Scaling laws in the α-XY model

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POLITÉCNICA

gisc

CBPF
Centro Brasileiro de Pesquisas Físicas

STATISTICAL MECHANHICS FOR COMPLEXITY
A CELEBRATION OF THE 80TH BIRTHDAY OF CONSTANTINO TSALLIS
Rio de Janeiro, 6 to 10 november 2023

C. Tsallis / **F.D. Nobre**

L. Boltzmann

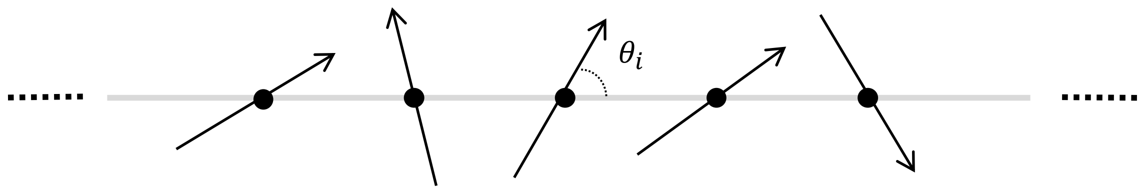
Let's go for 90, Constantino!

Outline

- HMF model
- α −XY model
  - Scaling of t_{QSS} with N and α/d
  - Scaling of t_{QSS} with U_c-U
- Ongoing work

Hamiltonian Mean Field model

- *N* planar rotators



$$\mathcal{H} = K + V = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\epsilon}{2N}\sum_{i,j=1}^{N}\left[1 - \cos(\theta_i - \theta_j)\right]$$

$\vec{m}_i \cdot \vec{m}_j$

- Spin of rotator *i* : $\vec{m}_i = (\cos\theta_i,\ \mathrm{sen}\,\theta_i)$
- Total magnetization: $\vec{M} = \frac{1}{N}\sum_{i=1}^{N}\vec{m}_i$
- $\epsilon > 0$: ferro; $\epsilon < 0$: antiferro

M. Antoni & S. Ruffo, PRE **52**, 2361 (1995)

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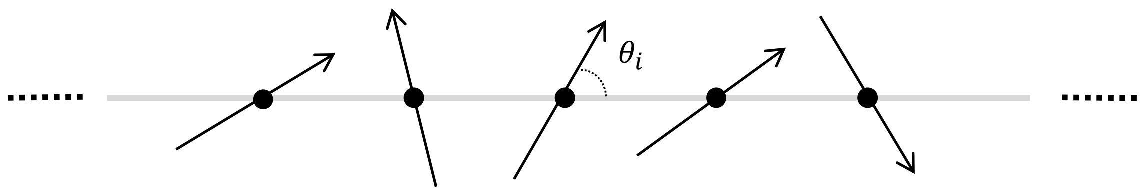
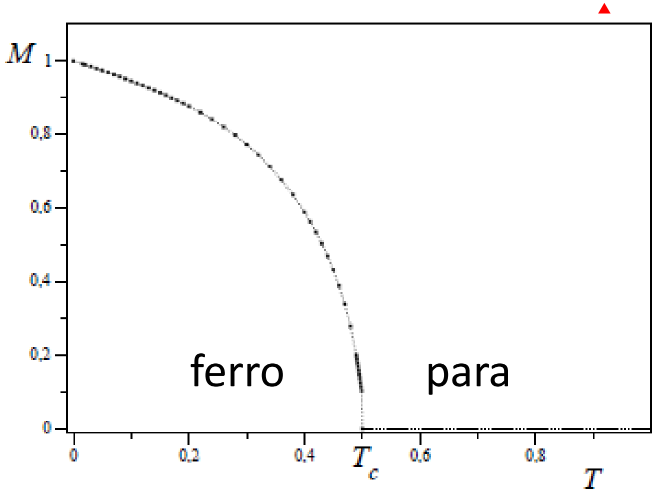


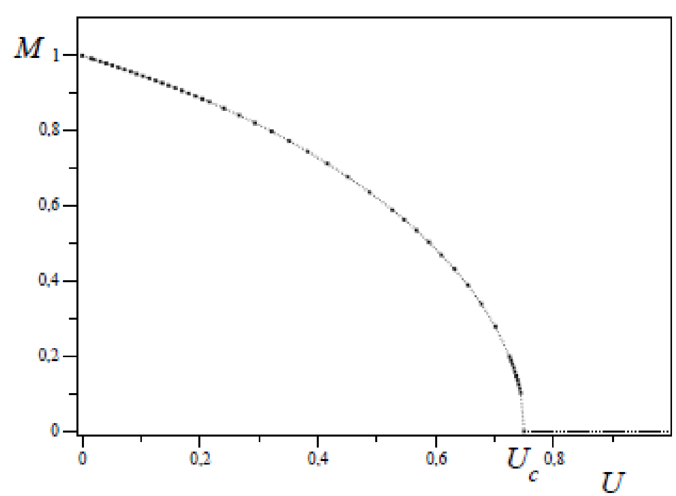
$$\mathcal{H} = K + V = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\epsilon}{2N}\sum_{i,j=1}^{N}\left[1 - \cos(\theta_i - \theta_j)\right]$$

- Equation of state: $U = \frac{T}{2} + \frac{\epsilon}{2}(1 - M^2)$

$T_c = \frac{1}{2}$ $U_c = \frac{3}{4}$



$T = \frac{2}{m}$



Hamiltonian Mean Field model

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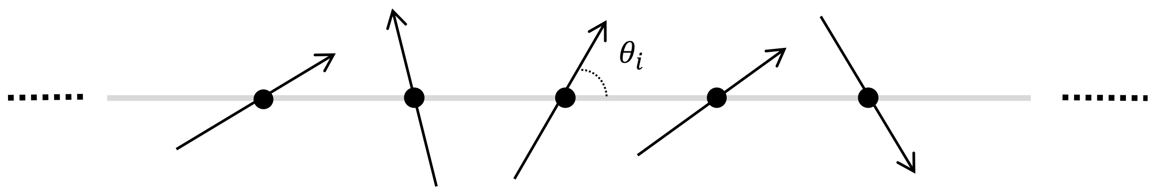

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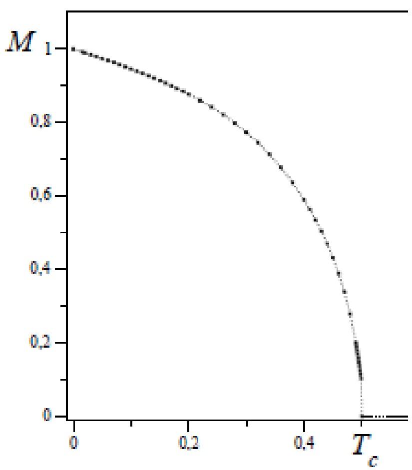

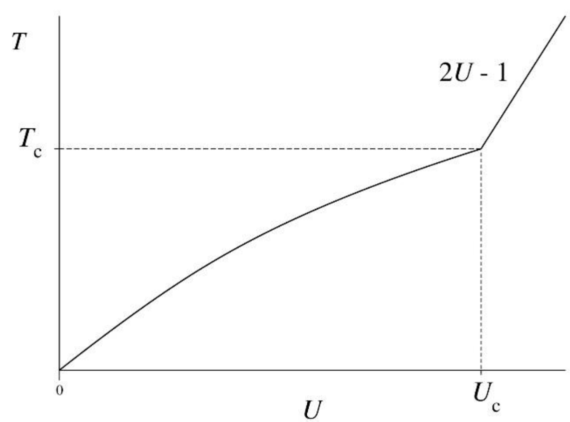

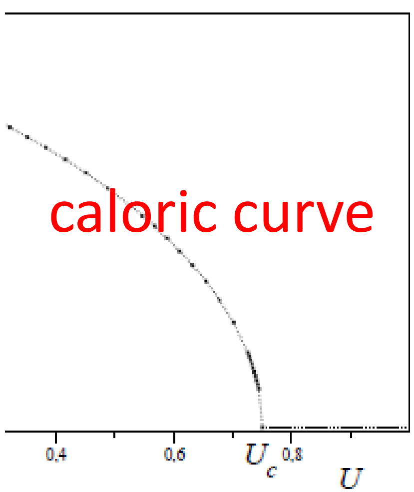

Hamiltonian Mean Field model

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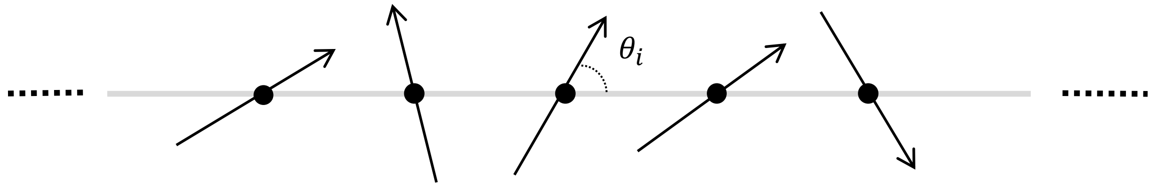

