REFLECTIONS OF HALF-A-CENTURY LIFE IN THE WORLD OF SCIENCE

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SANTA FE INSTITUTE

COMPLEXITY SCIENCE HUB VIENNA

Rio de Janeiro, November 2023

DEFINITIONS: *q-logarithm*: $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-exponential: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k\sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k\sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

Physics Letters A 165 (1992) 409–411
North-Holland

PHYSICS LETTERS A

Introduction of *"Tsallis entropy"* in the literature

On the irreversible nature of the Tsallis and Renyi entropies ☆

Ananias M. Mariz
Departamento de Física, UFRN, Campus Universitário, Caixa Postal 1641, 59072 Natal, RN, Brazil

Received 10 February 1992; accepted for publication 31 March 1992
Communicated by A.R. Bishop

We prove that the detailed balance hypothesis (i.e., $A_{ij}=A_{ji}$, where $\{A_{ij}\}$ are the transition probabilities, per unit time, between any two microscopic configurations i and j) implies *irreversibility* of both the recently introduced Tsallis entropy $S_q^T \equiv [k/(q-1)](1-\sum_{i=1}^{w} P_i^q)$ as well as the Renyi entropy $S_q^R \equiv [k/(1-q)] \ln(\sum_{i=1}^{w} P_i^q)$ $(q \in \mathbb{R})$. More precisely, for $q>0$, $q=0$ and $q<0$ we have respectively $dS/dt \geqslant 0$, $dS/dt=0$ and $dS/dt \leqslant 0$ $(S=S_q^T, S_q^R)$, where the equality holds for equilibrium.

J. Phys. A: Math. Gen. 27 (1994) 3663–3670. Printed in the UK

The fluctuation–dissipation theorem in the framework of the Tsallis statistics

Introduction of *"Tsallis statistics"* in the literature

A Chame and E V L de Mello
Instituto de Fisica, Universidade Federal Fluminense, Outeiro São João Batista s/nº—Centro 24.030.130—Niterói, RJ, Brazil

Received 28 September 1993

Abstract. In the framework of a generalized statistical mechanics introduced recently by Tsallis, we derive a generalized form of the fluctuation–dissipation theorem, which expresses a relation between extended susceptibilities and equilibrium fluctuations. To achieve this, we consistently propose a generalized functional form for the instantaneous distribution function. The present theorem recovers as particular cases the corresponding generalized relations already obtained for the specific heat in terms of the generalized energy fluctuations and for the susceptibility of a magnetic system under the action of a uniform magnetic field.

Physics Letters A 174 (1993) 384–386
North-Holland

PHYSICS LETTERS A

Stellar polytropes and Tsallis' entropy

A.R. Plastino and A. Plastino
Physics Department, National University La Plata, C.C. 67, 1900 La Plata, Argentina

Received 29 October 1992; accepted for publication 18 December 1992
Communicated by A.R. Bishop

It is shown that recourse to Tsallis' generalized entropy makes it possible to find sensible distribution functions for stellar polytropes, while that of Boltzmann yields unphysical distributions. Additionally, some constraints are imposed on Tsallis' entropy.

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\,\frac{S_q(A)}{k}\,\frac{S_q(B)}{k}$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

Equivalently, $S_q(A+B) = S_q(A) + S_q(B) + \boxed{\frac{1-q}{k}}\,S_q(A)\,S_q(B)$

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \;,\; i.e.,\;\; S(N) \propto N \;\; (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

$W \equiv$ total number of possibilities with nonzero probability, assumed to be equally probable

If $W(N) \sim \mu^N \quad (\mu > 1) \quad \Rightarrow S_{BG}(N) = k \ln W(N) \propto N$ $\quad$ *OK*!

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N$$ *OK*!

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k\ [\ln W(N)]^\delta \propto N^{\gamma\,\delta} \Rightarrow S_{\delta=1/\gamma}(N) \propto N$$ *OK*!

If $W(N) \sim D\ \ln N \quad (D > 0)$

$$\Rightarrow S_\lambda^C(N) = k\ [e^{\lambda W(N)} - e^\lambda]\ \Rightarrow S_\lambda^C(N) \sim kN^{\lambda D}$$

$$\Rightarrow S^C_{\lambda=1/D}(N) \propto N$$ *OK*!

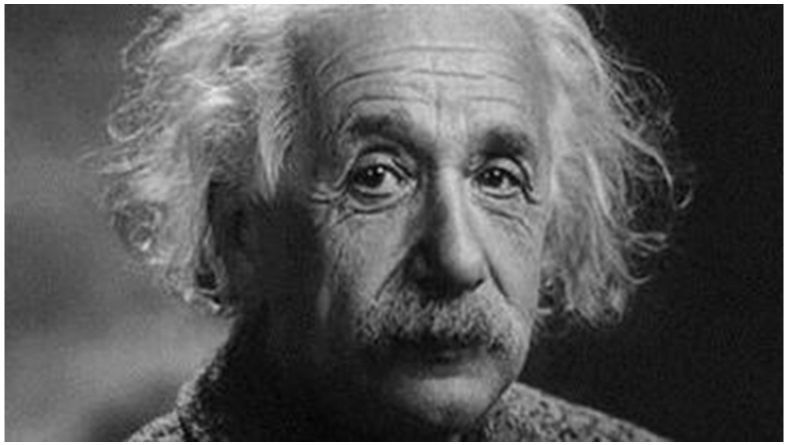
IMPORTANT: $\mu^N >> \nu^{N^\gamma} >> N^\rho >> \ln N \quad \text{if} \quad N >> 1$

All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy (*Anna Karenina*, 1875-1877)

SYSTEMS $W(N)$ (*equiprobable*)	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)	ENTROPY S_λ^{C} $(\lambda > 0)$ (NONADDITIVE)
$\sim A\,\mu^N$ $(A>0, \mu>1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$\sim B\,N^\rho$ $(B>0, \rho>0)$	NONEXTENSIVE	EXTENSIVE $(q = 1 - 1/\rho)$	NONEXTENSIVE	NONEXTENSIVE
$\sim C\nu^{N^\gamma}$ $(C>0, \nu>1, 0<\gamma<1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\delta = 1/\gamma)$	NONEXTENSIVE
$\sim D\,\ln N$ $(D>0)$	NONEXTENSIVE	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\lambda = 1/D)$

A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown. Albert Einstein (1949)



Eur. Phys. J. Special Topics **223**, 2161–2175 (2014)

DOI: 10.1140/epjst/e2014-02256-1

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Review

Thermodynamics is more powerful than the role to it reserved by Boltzmann-Gibbs statistical mechanics

C. Tsallis[1,2,a] and L.J.L. Cirto[1,b]

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[2] Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

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A Letters Journal Exploring the Frontiers of Physics

Highlights 2015

A compilation of the best papers published within the last year

STATISTICAL & QUANTUM MECHANICS, QUANTUM INFORMATION & COMPLEX NETWORKS

OPEN ACCESS

Boltzmann-Gibbs entropy is sufficient but not necessary for the likelihood factorization required by Einstein

Constantino Tsallis

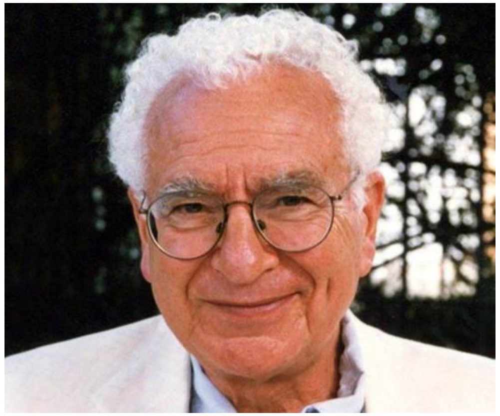
Constantino Tsallis and Hans J. Haubold

2015 *EPL* **110** 30005

In 1910 Einstein published a work on a crucial aspect of his understanding of the Boltzmann entropy. He essentially argued that the likelihood function of any system composed by two probabilistically independent subsystems *ought* to be factorizable into the likelihood functions of each of the subsystems. Consistently he was satisfied by the fact that the Boltzmann (additive) entropy fulfills this epistemologically fundamental requirement. We show here that entropies (e.g., the q-entropy on which nonextensive statistical mechanics is based) which generalize the BG one through violation of its well-known additivity can *also* fulfill the same requirement. This important fact sheds light on the very foundations of the connection between the micro- and macro-scopic worlds, and consistently supports that the classical thermodynamical Legendre structure is more powerful than the role to it reserved by the Boltzmann-Gibbs statistical mechanics.

Milan j. math. 76 (2008), 307–328
© 2008 Birkhäuser Verlag Basel/Switzerland
1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics



M. Gell-Mann

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
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(Received 10 November 2009; accepted 4 January 2010; published online 3 March 2010)

See also:

H.J. Hilhorst, JSTAT P10023 (2010)
M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)
M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)
A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)
A. Plastino and M.C. Rocca, Physica A **392**, 3952 (2013)
S. Umarov and C. T., J Phys A **49**, 415204 (2016)

SLOW CHEMICAL REACTION

RESEARCH BRIEFINGS | 01 March 2023

Rate of quantum-tunnelling reaction revealed

A physical phenomenon called quantum tunnelling is rare in chemical reactions, making it difficult to study theoretically and experimentally. The measurement of the tunnelling rate in a hydrogen reaction has enabled the verification of quantum-tunnelling calculations, providing a benchmark for testing future quantum calculations.

$$D^- + H_2 \rightarrow H^- + HD$$

Article

Tunnelling measured in a very slow ion–molecule reaction

https://doi.org/10.1038/s41586-023-05727-z

Received: 13 July 2022

Accepted: 12 January 2023

Published online: 1 March 2023

Check for updates

Robert Wild[1], Markus Nötzold[1], Malcolm Simpson[1], Thuy Dung Tran[1,2] & Roland Wester[1]✉

