed hypernetwork

ANOMALOUS SYNC

synchronization tongue



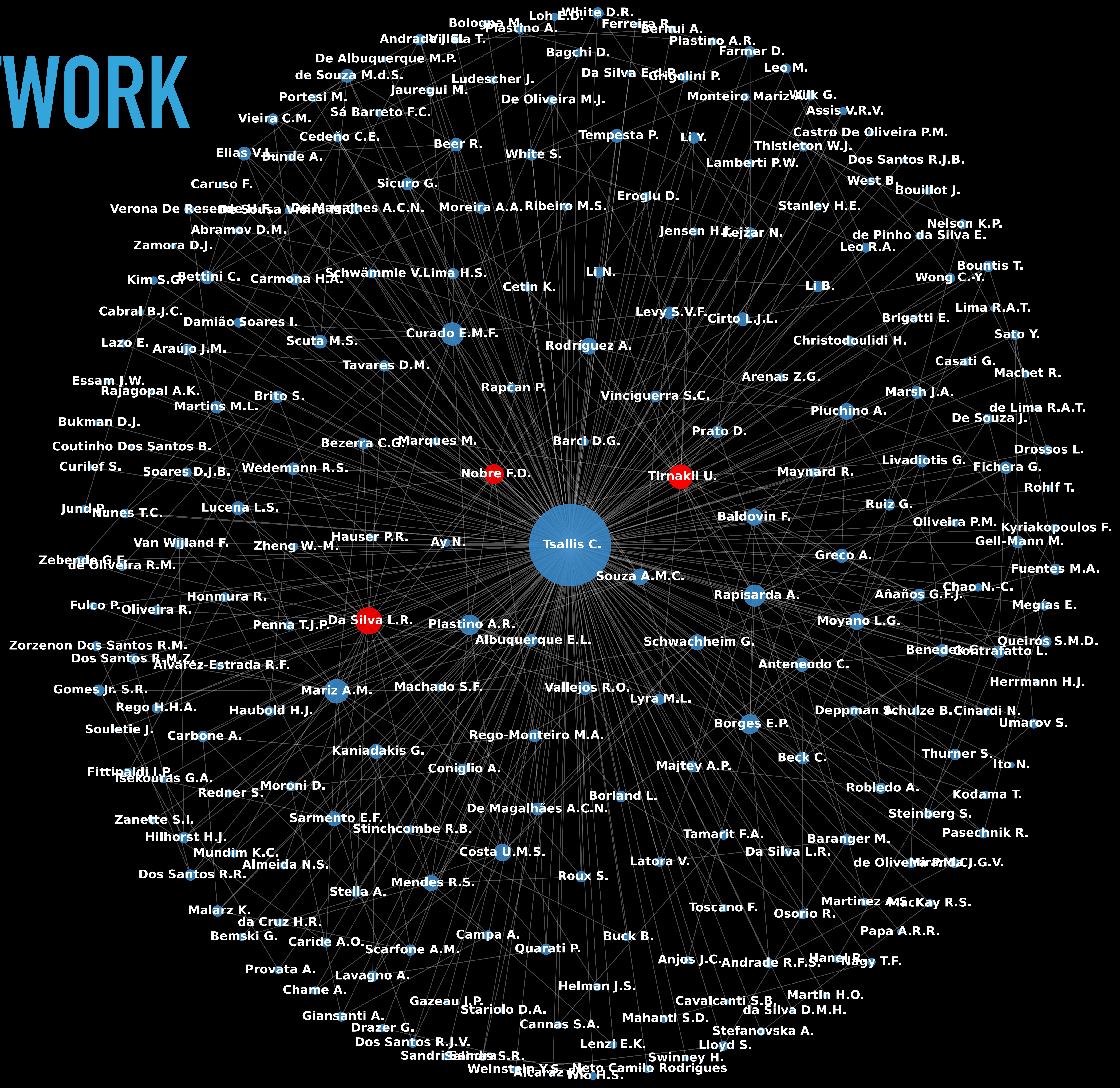
$$E = \frac{1}{T}\sum_{t=1}^{T} \phi(t)$$