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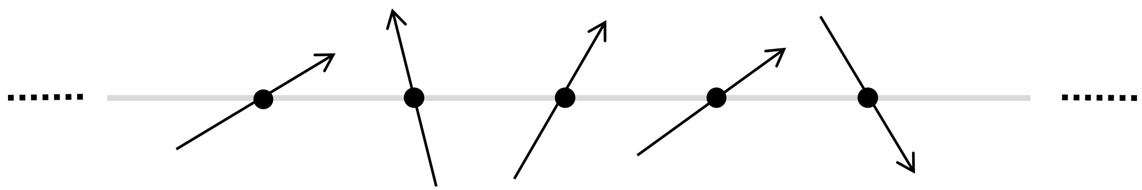
- Equation of state: $U = \frac{T}{2} + \frac{\epsilon}{2}(1 - M^2)$
- Equations of motion:

$$\dot{\theta}_i = \frac{\partial H}{\partial p_i} = p_i$$

$$\dot{p}_i = -\frac{\partial H}{\partial \theta_i} = -\frac{\epsilon}{N}\sum_{j=1}^{N} \operatorname{sen}(\theta_i - \theta_j) = \epsilon\, \vec{m}_i \times \vec{M}$$

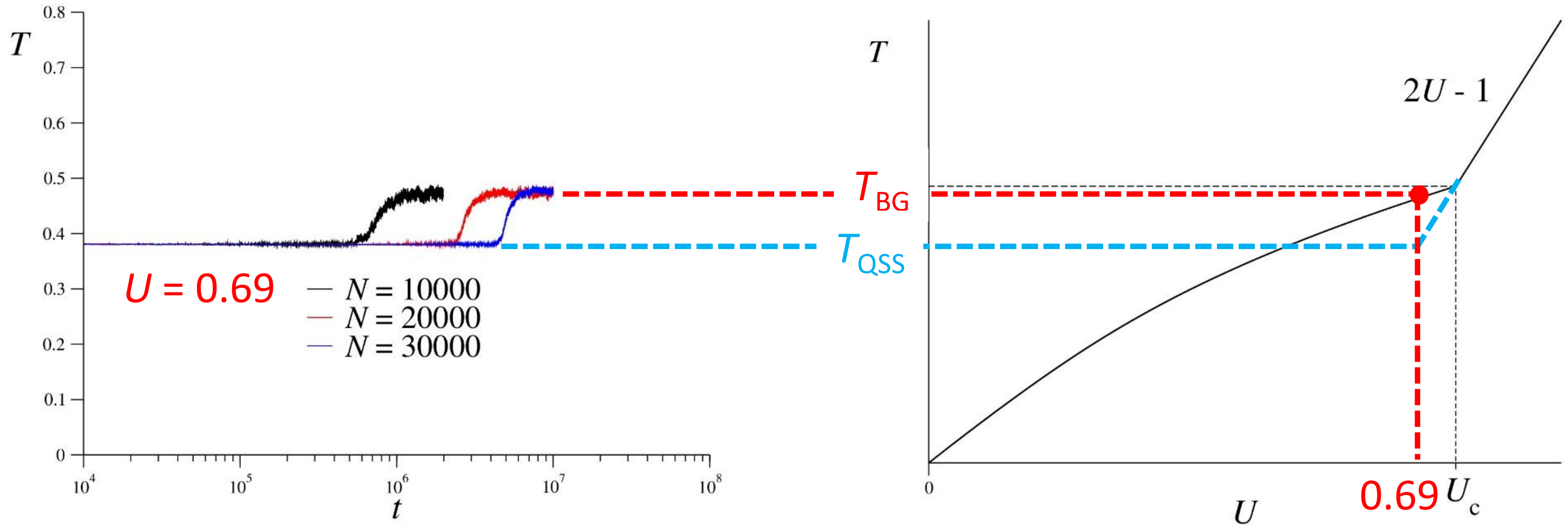
Hamiltonian Mean Field model

- *N* planar rotators



quasistationary state

$$T_{QSS} = \lim_{t\to\infty} \lim_{N\to\infty} T \neq \lim_{N\to\infty} \lim_{t\to\infty} T = T_{BG}$$



Hamiltonian Mean Field model

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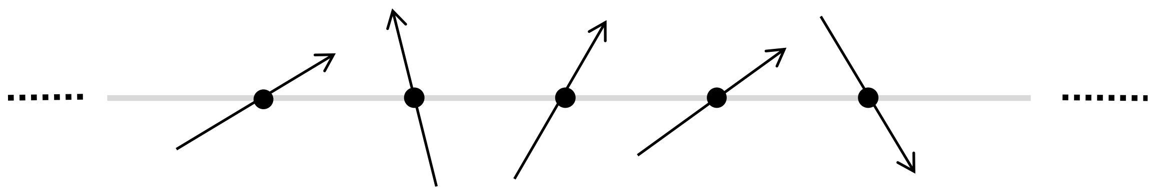

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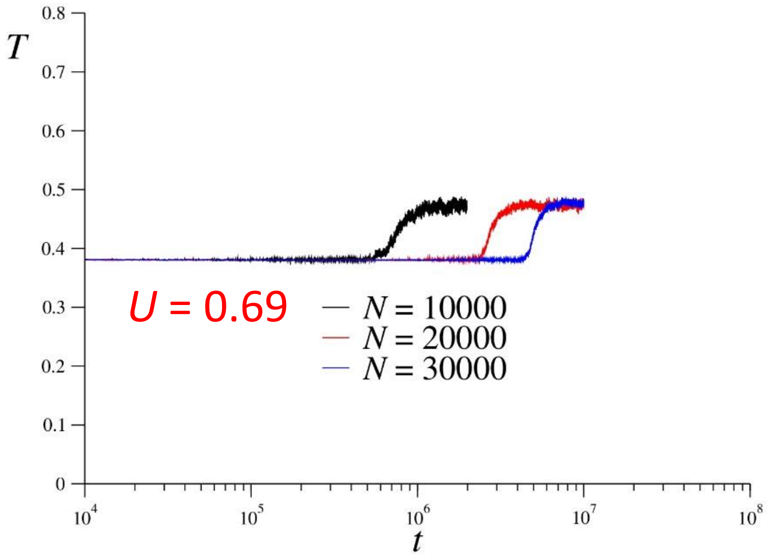
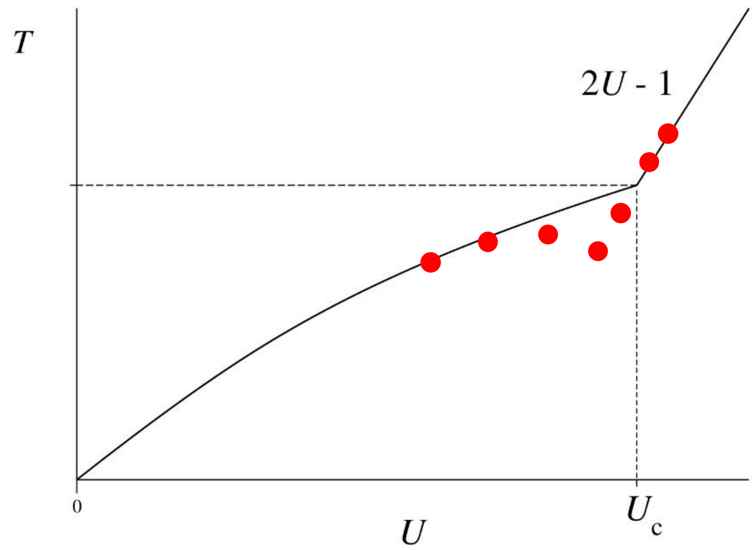