Quantum tunnellin
pathways are energ
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allow for accurate f
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$H_2 + D^- \rightarrow H^- + HD$, h
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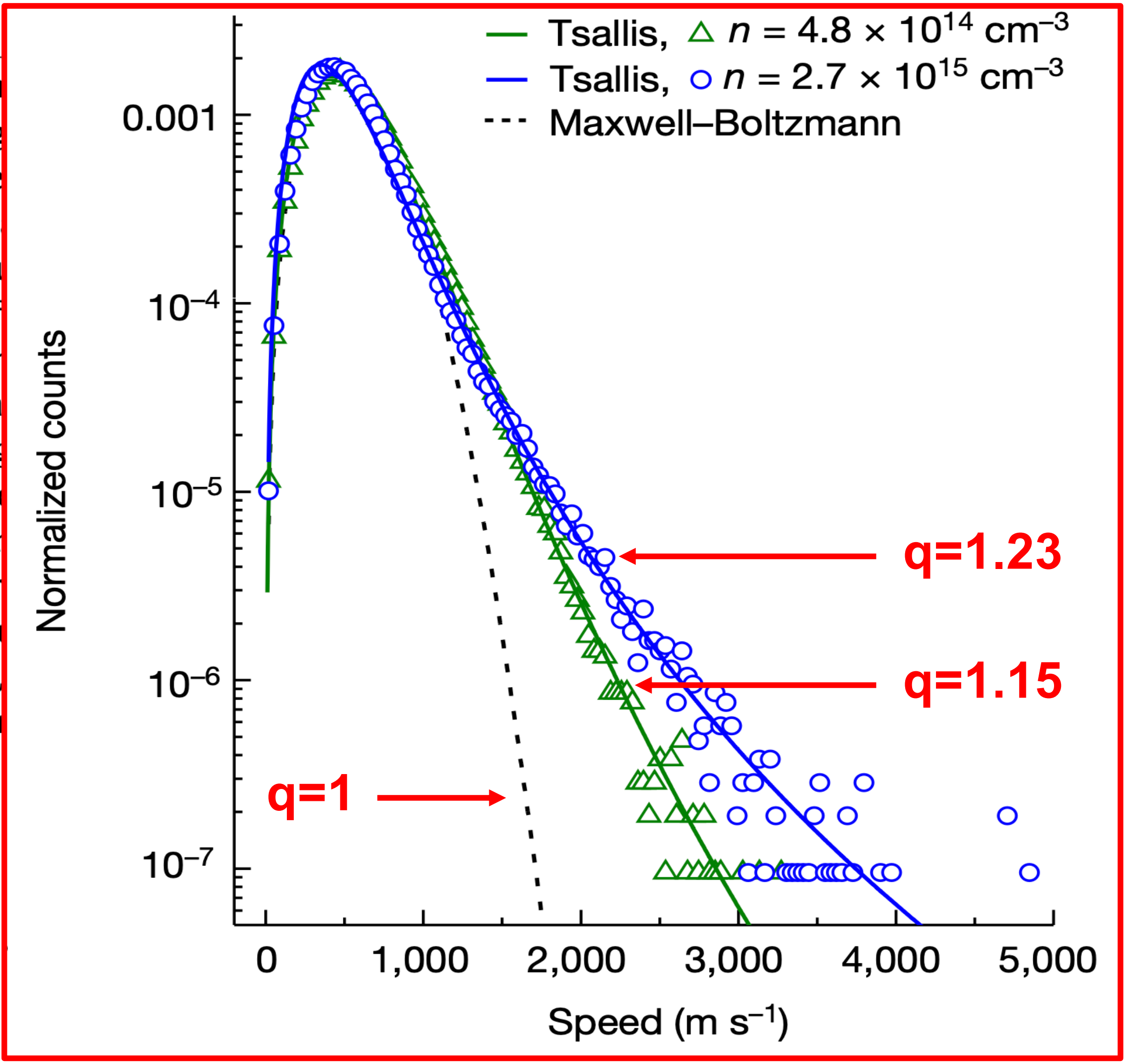
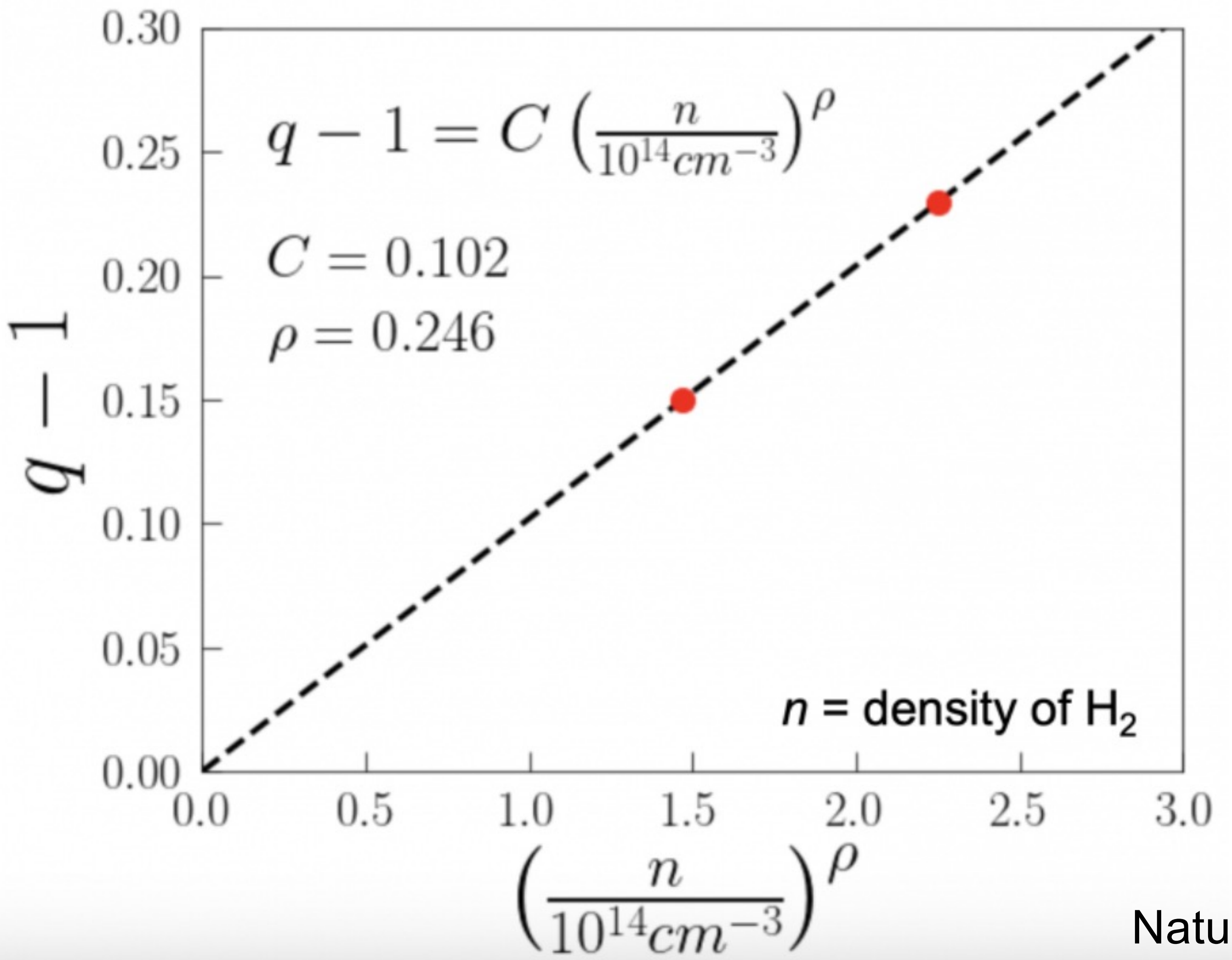


Constantino Tsallis

5 hours ago

Equation (3) [for further details, see for instance C. Tsallis, Introduction to Nonextensive Statistical Mechanics – Approaching a Complex World – Second Edition (Springer, 2023)] is illustrated in Fig. 3 (c). Prof. Roland Webster kindly shared with me the index q corresponding to the values of the hydrogen density n indicated in Fig. 3 (c). With this information, it is possible to construct the attached figure. The ideal gas limit (n=0) corresponds, as expected, to q=1, i.e., to Boltzmann-Gibbs statistical mechanics. Further experimental validation and/or theoretical approaches of the new connection (q-1) proportional n^{1/4} are naturally very welcome.



https://doi.org/10.1038/s41586-023-05727-z

Nature, 17 May 2023

LONG – RANGE INTERACTIONS

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

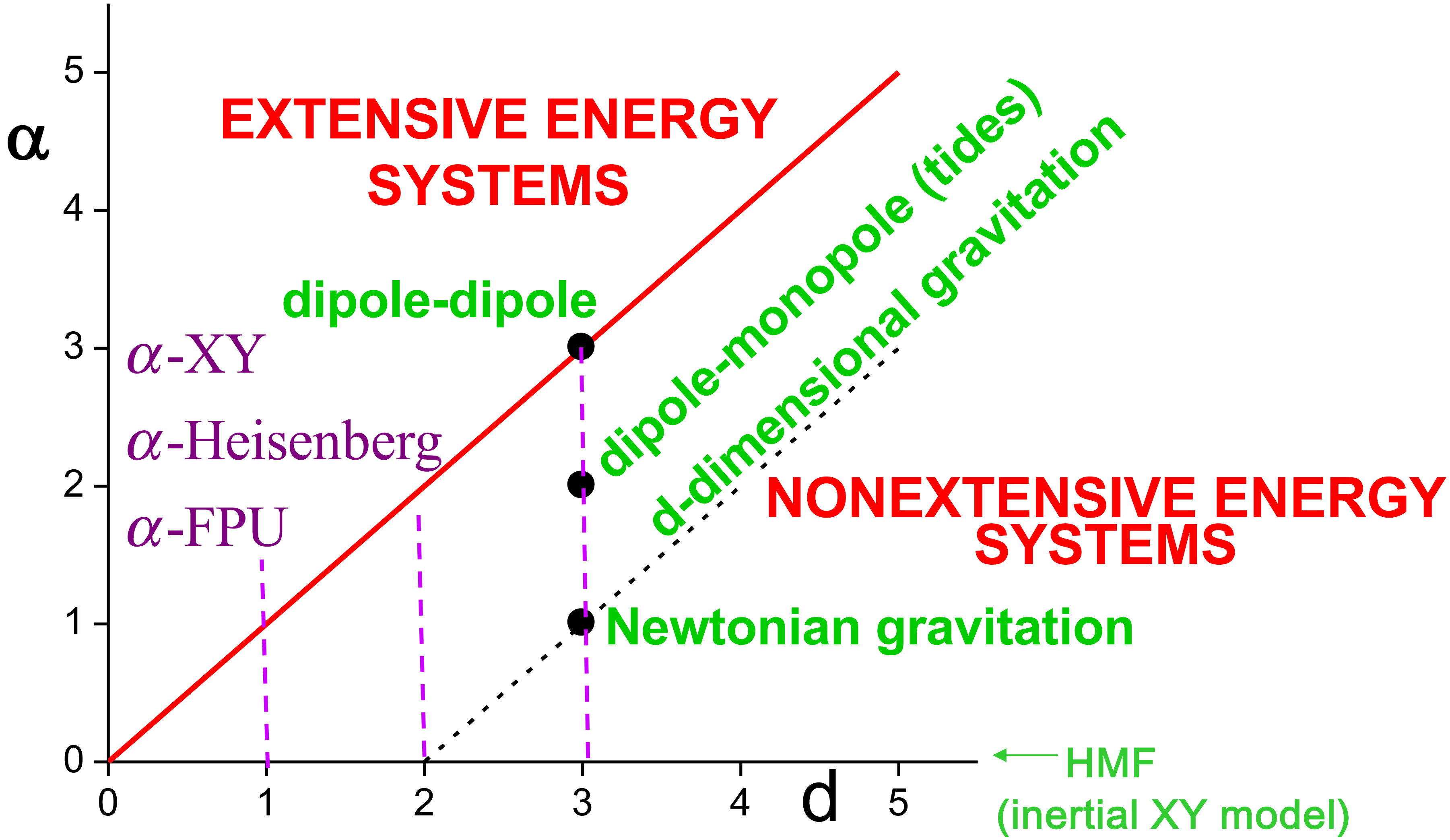
In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \quad (r \to \infty) \quad (A > 0,\ \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*