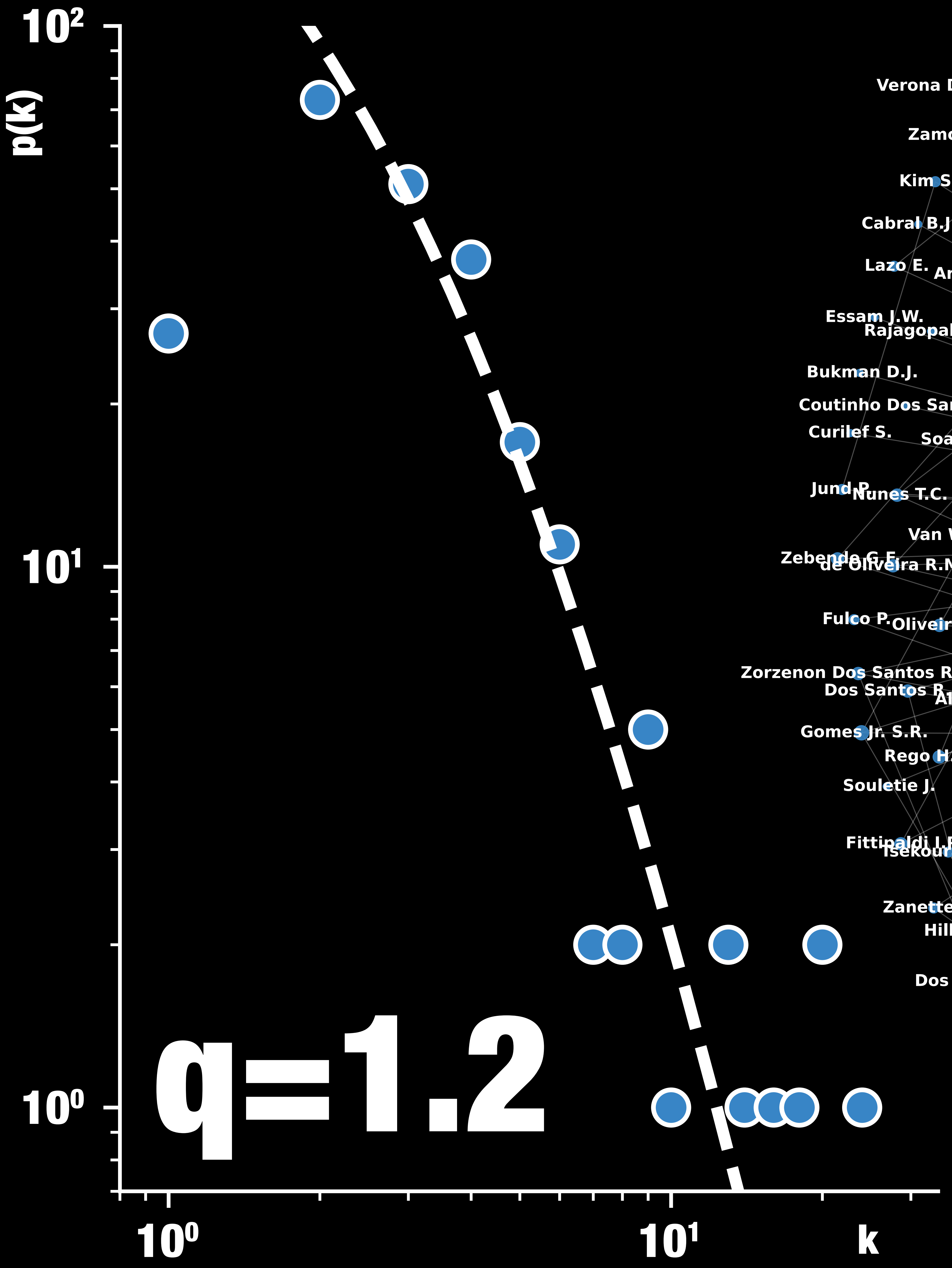
Nijholt, E., Ocampo-Espindola, JL., Eroglu, D., Kiss, IZ., Pereira, T. *Nature Communications, 2022;* 13:4849