negative specific heat



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- α –XY model
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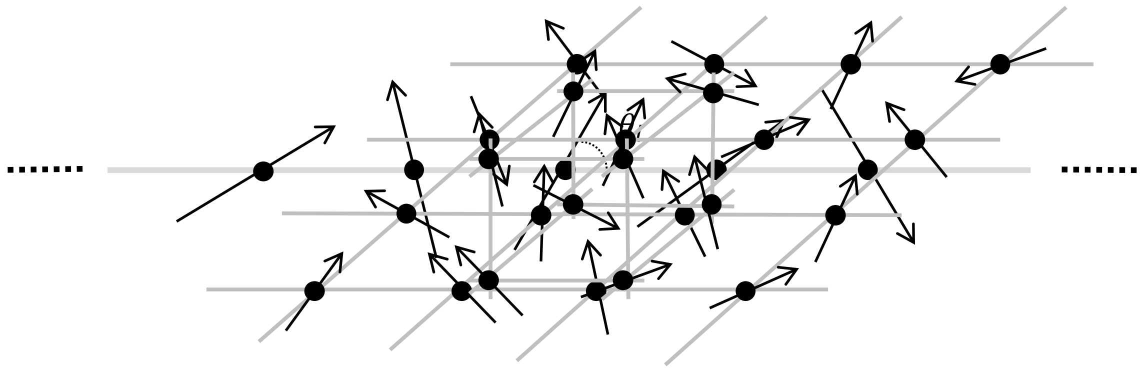
α –XY model

$$\mathcal{H} = K + V = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\epsilon}{2\tilde{N}} \sum_{i \neq j}^{N} \frac{1 - \cos(\theta_i - \theta_j)}{r_{ij}^{\alpha}}$$



C. Anteneodo & C. Tsallis, PRL **80**, 5313 (1998)

α –XY model

$$\mathcal{H} = K + V = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\epsilon}{2\tilde{N}}\sum_{i\neq j}^{N} \frac{1-\cos(\theta_i-\theta_j)}{r_{ij}^{\alpha}}$$

$$\tilde{N} = 1 + d\int_1^{N^{1/d}} \frac{r^{d-1}}{r^{\alpha}}\mathrm{d}r = \frac{N^{1-\alpha/d}-\alpha/d}{1-\alpha/d} \sim \begin{cases} \frac{N^{1-\alpha/d}}{1-\alpha/d}; & 0 \leqslant \alpha/d < 1 \\ 1+\ln N; & \alpha = d \\ \frac{\alpha/d}{\alpha/d-1}; & \alpha/d > 1 \end{cases}$$

- short range: $\alpha/d > 1$ $\tilde{N} \rightarrow$ const
- long range: $\alpha/d \leq 1$ $\tilde{N}$ diverges

- $\alpha \rightarrow \infty$ NN
- $\alpha = 0$ HMF

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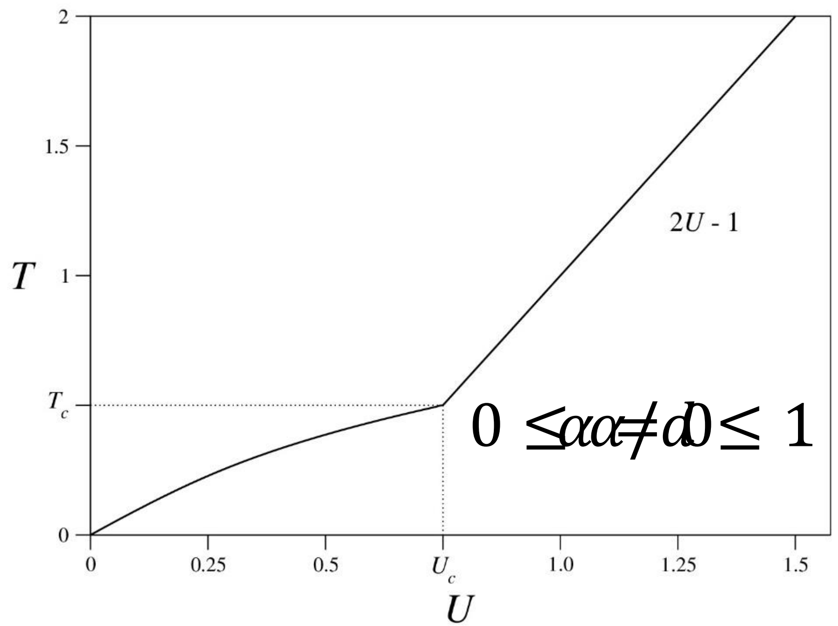
α −XY model

$$\mathcal{H} = K + V = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\epsilon}{2\tilde{N}}\sum_{i\neq j}^{N} \frac{1-\cos(\theta_i-\theta_j)}{r_{ij}^{\alpha}}$$

- caloric curve



α −XY model

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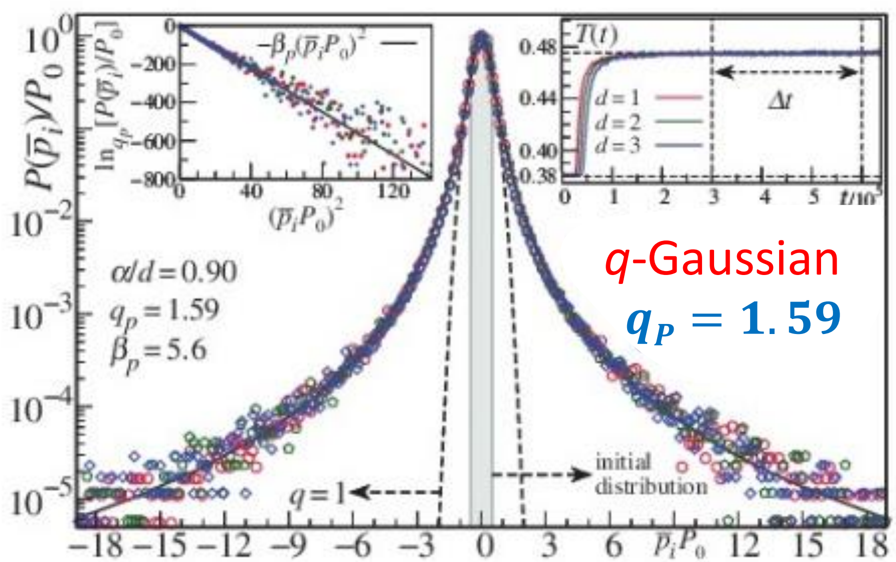
- Time-averaged momenta pdf:

$\boldsymbol{\alpha/d = 0.9}$



L. J. Cirto, A. Rodríguez, F. D. Nobre & C. Tsallis, EPL **123,** 30003 (2018)

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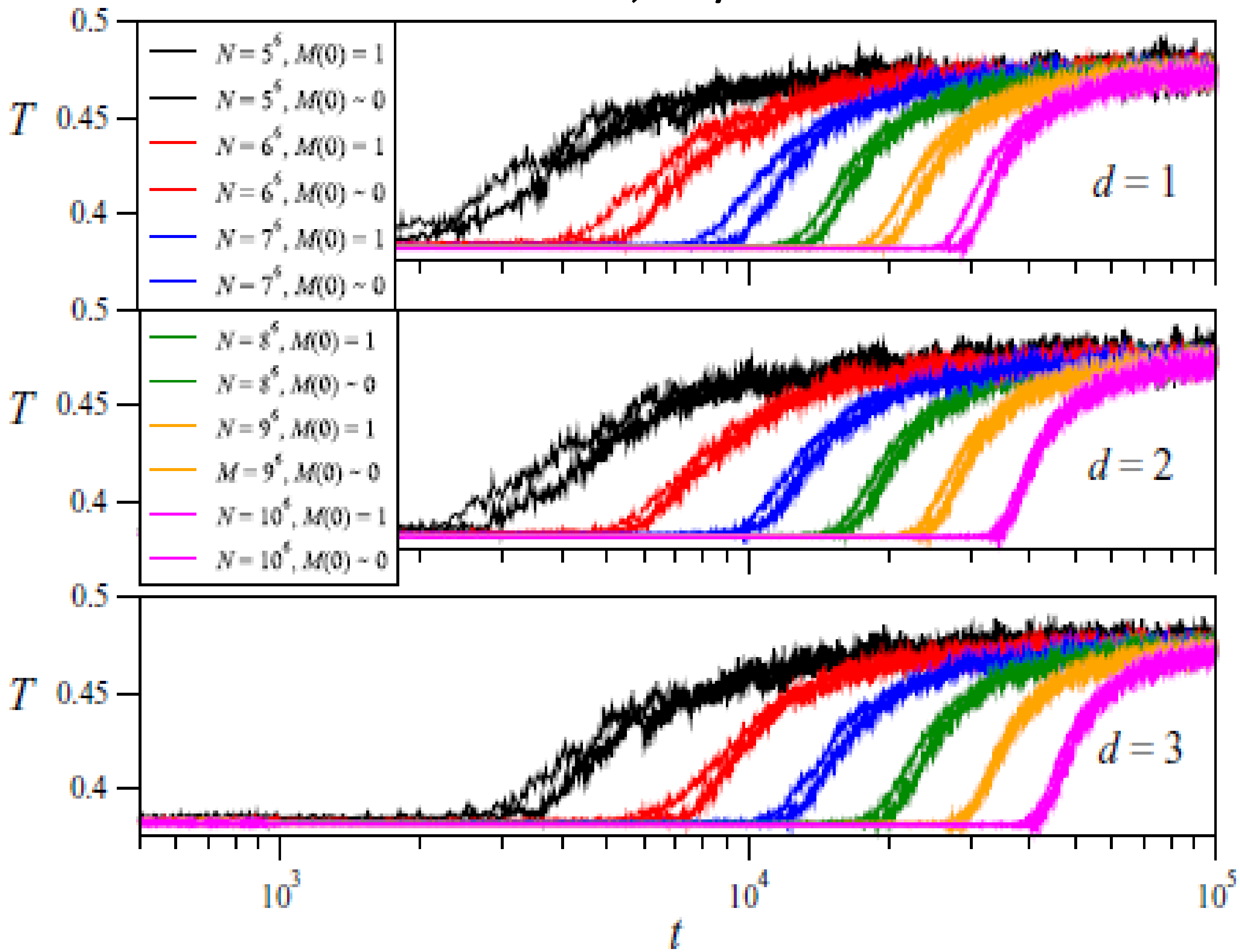
Scaling of t_{QSS} with N