A Letters Journal Exploring
the Frontiers of Physics

August 2018

EPL, **123** (2018) 30003
doi: 10.1209/0295-5075/123/30003

www.epljournal.org

Validity and failure of the Boltzmann weight

L. J. L. Cirto[1], A. Rodríguez[2], F. D. Nobre[1,3] and C. Tsallis[1,3,4,5]

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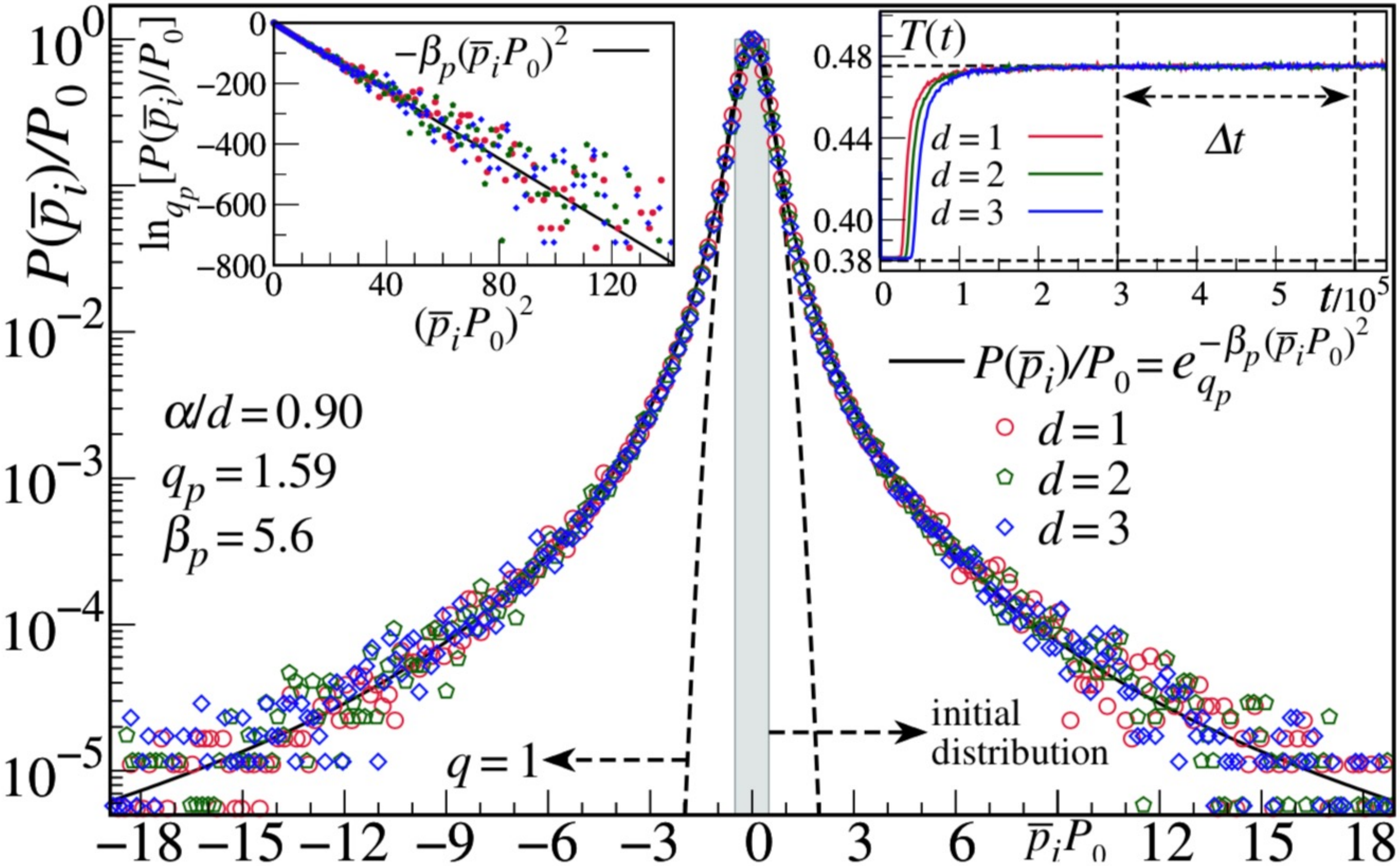
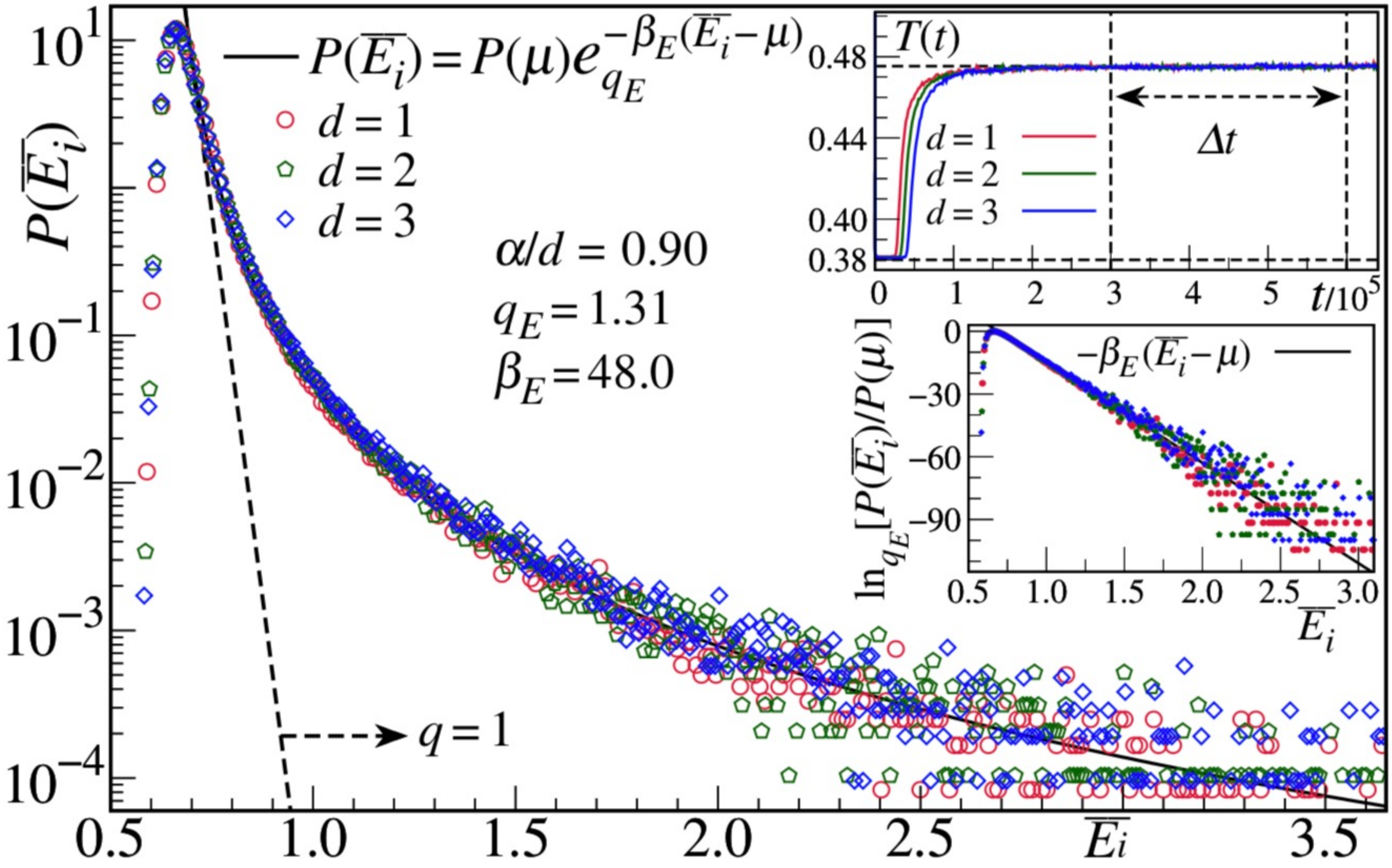
[2] *Departamento de Matemática Aplicada a la Ingeniería Aeroespacial, Universidad Politécnica de Madrid Plaza Cardenal Cisneros s/n, 28040 Madrid, Spain*
[3] *National Institute of Science and Technology for Complex Systems - Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[4] *Santa Fe Institute - 1399 Hyde Park Road, Santa Fe, 87501 NM, United States*
[5] *Complexity Science Hub Vienna - Josefstädter Strasse 39, 1080 Vienna, Austria*

d - DIMENSIONAL XY MODEL

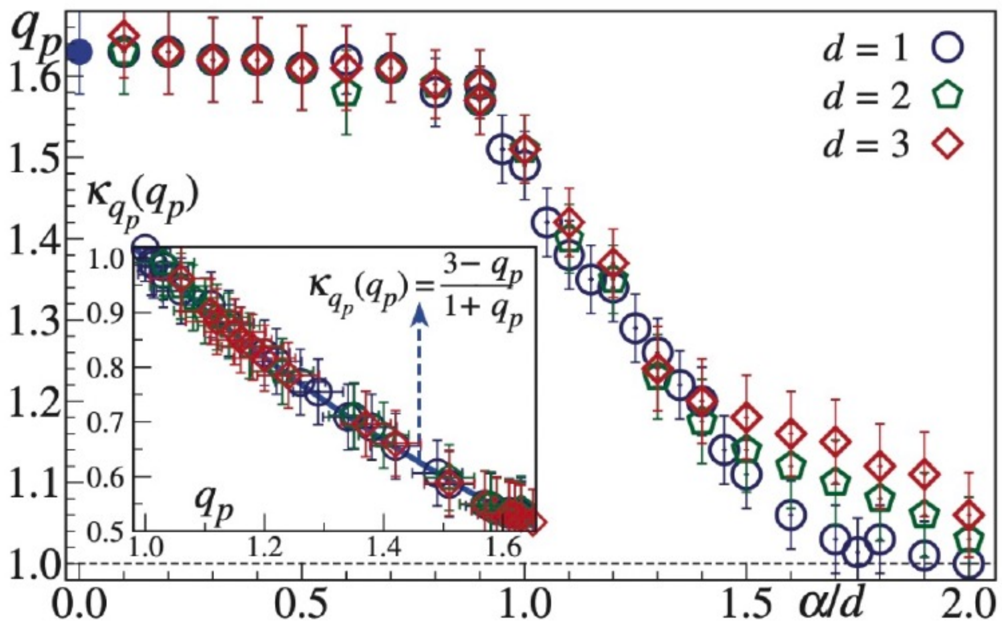


L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)

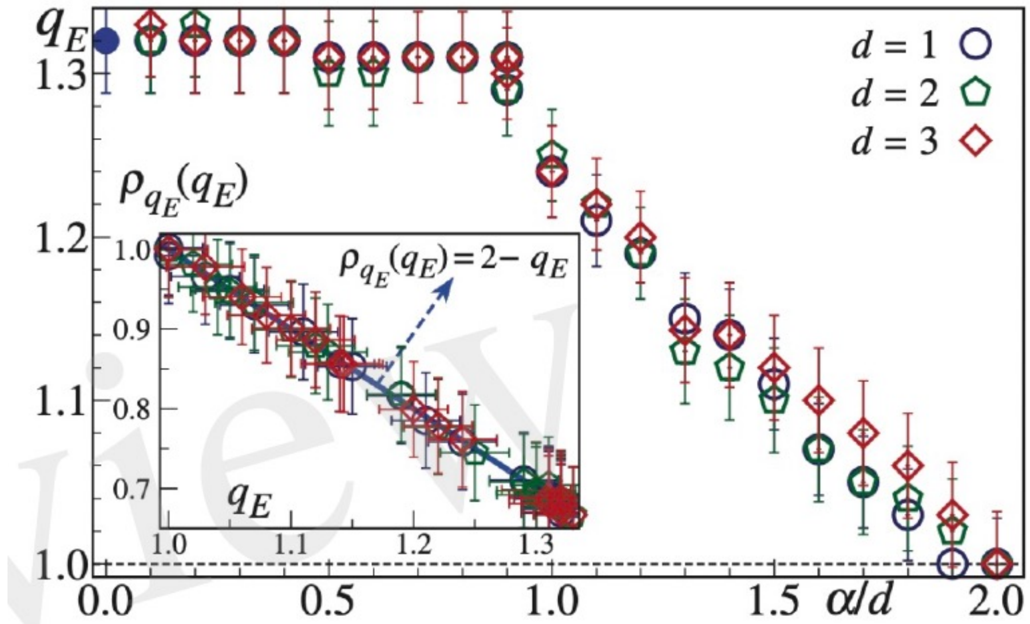
d - DIMENSIONAL XY MODEL



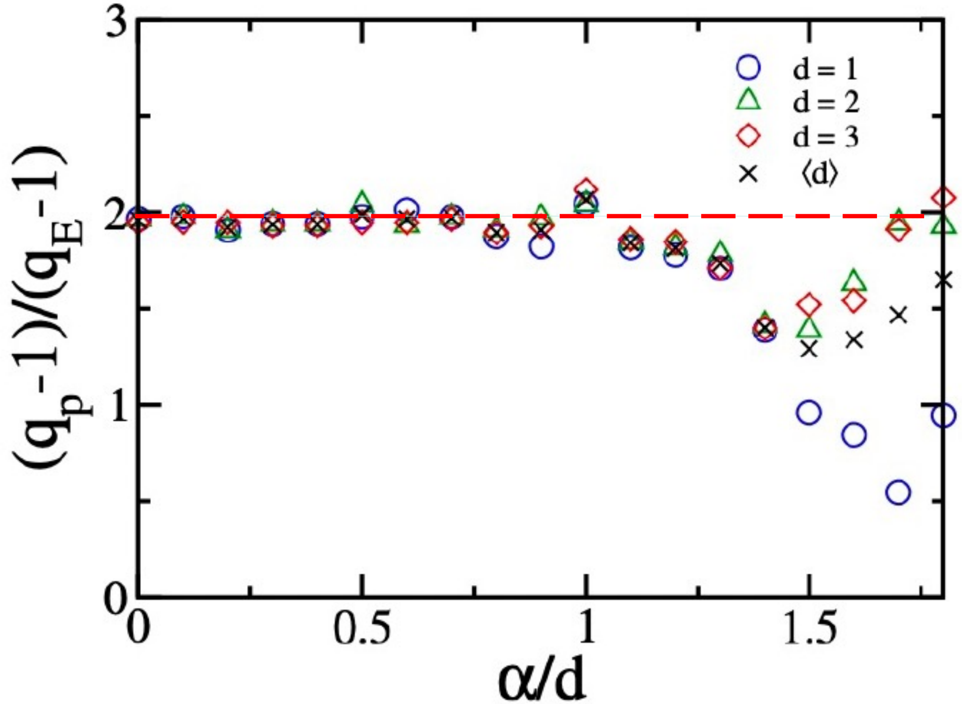
L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)



(a)



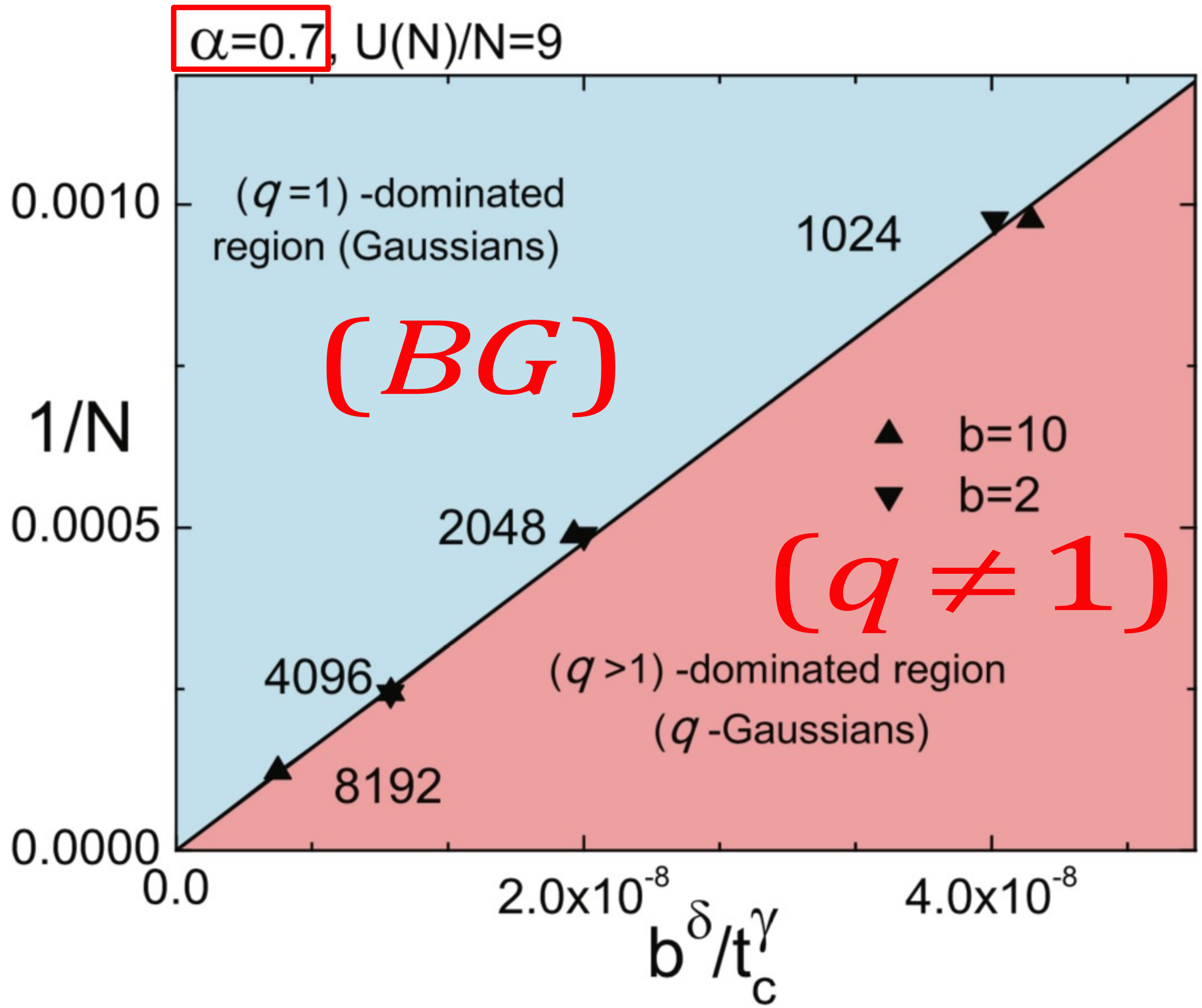
A. Rodriguez, A. Pluchino, U. Tirnakli, A. Rapisarda and C. T.
Symmetry **15**, 444 (2023)