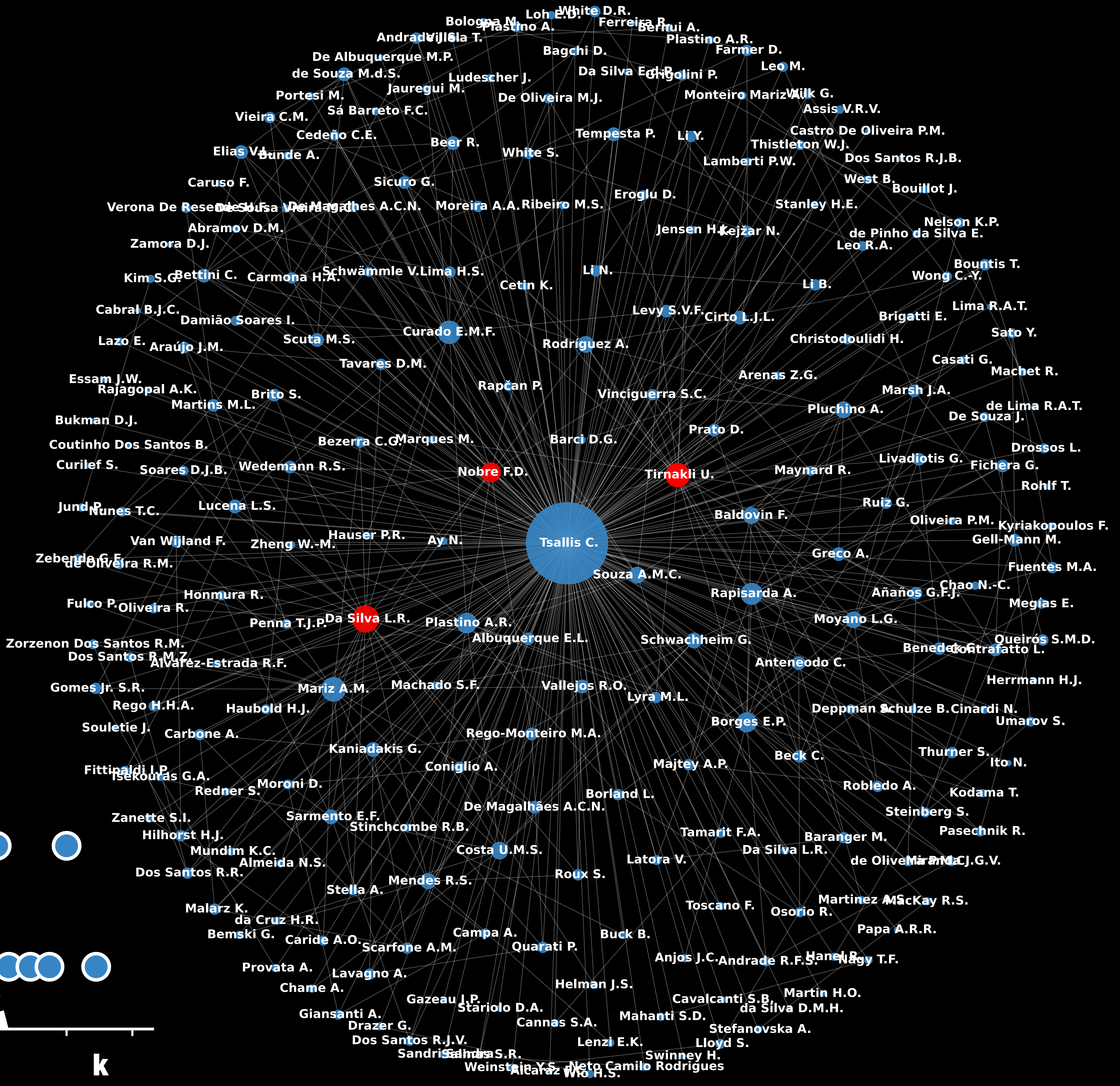
I AM NOT ONLY RECONSTRUCTING BRAIN!