- t_{QSS} increases with N and with d; $t_{QSS}(M(0)\sim 0) > t_{QSS}(M(0) = 1)$

$U = 0.69$, $\alpha/d = 0.9$



- system size: $N = L^d$

$$5^6 = 15625 = 125^2 = 25^3$$
$$6^6 = 46656 = 216^2 = 36^3$$
$$7^6 = 117649 = 343^2 = 49^3$$
$$8^6 = 262144 = 512^2 = 64^3$$
$$9^6 = 531441 = 729^2 = 81^3$$
$$10^6 = 1000000 = 1000^2 = 100^3$$

- Initial conditions

$M(0) = 1$: $\theta_i = 0 \ \forall\, i$

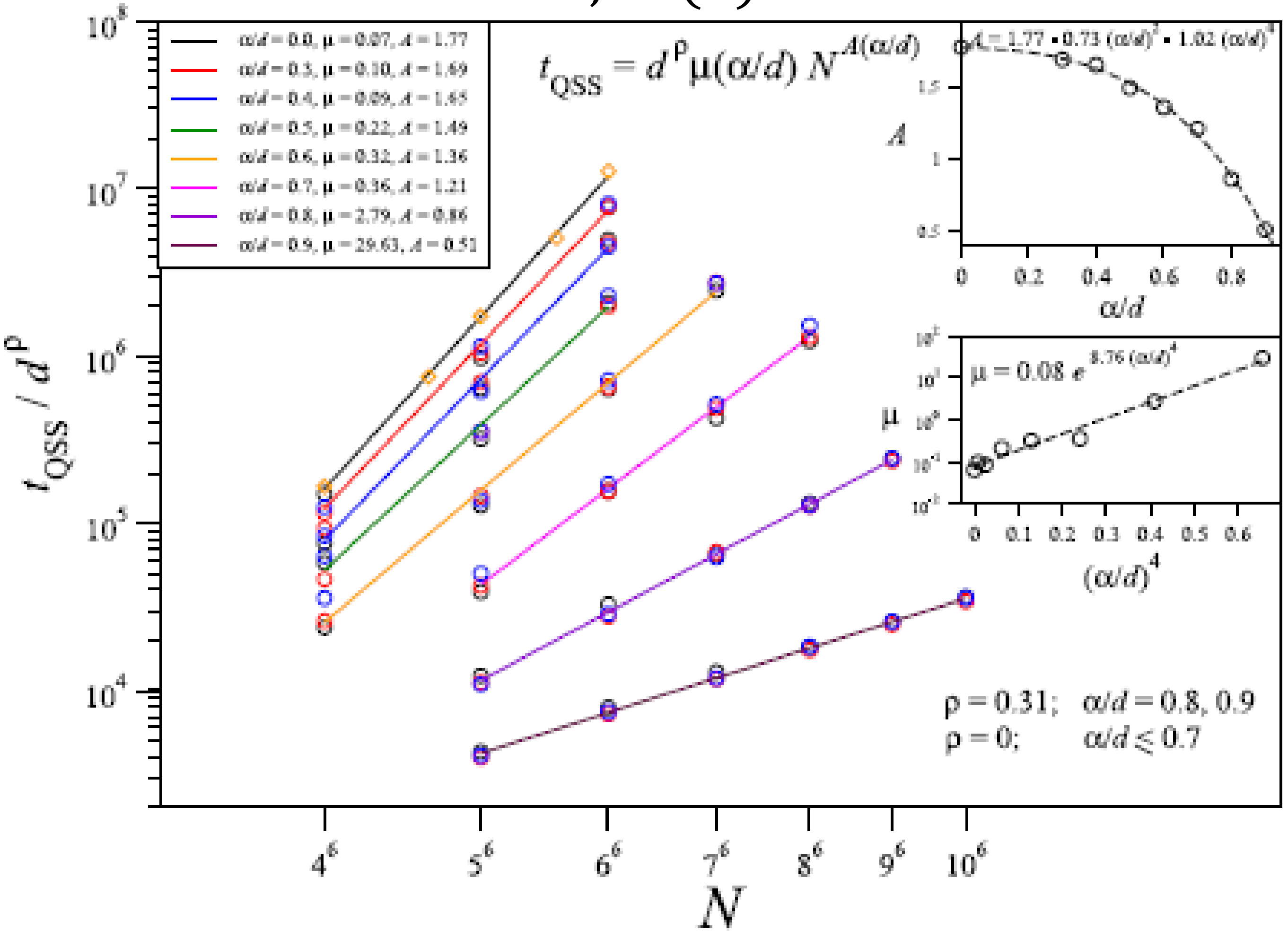
$M(0) \sim 0$: $\theta_i \in [0, 2\pi]$

A. Rodríguez, F. D. Nobre & C. Tsallis, PRE **103,** 042110 (2021)

Scaling of t_{QSS} with N

- t_{QSS} increases with N and with d: $t_{\mathrm{QSS}} = d^{\rho}\mu(\alpha/d)N^{A(\alpha/d)}$