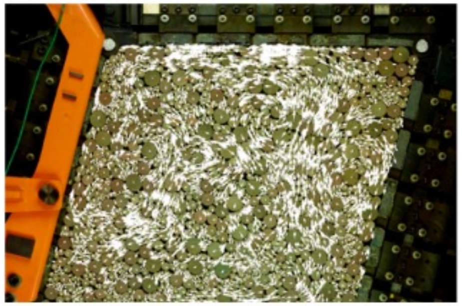
EPL, **108** (2014) 40006
doi: 10.1209/0295-5075/108/40006 www.epljournal.org

Fermi-Pasta-Ulam model with long-range interactions: Dynamics and thermostatistics

H. Christodoulidi[1], C. Tsallis[2,3] and T. Bountis[1]



GRANULAR MATTER



EDITORS' SUGGESTION

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

The velocity distribution of sheared granular media shows unexpected similarities with turbulent fluid flows.

Gaël Combe, Vincent Richefeu, Marta Stasiak, and Allbens P.F. Atman

Phys. Rev. Lett. **115**, 238301 (2015)

PRL **115,** 238301 (2015) PHYSICAL REVIEW LETTERS week ending 4 DECEMBER 2015

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

Gaël Combe,* Vincent Richefeu, and Marta Stasiak
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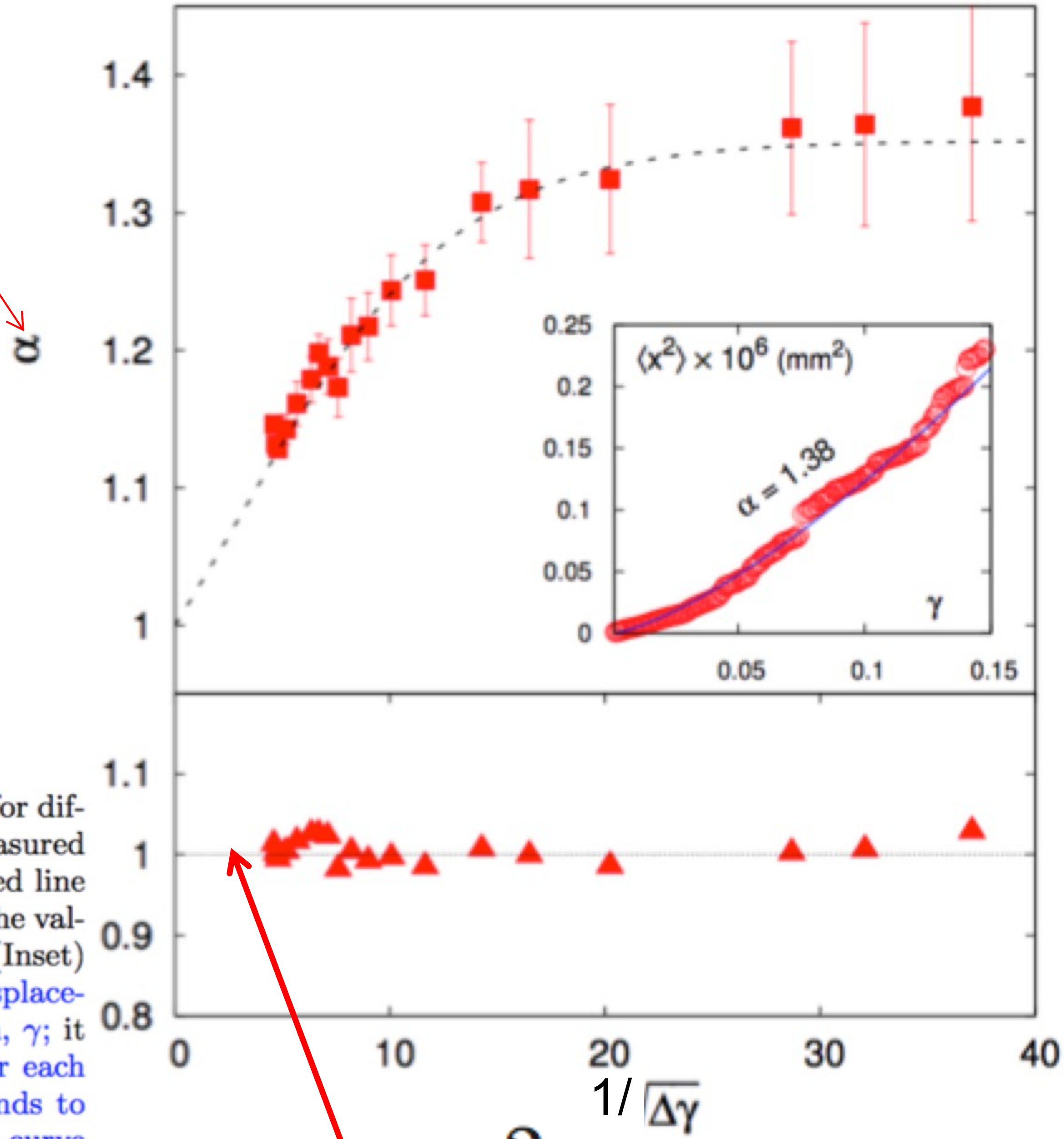
Avenida Amazonas 7675, 30510-000 Belo Horizonte-MG, Brazil
(Received 28 July 2015; published 1 December 2015)

$$\langle x^2 \rangle \propto t^{\alpha}$$



Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

FIG. 4. Verification of the Tsallis-Bukman scaling law for different regimes of diffusion. (top) Evolution of the measured diffusion exponent α as a function of $1/\sqrt{\Delta\gamma}$ the dashed line is a direct application of the scaling law from the fit of the values shown in Fig. 3, $\alpha(1/\sqrt{\Delta\gamma}) = 2/[3 - q(1/\sqrt{\Delta\gamma})]$. (Inset) a typical diffusion curve showing the mean square displacement fluctuations, $\langle x^2 \rangle$, in function of the shear strain, γ; it allows the assessment of the diffusion exponent, α, for each strain window tested. In the case shown, it corresponds to the smallest strain window, the rightmost point in the curve at the main panel. Note that for a constant strain rate, γ is proportional to time. (Bottom) Measure of the deviation of the data relative to the scaling law prediction, as a function of $1/\sqrt{\Delta\gamma}$, showing an agreement on the order of $\pm 2\%$.

$$\alpha = \frac{2}{3 - q}$$

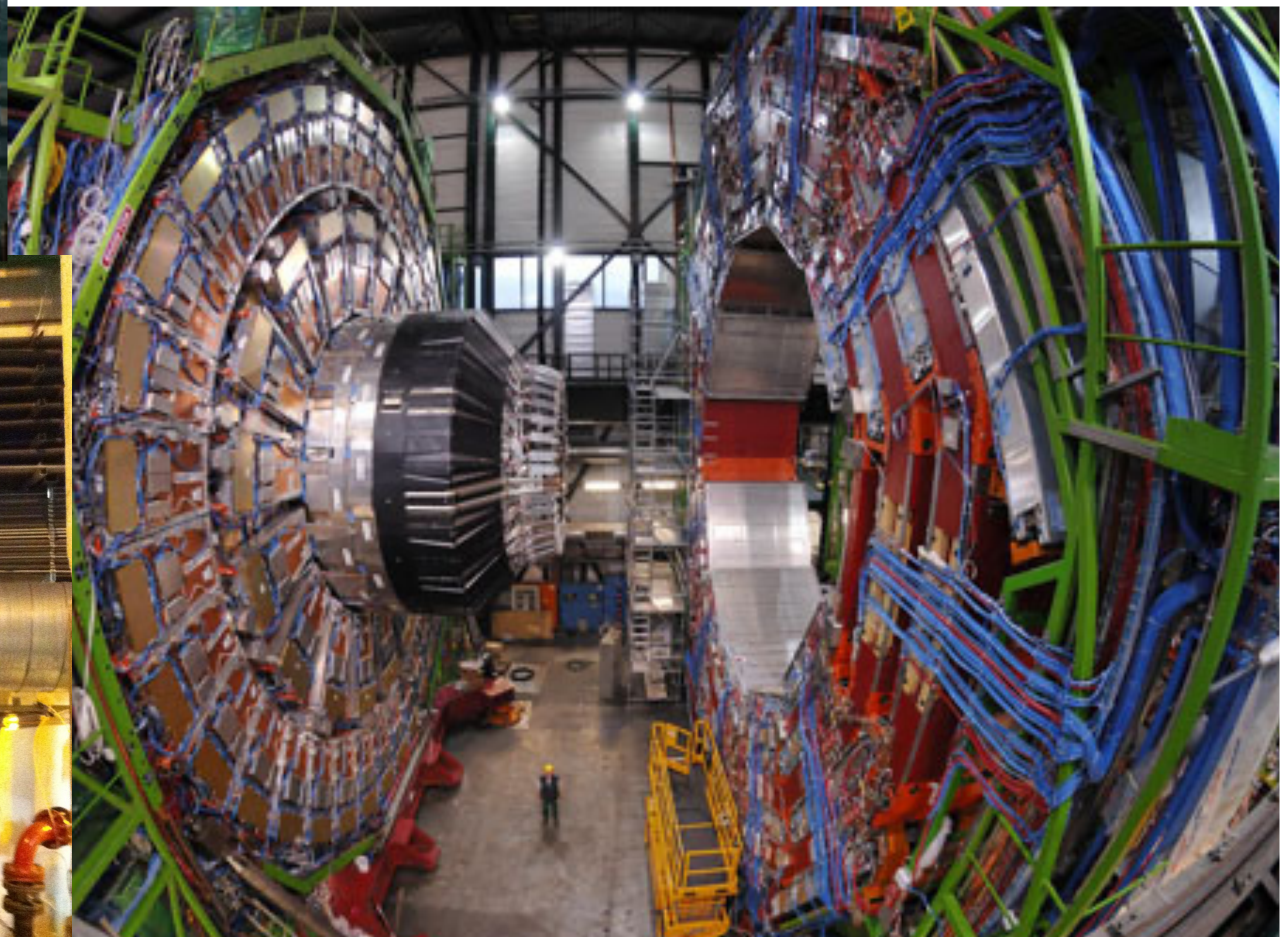
CT and DJ Bukman, PRE **54** (1996) R2197

HIGH ENERGY COLLISIONS

LHC (Large Hadron Collider)