ALSO?

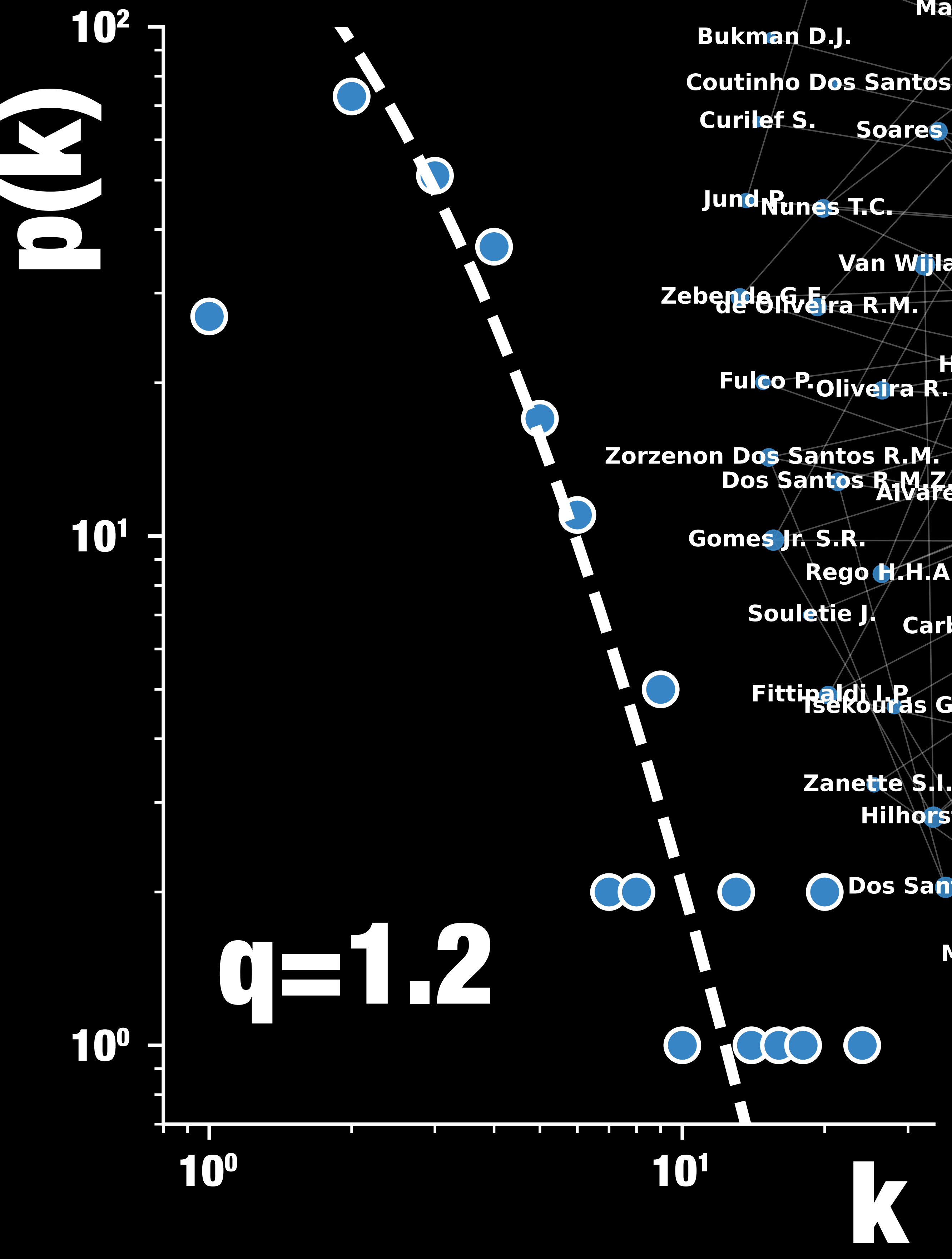
TSALLIS NETWORK

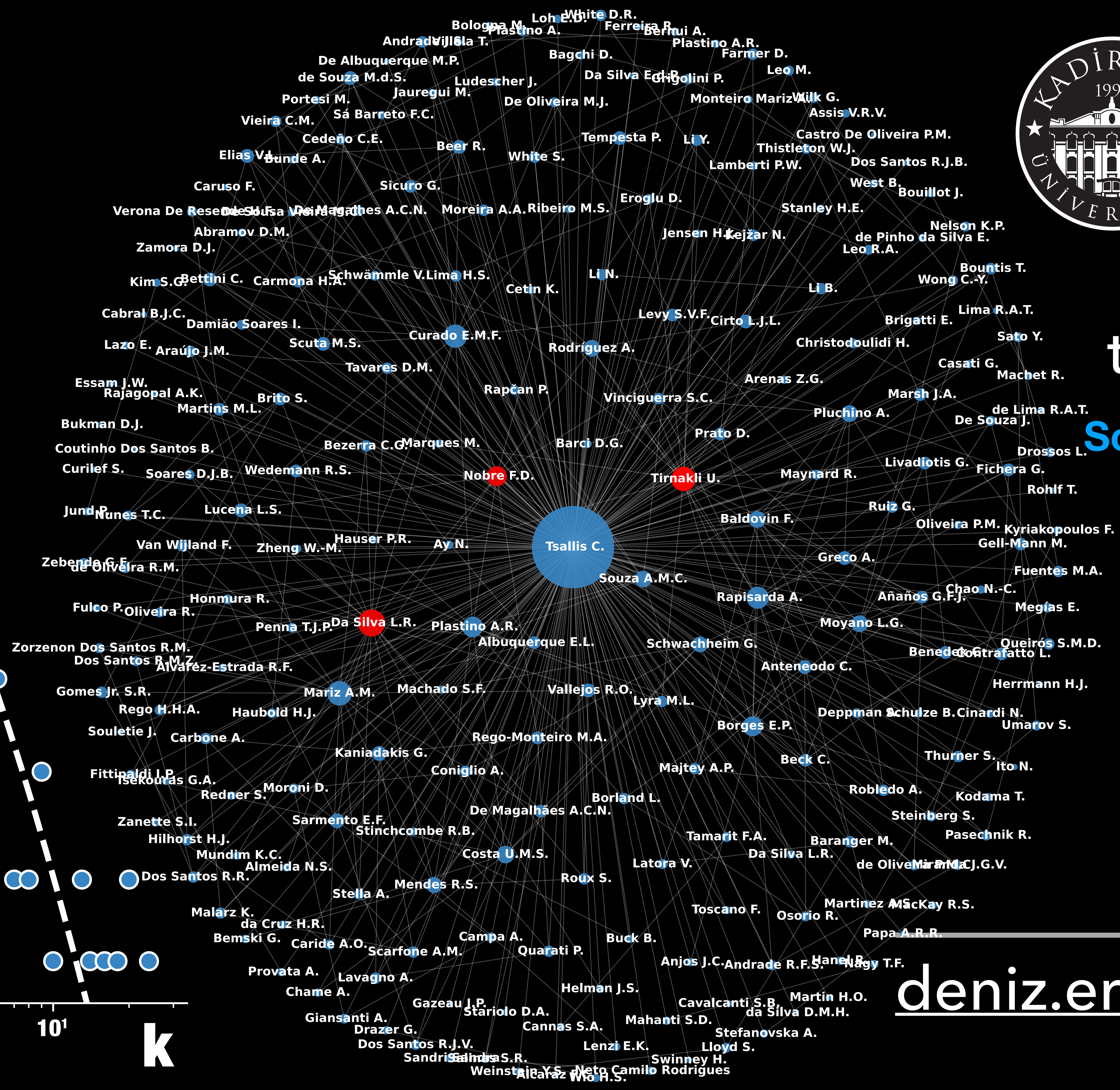



p(k)
10²
10¹
10⁰
10⁰
10¹
k
q=1.2


Tsallis C.
Nobre F.D.
Tirnakli U.
Da Silva L.R.

THANKS!





Network-Oriented Dynamics and Data Science

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FINALLY!!!
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