$U = 0.69,\ M(0) \sim 0$



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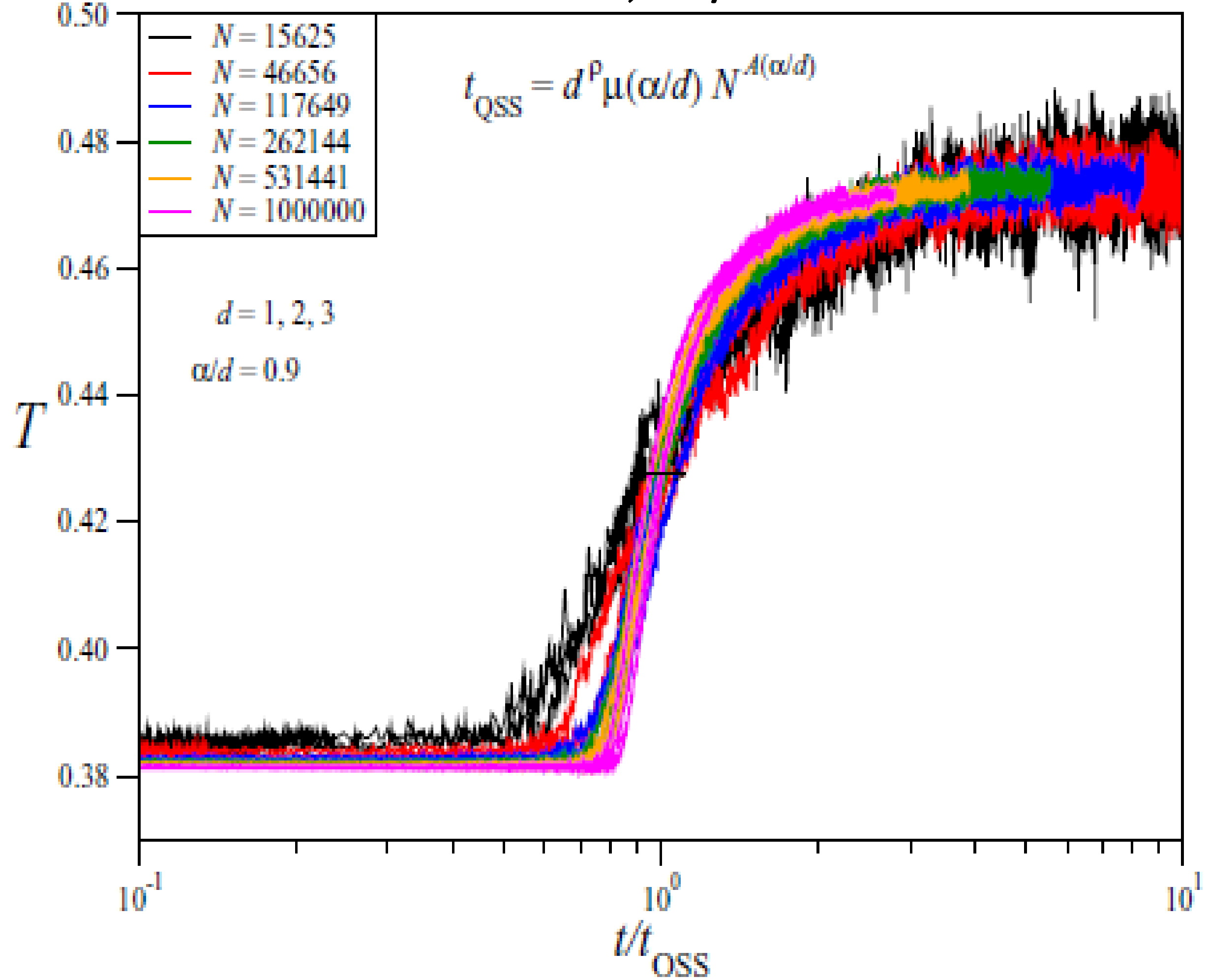
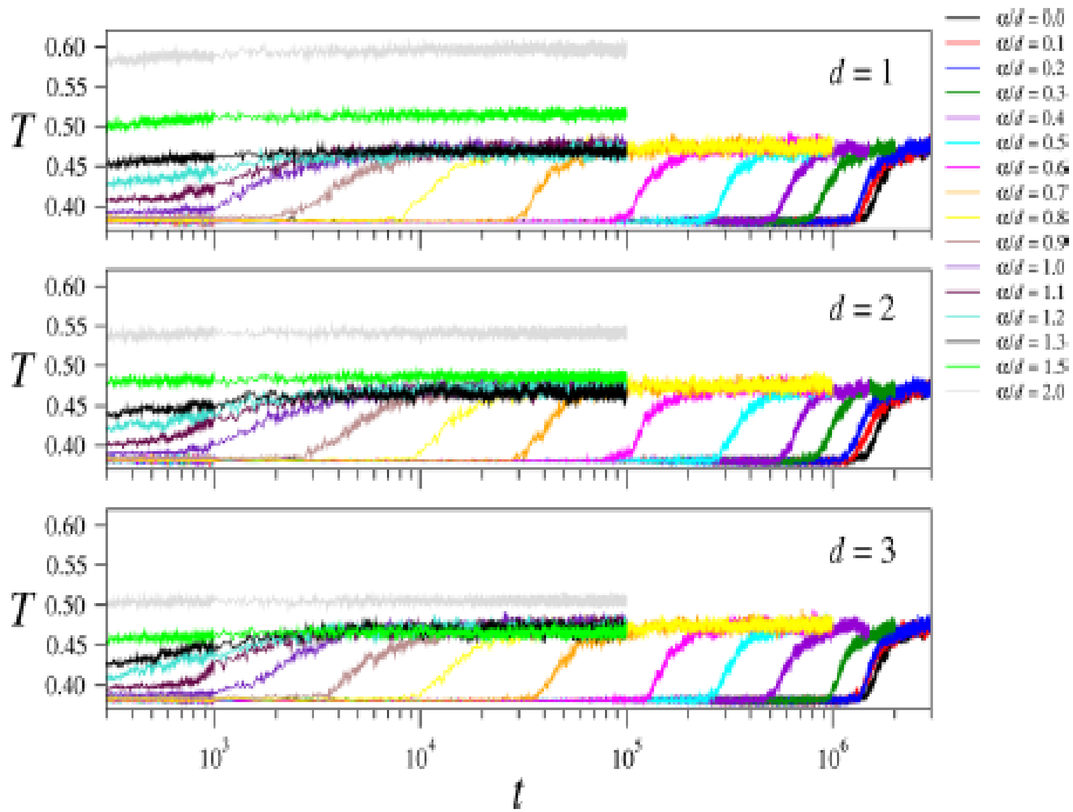


Scaling of t_{QSS} with α/d

- t_{QSS} decreases with α/d

$$U = 0.69, N = 15625$$



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Scaling of t_{QSS} with α/d

- t_{QSS} decreases with α/d: $t_{QSS} = d^{\rho}\, \nu(N) \exp[-B(N)(\alpha/d)^2]$

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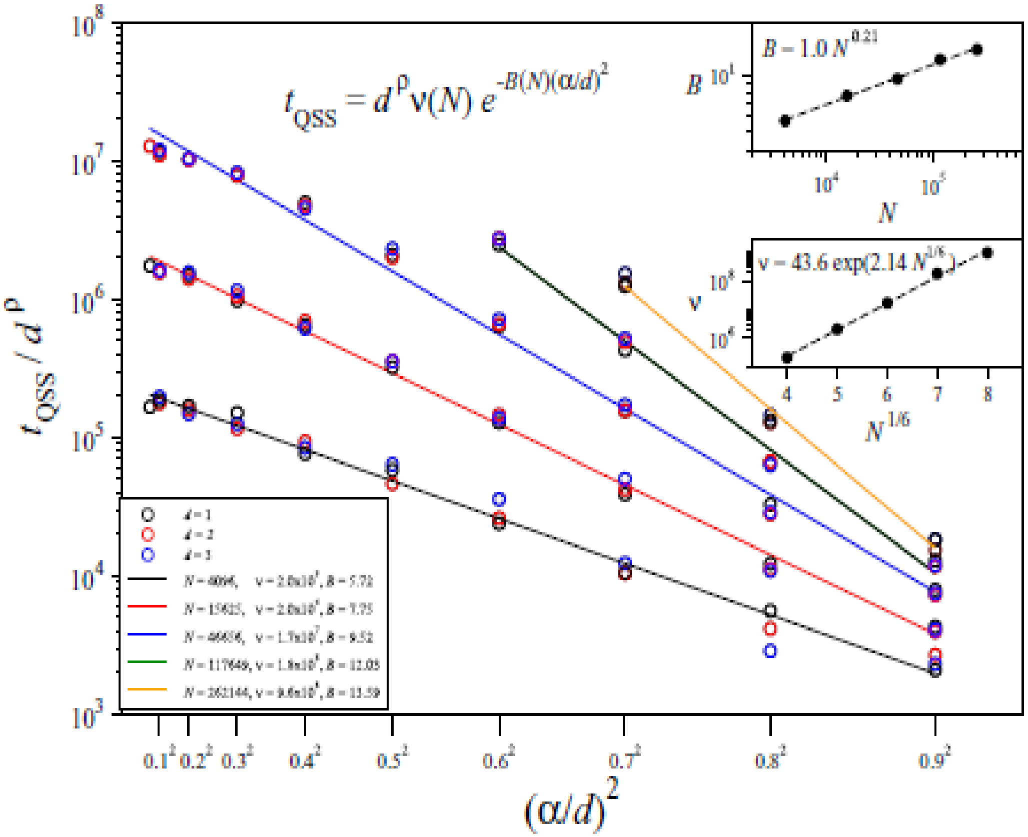