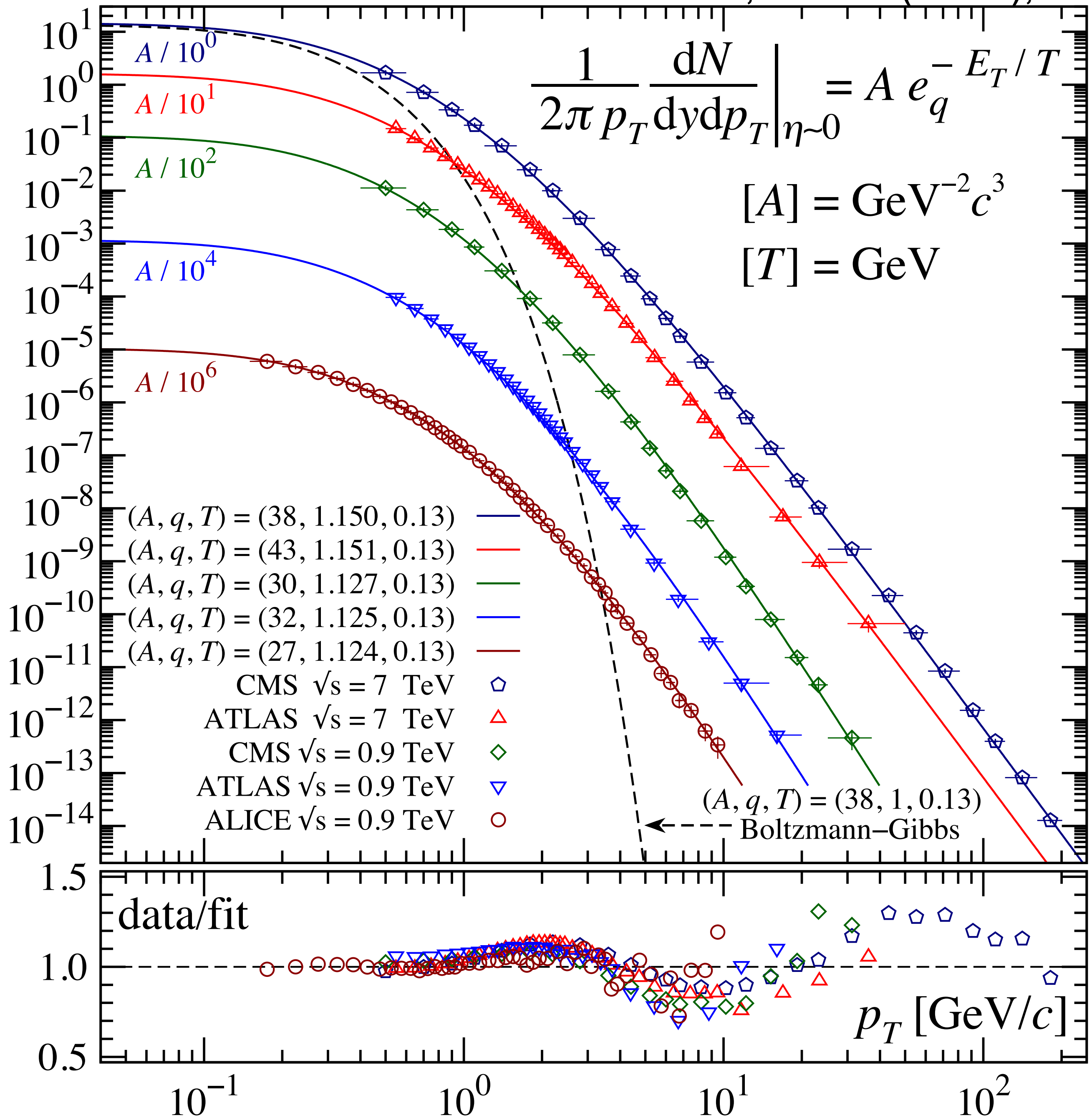
CMS, ALICE, ATLAS and LHCb detectors

~ 4000 scientists/engineers from ~ 200 institutions of ~ 50 countries



SIMPLE APPROACH: TWO-DIMENSIONAL SINGLE RELATIVISTIC FREE PARTICLE

C.Y. Wong, G. Wilk, L.J.L. Cirto and C. T.,
EPJ Web of Conferences **90**, 04002 (2015), and PRD **91**, 114027 (2015)



$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$

LHC/CERN proton-proton collisions

$q = 1.14 \pm 0.015$

$T = 0.13$ GeV

pion π^+ mass $= 0.1396$ GeV

pion π^0 mass $= 0.1350$ GeV

Equilibrium Distribution of Heavy Quarks in Fokker-Planck Dynamics

D. Brian Walton[1,*] and Johann Rafelski[2,†]
[1]Program in Applied Mathematics, University of Arizona, Tucson, Arizona 85721
[2]Physics Department, University of Arizona, Tucson, Arizona 85721
(Received 8 July 1999)

We obtain an explicit generalization, within Fokker-Planck dynamics, of Einstein's relation between drag, diffusion, and the equilibrium distribution for a spatially homogeneous system, considering both the transverse and longitudinal diffusion for dimension $n > 1$. We provide a complete characterization of the equilibrium distribution in terms of the drag and diffusion transport coefficients. We apply this analysis to charm quark dynamics in a thermal quark-gluon plasma for the case of collisional equilibration.

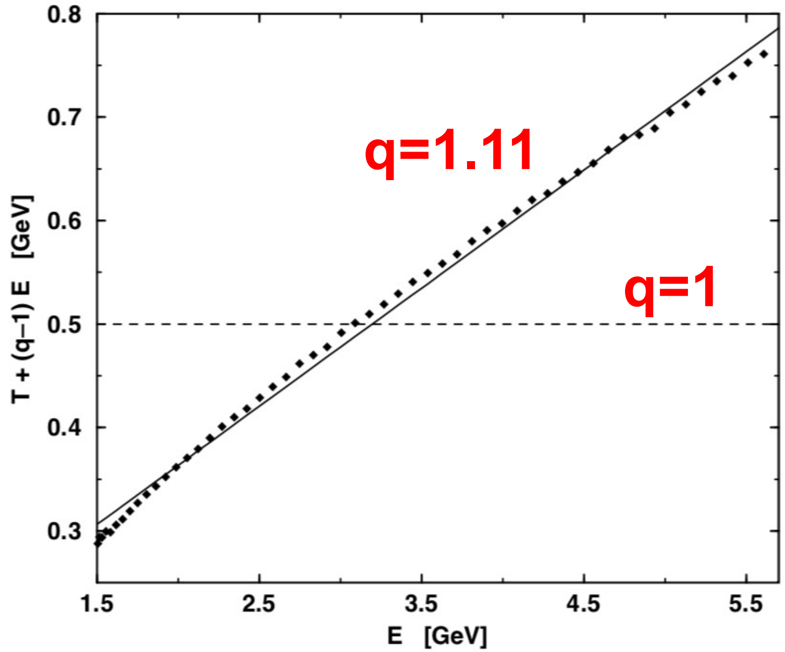


FIG. 1. Calculated data (diamonds) and linear fit for the ratio in Eq. (25) for a charmed quark $m_c = 1.5$ GeV thermalizing in gluon background at $T_b = 500$ MeV. Dashed line: result expected for a Boltzmann-Jüttner distribution, $T = T_b$.

PHYSICAL REVIEW D **101,** 034019 (2020)

Fractals, nonextensive statistics, and QCD

Airton Deppman,[1,2,*] Eugenio Megías,[2] and Debora P. Menezes[3]

[1]*Instituto de Física, Rua do Matão 1371-Butantã, São Paulo-SP, CEP 05508-090 Brazil*
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(Received 15 January 2020; accepted 24 January 2020; published 19 February 2020)

In this work, we analyze how scaling properties of Yang-Mills field theory manifest as self-similarity of truncated n-point functions by scale evolution. The presence of such structures, which actually behave as fractals, allows for recurrent nonperturbative calculation of any vertex. Some general properties are indeed independent of the perturbative order, what simplifies the nonperturbative calculations. We show that for sufficiently high perturbative orders a statistical approach can be used, the nonextensive statistics is obtained, and the Tsallis index, q, is deduced in terms of the field theory parameters. The results are applied to QCD in the one-loop approximation, where q can be calculated, resulting in a good agreement with the value obtained experimentally. We discuss how this approach allows us to understand some intriguing experimental findings in high energy collisions, as the behavior of multiplicity against collision energy, long-tail distributions, and the fractal dimension observed in intermittency analysis.

First-principle Yang-Mills/QCD grounds yields

$$\frac{1}{q-1} = \frac{11}{3} N_c - \frac{2}{3} N_f$$ (Deppman, Megias and Menezes PRD 2020)

where $N_c \equiv$ number of colors

$N_f \equiv$ number of flavors

hence

$$(N_c, N_f) = (3,6) \;\Rightarrow\; q = \frac{8}{7} \simeq 1.14$$ **SU(6)**

(Deppman, Megias and Menezes PRD 2020)

$$(N_c, N_f) = (3,3) \;\Rightarrow\; q = \frac{10}{9} \simeq 1.11$$ **SU(3)**

(Walton and Rafelski PRL 2000; C.T. 2022)

Evidence of fractal structures in hadrons

Rafael P. Baptista,[1] Lucas Q. Rocha,[1] D. P. Menezes,[2] Luis A. Trevisan,[3] Constantino Tsallis,[4, 5, 6] and Airton Deppman[1]

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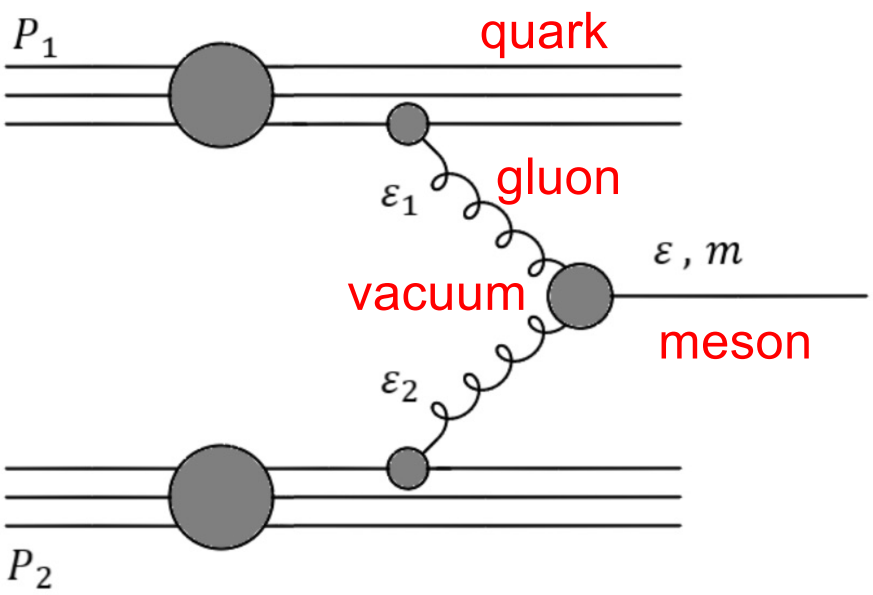
(Dated: September 11, 2023)