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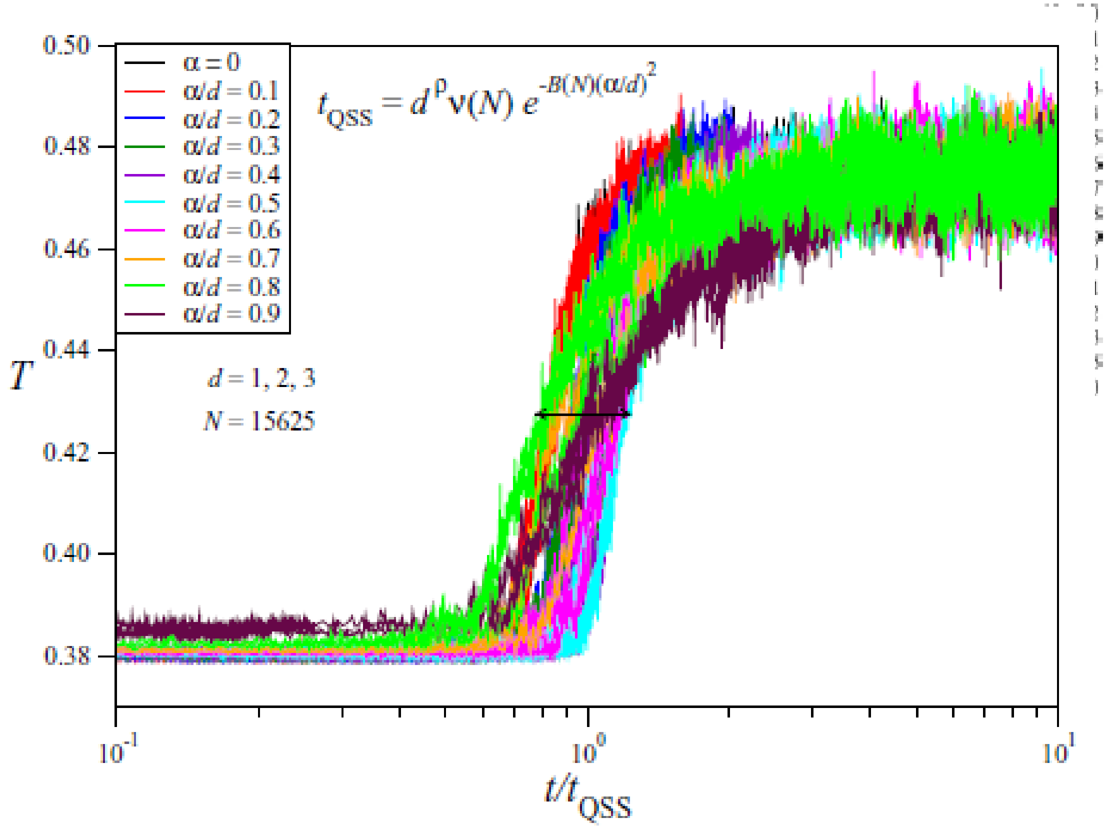

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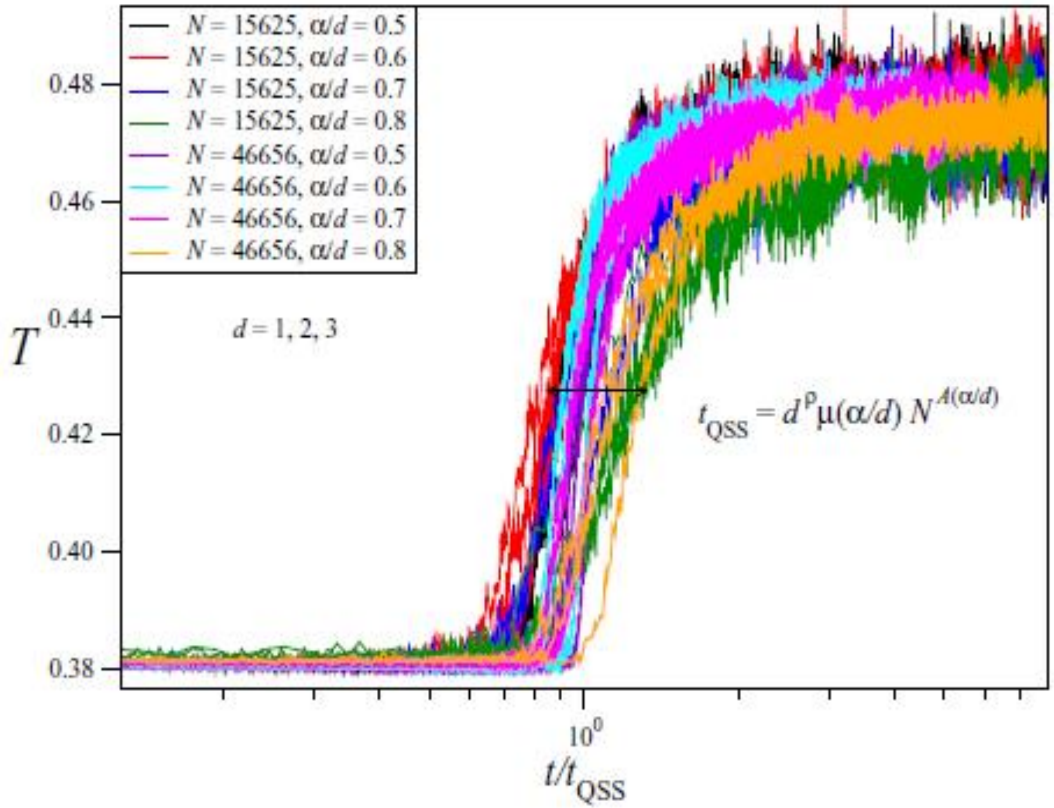
Scaling of t_{QSS} with α/d and N

- t_{QSS} decreases with α/d: $t_{QSS} =$
- t_{QSS} increases with N and d: $t_{QSS} = d^{\rho}\mu(\alpha/d)N$

$$t_{QSS}(N,\alpha,d) \propto N^{A(\alpha/d)} e^{-B(N)(\alpha/d)^2}$$

$U = 0.69$



A. Rodríguez, F. D. Nobre & C. Tsallis, PRE **103,** 042110 (2021)

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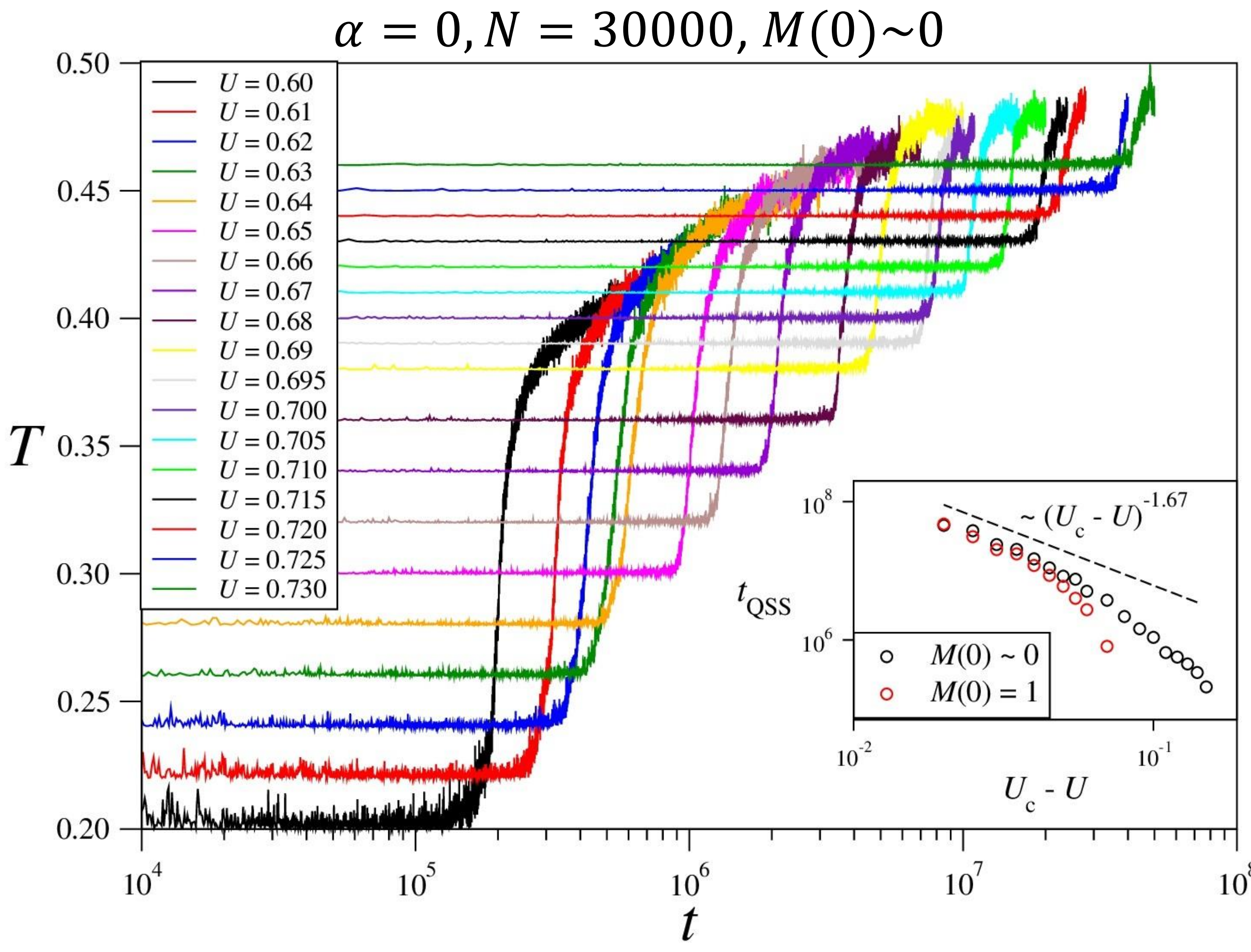
Scaling of t_{QSS} with U_c-U