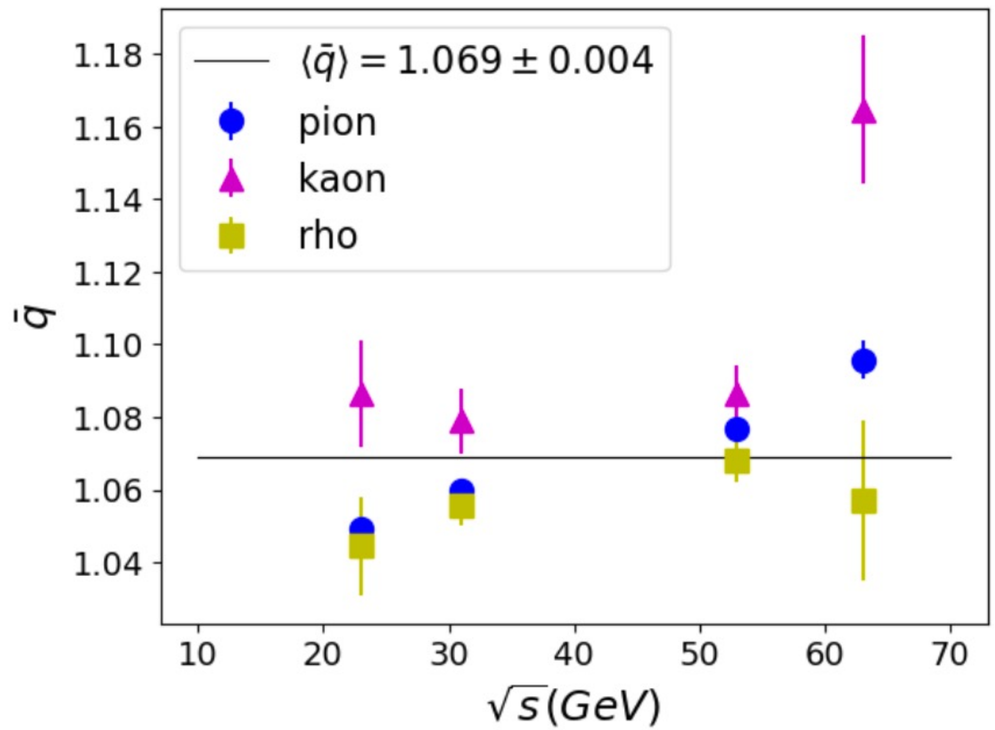
arXiv:2308.16888v1 [hep-th] 31 Aug 2023



$$\frac{1}{\bar{q}-1} = \frac{2}{q-1}$$

hence $q = 8/7 \sim 1.14$

→ $\bar{q} = 15/14 \sim 1.07$



RANDOM NETWORKS

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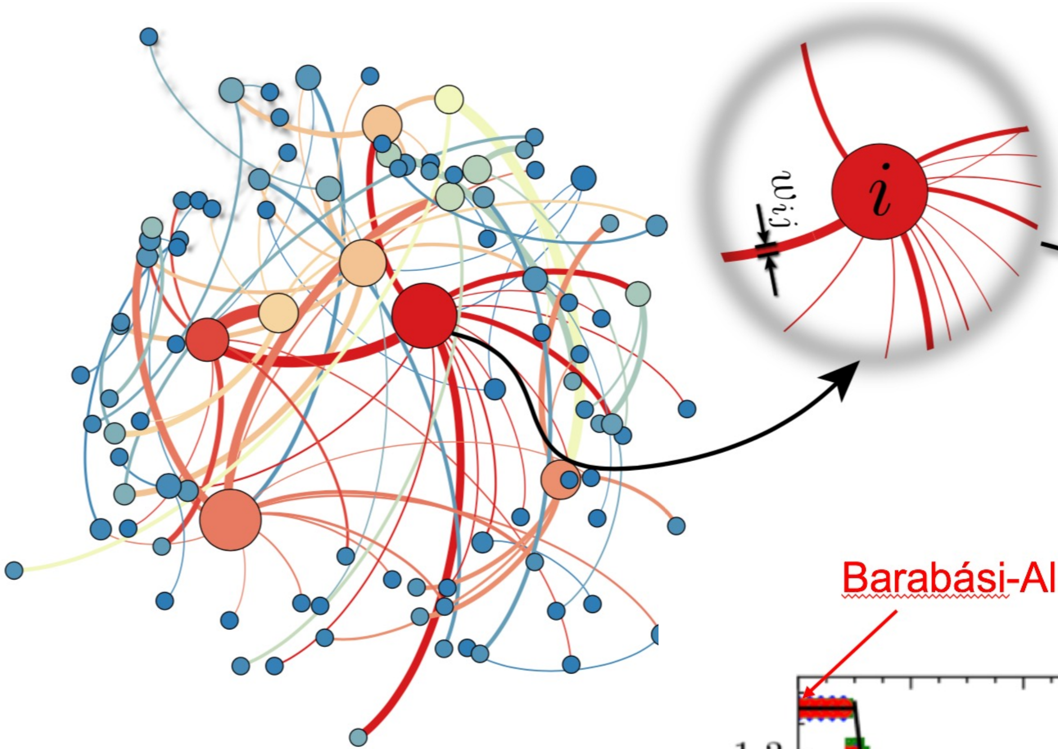
scientific reports

Connecting complex networks to nonadditive entropies

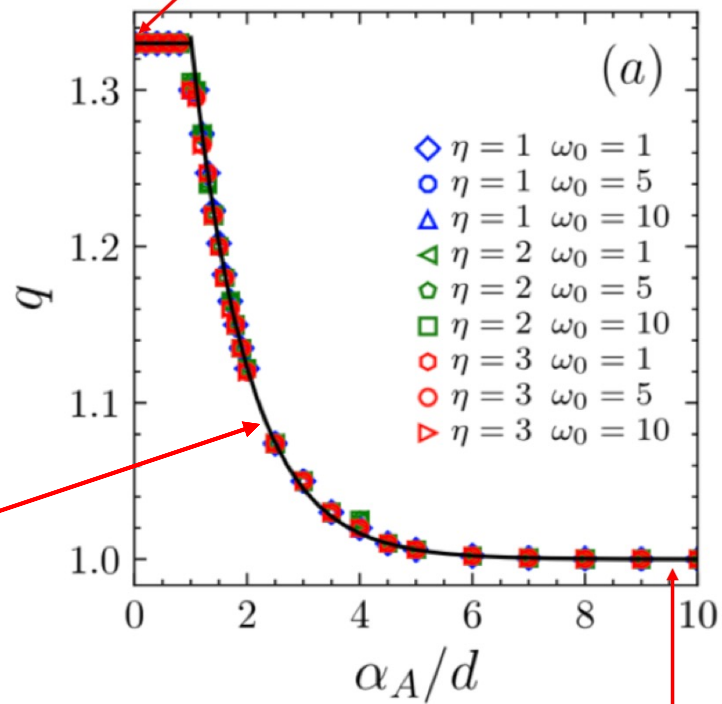
R. M. de Oliveira[1], Samuraí Brito[2]✉, L. R. da Silva[1,3] & Constantino Tsallis[3,4,5,6]

11, 1130 (2021)



$$\varepsilon_i = \sum_j w_{ij}/2$$

Barabási-Albert



$$q = 1 + \frac{1}{3} e^{1-\alpha_A/d}$$

Erdos-Renyi
(random graphs)

LOW-DIMENSIONAL

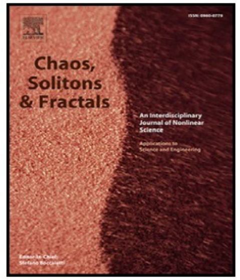
NONLINEAR DYNAMICAL SYSTEMS

Chaos, Solitons and Fractals 171 (2023) 113431

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

journal homepage: www.elsevier.com/locate/chaos



Time evolution of nonadditive entropies: The logistic map

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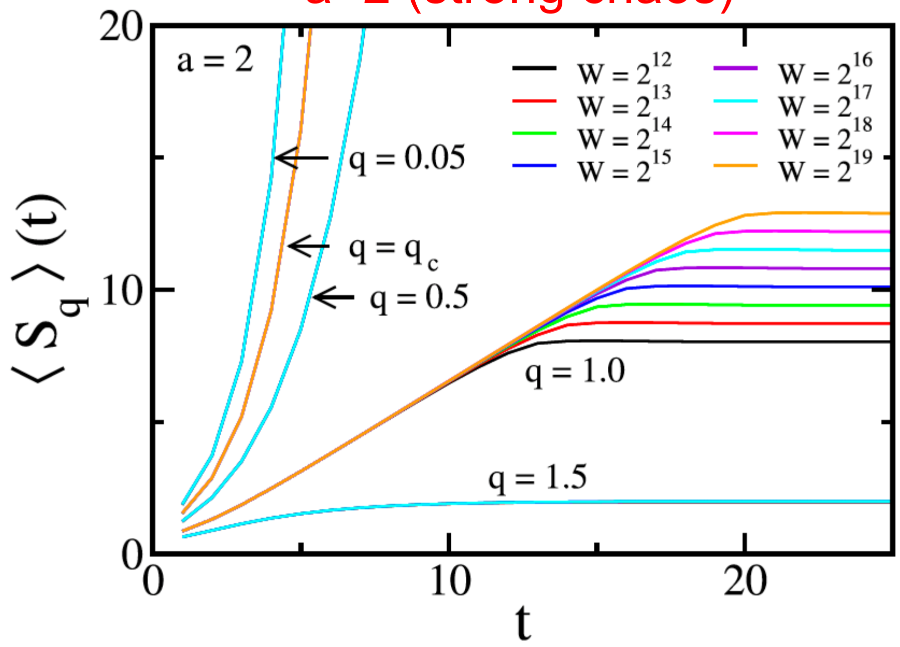
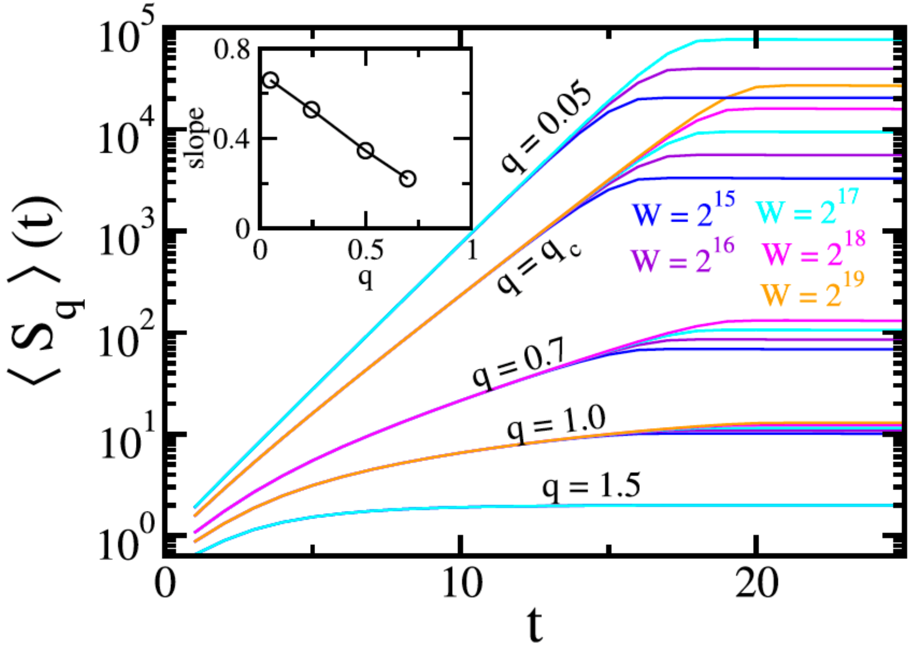
[d] Complexity Science Hub Vienna, Josefstädter Strasse 39, 1080, Vienna, Austria
[e] Instituto de Fisica, Universidade Federal da Bahia, Salvador-BA 40170-115, Brazil

a=2 (strong chaos)



a=a_c (Feigenbaum point)



EDGE OF CHAOS OF THE LOGISTIC MAP: Finite entropy production (Pesin)

(Using result in http://converge.to/feigenbaum)

$$q = 1 - \frac{\ln 2}{\ln \alpha_F} =$$

M. L. Lyra and C. T. , Phys Rev Lett 80, 53 (1998)

0.24448770134128206619877042340468040523444693549005767367036509863277496727665586657551562268575407062883496403827283060636001
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50988746163

(10026 meaningful digits)

A. Vieira (2022)

GRAVITATIONAL SYSTEMS

PHYSICAL REVIEW D VOLUME 13, NUMBER 2 15 JANUARY 1976

Black holes and thermodynamics*

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and Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England
(Received 30 June 1975)

A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

Eur. Phys. J. C (2013) 73:2487
DOI 10.1140/epjc/s10052-013-2487-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Black hole thermodynamical entropy

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Abstract As early as 1902, Gibbs pointed out that systems whose partition function diverges, e.g. gravitation, lie outside the validity of the Boltzmann–Gibbs (BG) theory. Consistently, since the pioneering Bekenstein–Hawking results, physically meaningful evidence (e.g., the holographic principle) has accumulated that the BG entropy S_{BG} of a $(3+1)$ black hole is proportional to its area L^2 (L being a characteristic linear length), and not to its volume L^3. Similarly it exists the *area law*, so named because, for a wide class of strongly quantum-entangled d-dimensional systems, S_{BG} is proportional to $\ln L$ if $d=1$, and to L^{d-1} if $d>1$, instead of being proportional to L^d $(d \geq 1)$. These results violate the extensivity of the thermodynamical entropy of a d-dimensional system. This thermodynamical inconsistency disappears if we realize that the thermodynamical entropy of such nonstandard systems is *not* to be identified with the BG *additive* entropy but with appropriately generalized *nonadditive* entropies. Indeed, the celebrated usefulness of the BG entropy is founded on hypothesis such as relatively weak probabilistic correlations (and their connections to ergodicity, which by no means can be assumed as a general rule of nature). Here we introduce a generalized entropy which, for the Schwarzschild black hole and the area law, can solve the thermodynamic puzzle.

$$\ln_{q,\delta} z \equiv (\ln_q z)^{\delta} = \left(\frac{z^{1-q} - 1}{1-q} \right)^{\delta} \qquad e^{z}_{q,\delta} \equiv e_q^{z^{1/\delta}} = \left[1 + (1-q)\, z^{1/\delta} \right]^{\frac{1}{1-q}}$$

$$S_{q,\delta} = k \sum_i p_i \ln_{q,\delta} \frac{1}{p_i}$$

(C. T. and L.J.L. Cirto 2013, H.S. Lima and C. T. 2020)

$\delta = 1$ $\qquad$ $q = 1$

$$S_{q,1} \equiv S_q = k \sum_i p_i \ln_q \frac{1}{p_i}$$

(C. T. 1988)

$$S_{1,\delta} \equiv S_{\delta} = k \sum_i p_i \left(\ln \frac{1}{p_i} \right)^{\delta}$$

(C. T. 2009)

$q = 1$ $\qquad$ $\delta = 1$

$$S_{BG} = k \sum_i p_i \ln \frac{1}{p_i}$$

DARK MATTER NEUTRINOS

IceCube Neutrino Obervatory (South Pole)

Eur. Phys. J. C (2022) 82:1123
https://doi.org/10.1140/epjc/s10052-022-11113-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Tsallis cosmology and its applications in dark matter physics with focus on IceCube high-energy neutrino data

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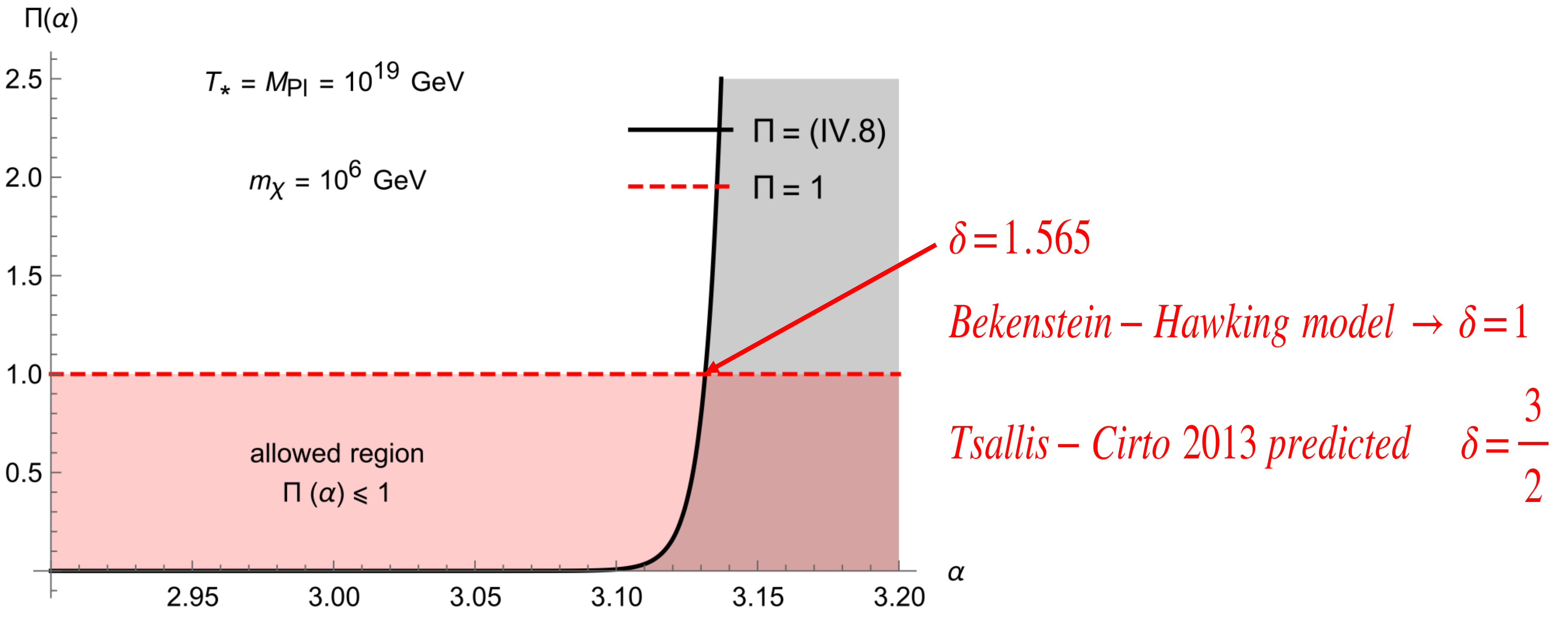
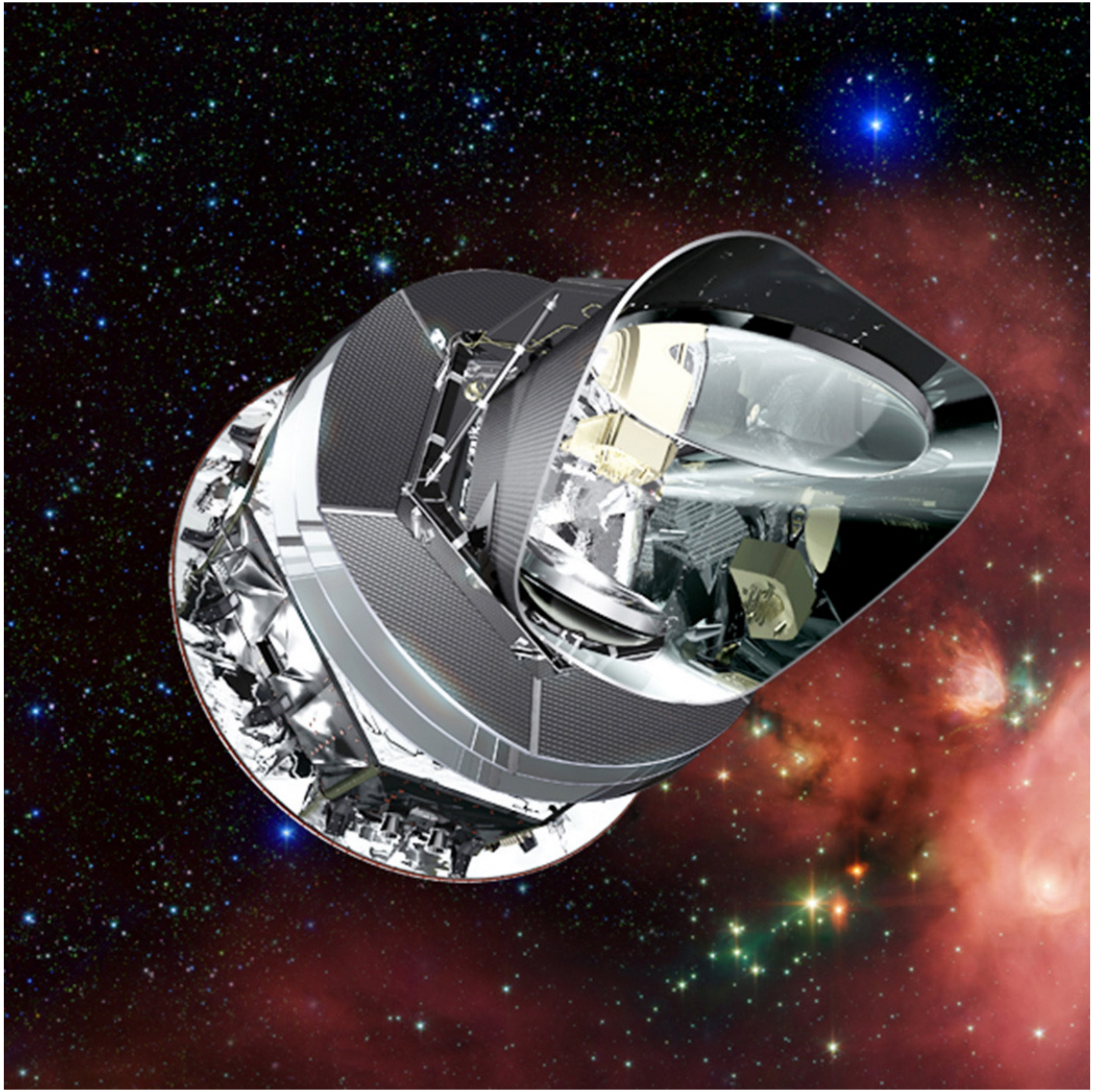


Planck Observatory / ESA



General Relativity and Gravitation (2023) 55:57
https://doi.org/10.1007/s10714-023-03104-9

RESEARCH ARTICLE

Search for neutrino masses in the Barrow holographic dark energy cosmology with Hubble horizon as IR cutoff

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Received: 6 June 2022 / Accepted: 7 March 2023

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Table 1 Observational constraints at 68% on main and derived parameters of the IDE+ $\sum m_\nu$ scenario

Model	Δ	H_0	Ω_m	$\sum m_\nu$	N_{eff}	h
Pantheon	$1.82^{+.17}_{-.42}$	$69.62^{+0.14}_{-0.14}$	$0.288^{+0.029}_{-0.028}$	< 0.183	$3.02^{+0.17}_{-0.17}$	$0.6962^{+0.0014}_{-0.0014}$
Union2	$1.83^{+.18}_{-.43}$	$69.68^{+0.26}_{-0.26}$	$0.289^{+0.041}_{-0.025}$	< 0.161	$2.83^{+0.2}_{-0.2}$	$0.6968^{+0.0026}_{-0.0026}$
CC	$1.31^{+1.41}_{-0.51}$	66.71^{+1}_{-1}	$0.288^{+0.008}_{-0.008}$	< 0.121	$2.95^{+0.11}_{-0.12}$	$0.6671^{+0.01}_{-0.01}$
Pantheon+cc+Union2	$1.74^{+.25}_{-.17}$	$69.86^{+0.17}_{-0.17}$	$0.280^{+0.1}_{-0.1}$	< 0.134	$2.92^{+0.12}_{-0.12}$	$0.6986^{+0.0017}_{-0.0017}$

The parameter H_0 is in the units of $km/sec/Mpc$, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV

$$Planck\ Collaboration\ (2018) + Approach\ A \Rightarrow \Delta = 1.74 \Rightarrow \delta = 1 + \frac{\Delta}{2} = 1.87 > \frac{3}{2}$$

Table 2 Observational constraints at 68% on main and derived parameters of the IDE+ $\sum m_\nu$ scenario

Model	Δ	H_0	Ω_m	$\sum m_\nu$	N_{eff}	β	h
Pantheon	$.25^{+.12}_{-.19}$	$69.51^{+0.3}_{-0.5}$	$0.298^{+0.01}_{-0.011}$	< 0.153	$3.01^{+0.17}_{-0.12}$	$-.2$	$0.6951^{+0.003}_{-0.005}$
Union2	$.32^{+.18}_{-.12}$	$69.98^{+0.15}_{-0.13}$	$0.296^{+0.016}_{-0.013}$	< 0.165	$2.98^{+0.2}_{-0.2}$	$-.19$	$0.6998^{+0.0015}_{-0.0013}$
CC	$.3^{+.12}_{-0.12}$	$67.21^{+.3}_{-.3}$	$0.285^{+0.03}_{-0.02}$	< 0.275	$2.8^{+0.22}_{-0.22}$	$-.14$	$0.6721^{+0.003}_{-0.003}$
Pantheon+cc+Union2	$.52^{+.1}_{-.08}$	$69.46^{+0.4}_{-0.4}$	$0.276^{+0.006}_{-0.005}$	< 0.152	$3.05^{+0.13}_{-0.13}$	$-.15$	$0.6946^{+0.004}_{-0.004}$

The parameter H_0 is in the units of $km/sec/Mpc$, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV

$$Planck\ Collaboration\ (2018) + Approach\ B \Rightarrow \Delta = 0.52 \Rightarrow \delta = 1 + \frac{\Delta}{2} = 1.26 < \frac{3}{2}$$

$$\frac{1.87 + 1.26}{2} \pm 0.1 = 1.565 \pm 0.1\ !$$

entropy

MDPI

Article

Constraints on Tsallis Cosmology from Big Bang Nucleosynthesis and the Relic Abundance of Cold Dark Matter Particles

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Abstract: By employing Tsallis' extensive but non-additive δ-entropy, we formulate the first two laws of thermodynamics for gravitating systems. By invoking Carathéodory's principle, we pay particular attention to the integrating factor for the heat one-form. We show that the latter factorizes into the product of thermal and entropic parts, where the entropic part cannot be reduced to a constant, as is the case in conventional thermodynamics, due to the non-additive nature of S_δ. The ensuing two laws of thermodynamics imply a Tsallis cosmology, which is then applied to a radiation-dominated universe to address the Big Bang nucleosynthesis and the relic abundance of cold dark matter particles. It is demonstrated that the Tsallis cosmology with the scaling exponent $\delta \sim 1.499$ (or equivalently, the anomalous dimension $\Delta \sim 0.0013$) consistently describes both the abundance of cold dark matter particles and the formation of primordial light elements, such as deuterium ^2H and helium ^4He. Salient issues, including the zeroth law of thermodynamics for the δ-entropy and the lithium ^7Li problem, are also briefly discussed.