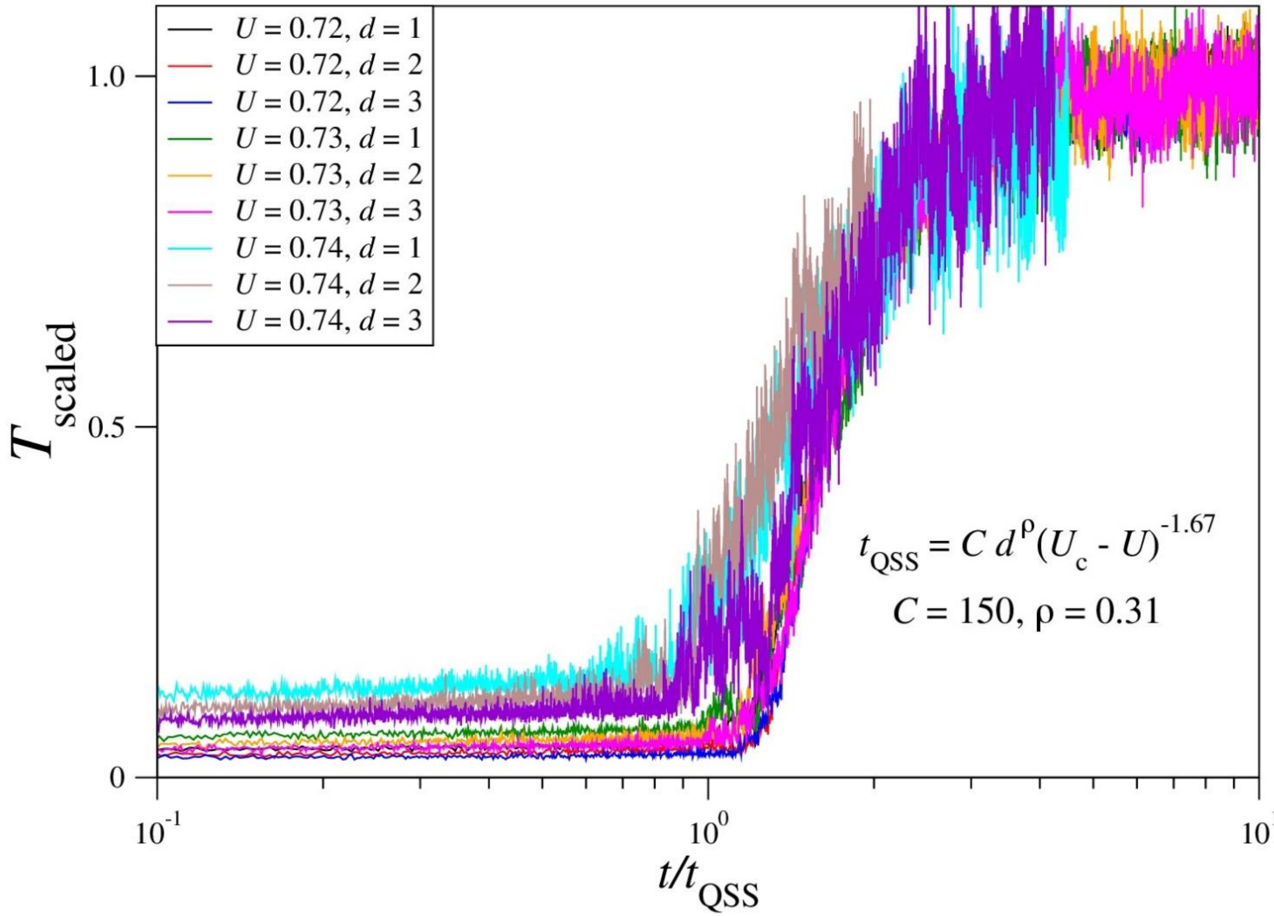


A. Rodríguez, F. D. Nobre & C. Tsallis, PRE **104,** 014144 (2021)

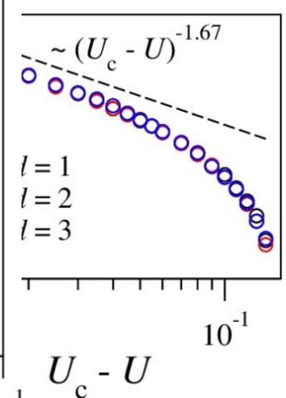
Scaling of t_{QSS} with U_c-U

- t_{QSS} increases with U_c-U and d: $t_{QSS} \propto (U_c - U)^{-\xi} \quad (U_c = 3/4,\ \xi \simeq 1.67)$

$$\alpha/d = 0.9, N = 262144$$



$$T_{\text{scaled}} = \frac{T - T_{\text{QSS}}}{T_{\text{BG}} - T_{\text{QSS}}}$$



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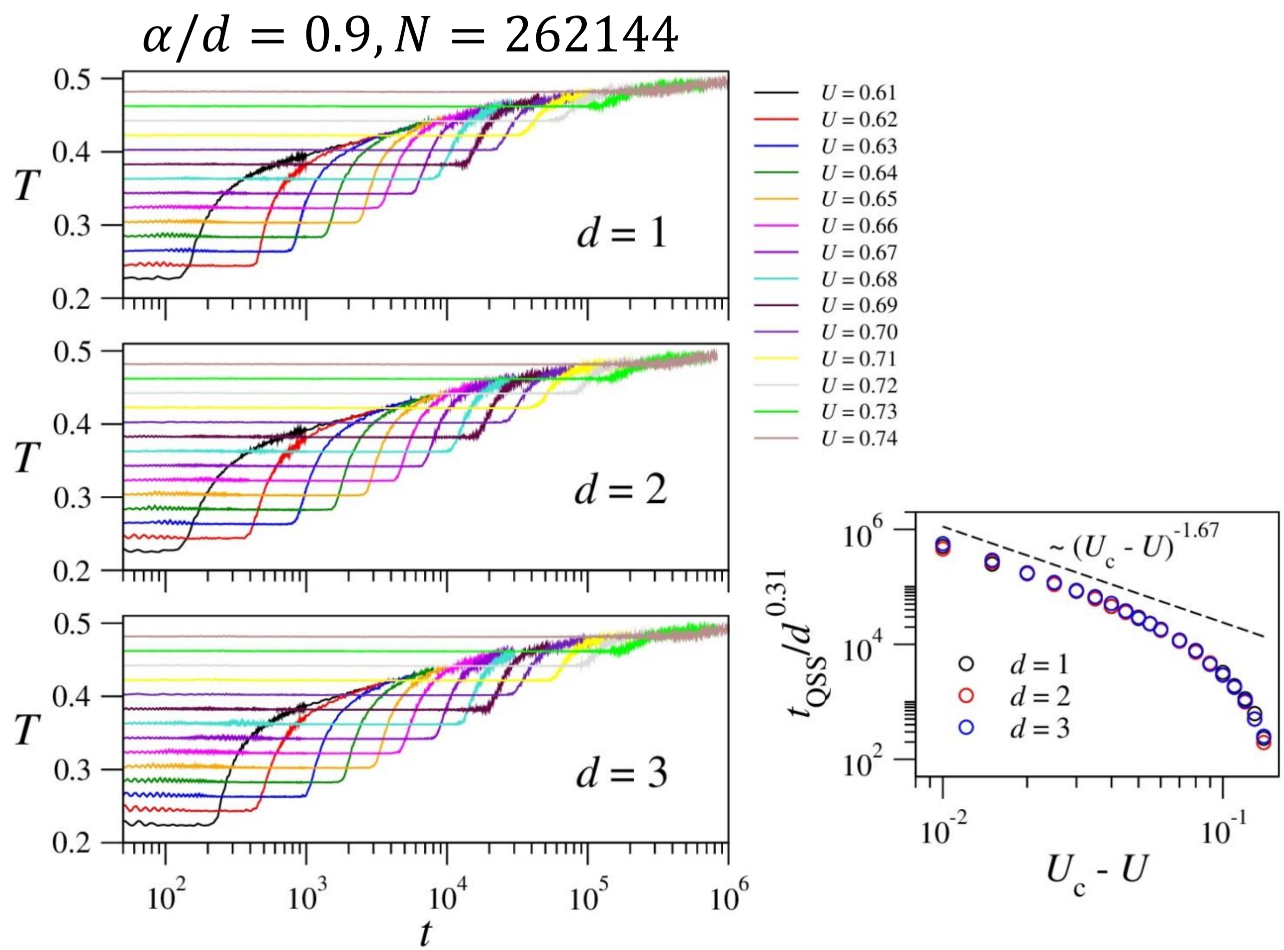