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Efficient classification of COVID-19 CT scans by using q-transform model for feature extraction

Razi J. Al-Azawi[1], Nadia M.G. Al-Saidi[2], Hamid A. Jalab[3], Hasan Kahtan[4] and Rabha W. Ibrahim[5]

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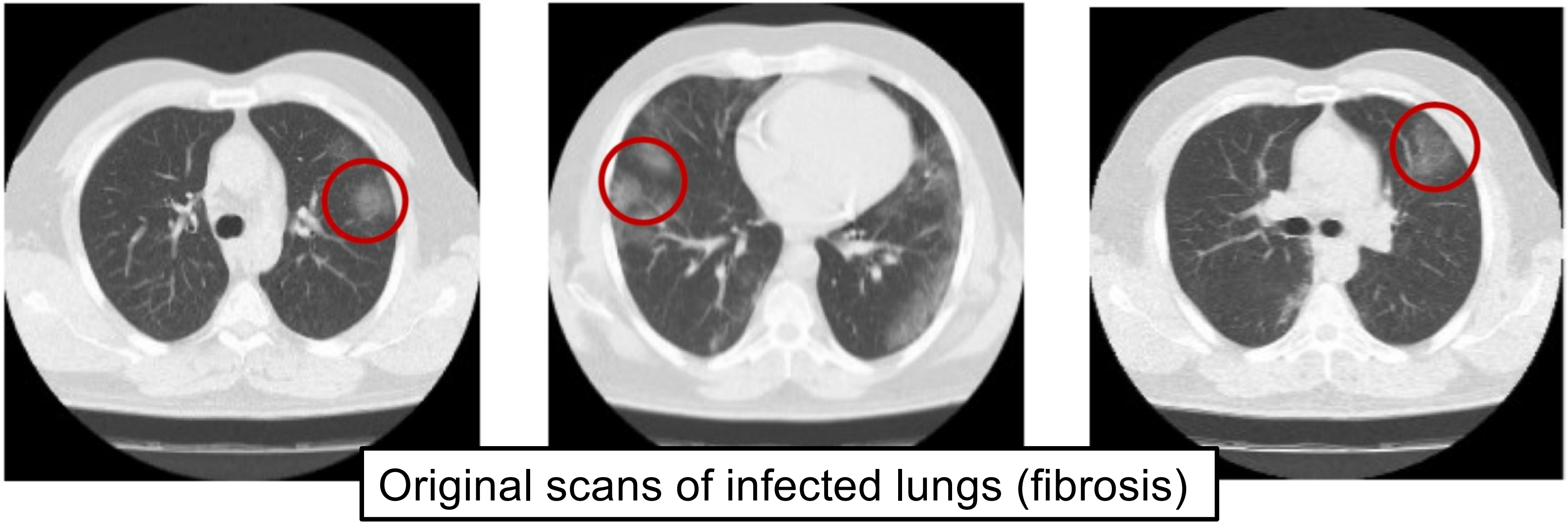
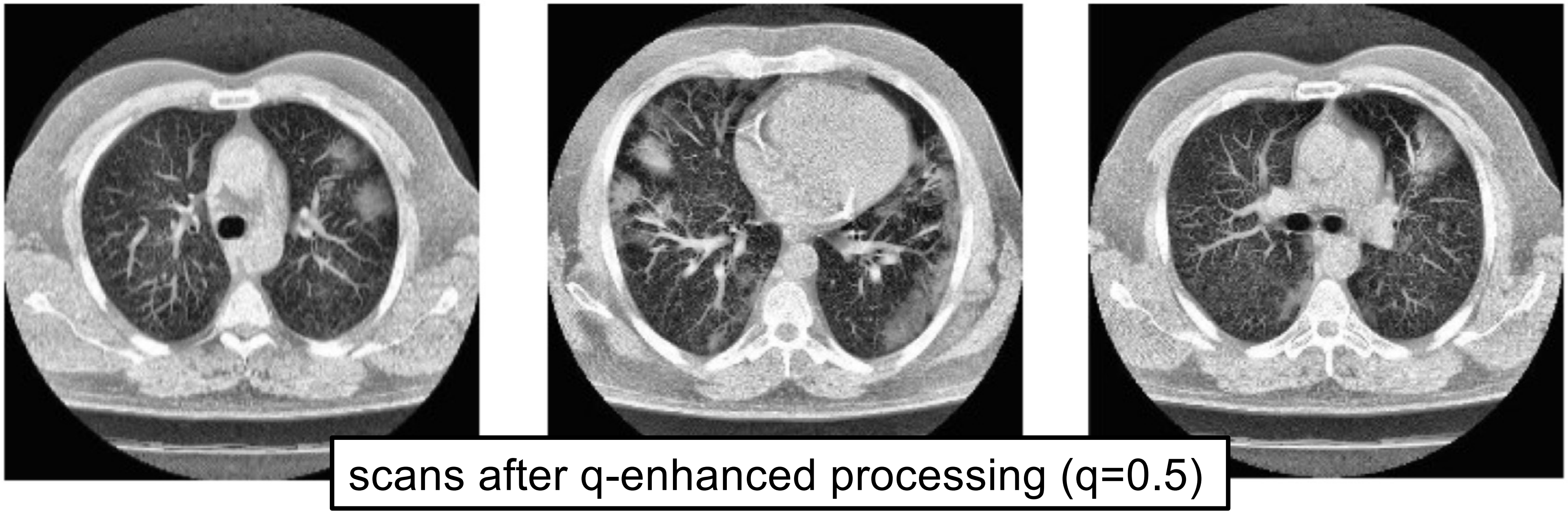
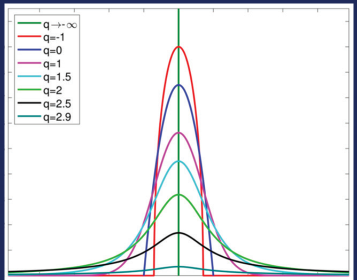
(15 June 2021)


Original scans of infected lungs (fibrosis)

(a)


scans after q-enhanced processing (q=0.5)

The book is devoted to the mathematical foundations of nonextensive statistical mechanics. This is the first book containing the systematic presentation of the mathematical theory and concepts related to nonextensive statistical mechanics, a current generalization of Boltzmann-Gibbs statistical mechanics introduced in 1988 by one of the authors and based on a nonadditive entropic functional extending the usual Boltzmann-Gibbs-von Neumann-Shannon entropy. Main mathematical tools like the q-exponential function, q-Gaussian distribution, q-Fourier transform, q-central limit theorems, and other related objects are discussed rigorously with detailed mathematical rational. The book also contains recent results obtained in this direction and challenging open problems. Each chapter is accompanied with additional useful notes including the history of development and related bibliographies for further reading.



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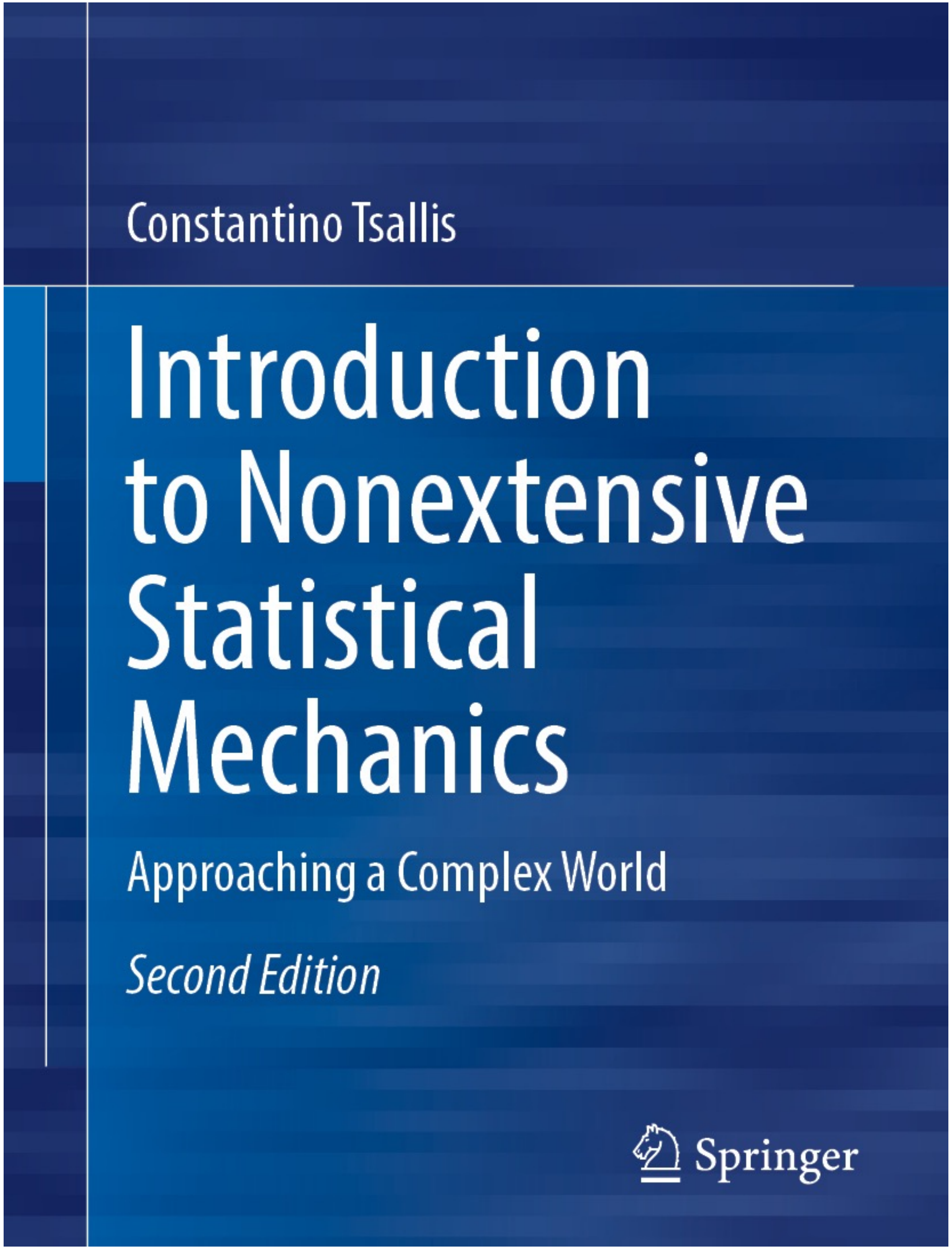


Second Edition
(Springer 2023)

TAKE-HOME MESSAGE

simple systems

$\leftrightarrow$ *local* space – time correlations (e.g., exponential decays)

$\leftrightarrow$ Boltzmann – Gibbs – von Neumann – Shannon *additive* entropic functional S_{BG}

$\leftrightarrow$ *Boltzmann – Gibbs* statistical mechanics

complex systems

$\leftrightarrow$ *nonlocal* space – time correlations (e.g., power – law decays)

$\leftrightarrow$ *nonadditive* entropic functionals $\left(e.g.,\ S_q\ ,\ S_\delta \right)$

$\leftrightarrow$ *generalized* statistical mechanics

C. T., Frontiers in Complex Systems (2023)

local correlations
(S_{BG} entropic functionnal)

nonlocal correlations
(nonadditive entropic functionnal)

Yo soy yo y mi circunstancia.

José Ortega y Gasset
(1883-1955)

SEMIOTICS

Charles Sanders Peirce (1839 - 1914)

Logical inferences

Deduction: All stones in box A are black.
Stone S_i is from box A.

Stone S_i is black.

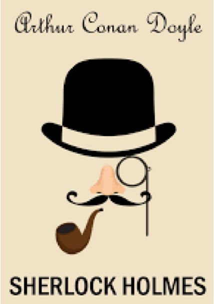
Induction: Stone S_1 is from box A and it is black.
Stone S_2 is from box A and it is black.
Stone S_3 is from box A and it is black.
…

All stones in box A are black.

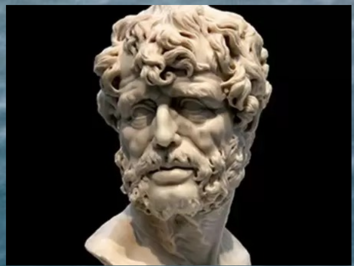
[Giuseppe Peano (1858-1932)]

Abduction: All stones in box A are black.
Stone S_i is black.

Stone S_i is from box A.

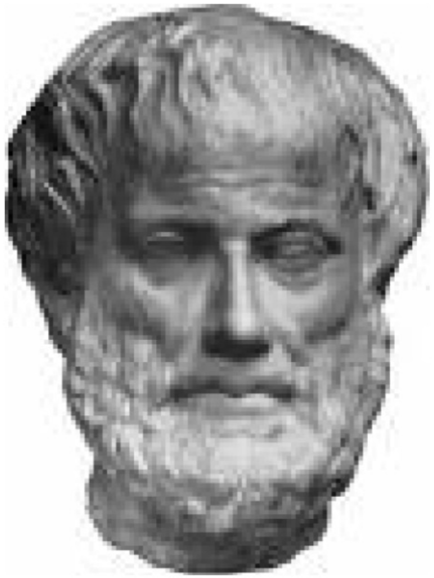


There is no favorable wind for the sailor who does not know to what port he is going



Lucius Annaeus Seneca
(4 BD – 65 AD)

Institute for Advanced Study - Princeton



[384-322 BC]

ARISTOTLE

Poetry is more elevated and more philosophical than history; for poetry expresses the universal, and history only the particular. History tells us the events as they happened, whereas poetry tells them as they could or should have happened.


Σχολή Αριστοτέλους
School of Aristoteles

PRAMANA
— journal of
physics

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Boltzmann and Einstein: Statistics and dynamics – An unsolved problem

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1. **Boltzmann's struggle**
2. **Einstein**
3. **Tsallis**
4. **Superstatistics [23]**

Will the essence of the present molecular theory, in spite of all amplifications and modifications, yet remain, or will one day an atomism totally different from the present prevail. . . . (Or, I could add, will entropy and statistics continue to play their present dominant role or will dynamics become more essential for the description of the properties of macroscopic systems?)" Boltzmann concludes his lecture by: "Indeed interesting questions! One almost regrets to have to die long before they are settled.

O! immodest mortal! Your fate is the joy of watching the ever-shifting battle!" (not to see its outcome).

Men of science have made abundant mistakes of every kind; their knowledge has improved only because of their gradual abandonment of ancient errors, poor approximations, and premature conclusions.

George Sarton
American historian of science
(1884-1956)

Nunca es tarde si la dicha es buena!

Let me paraphrase Tolstoy in his fascinating Anna Karenina:

> ***Detractors are all alike.***
> ***Collaborators and real friends are so in unique manners, which are all theirs.***

La gratitude est la mémoire du cœur!