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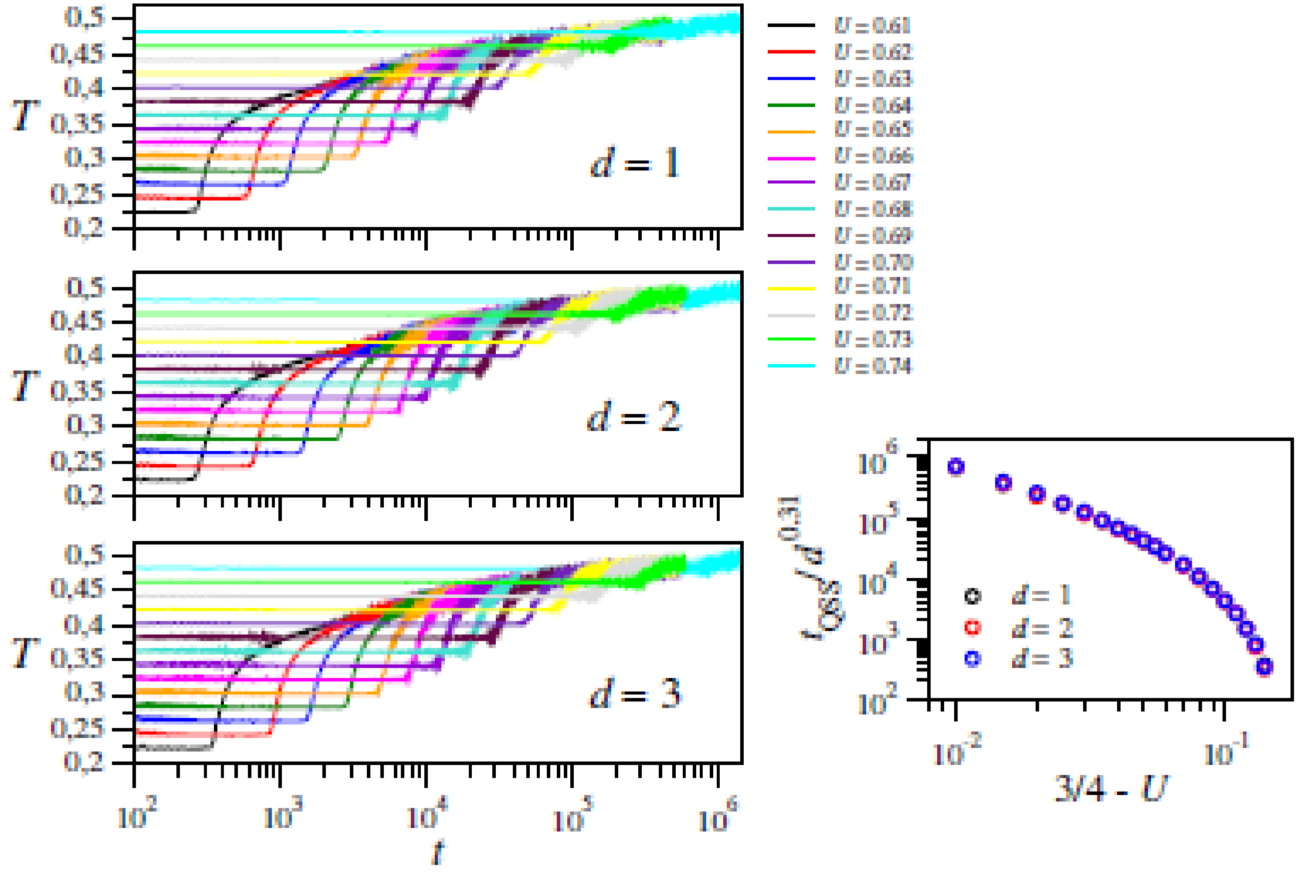
$\alpha/d = 0.9, N = 531444$



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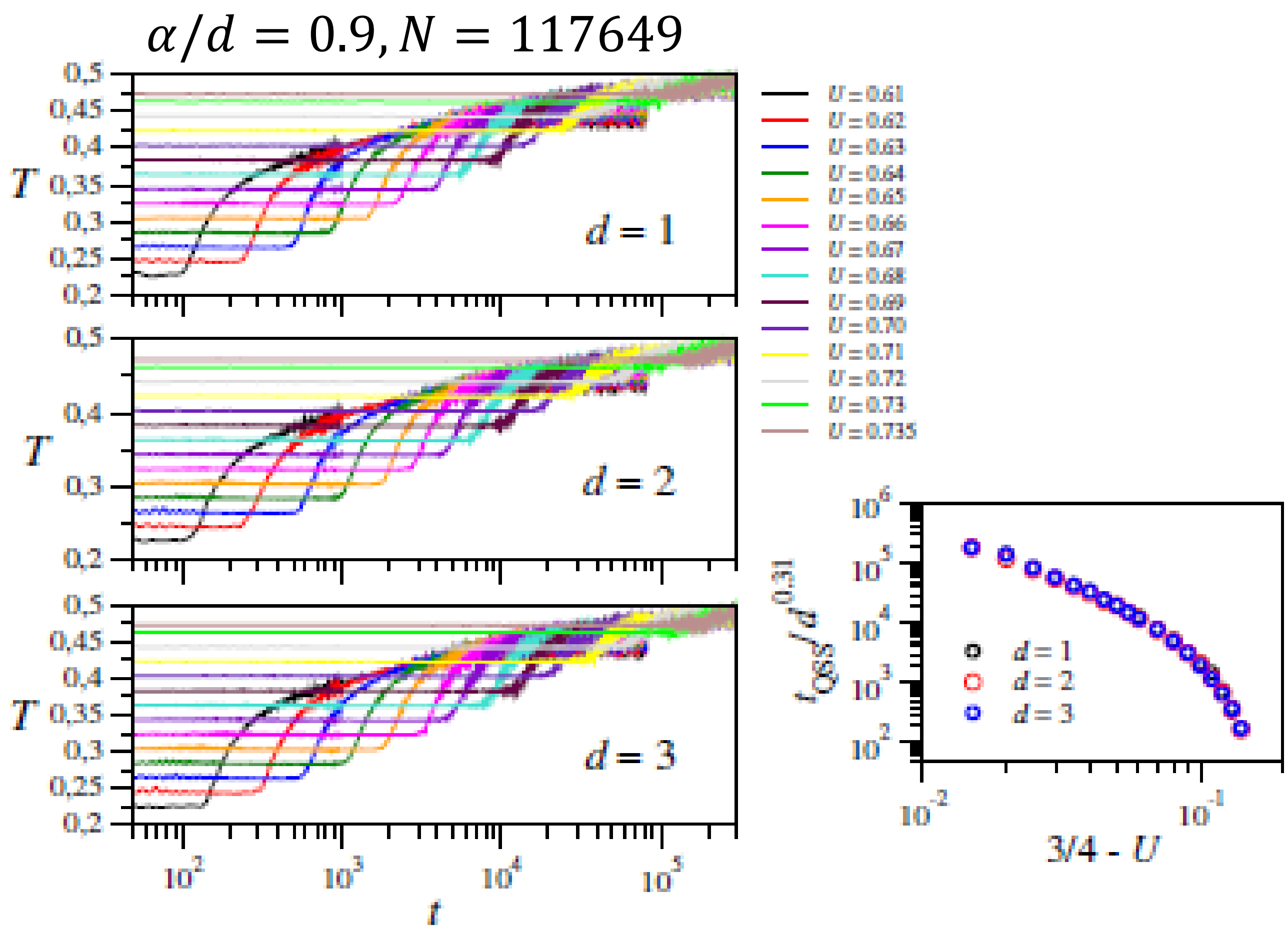

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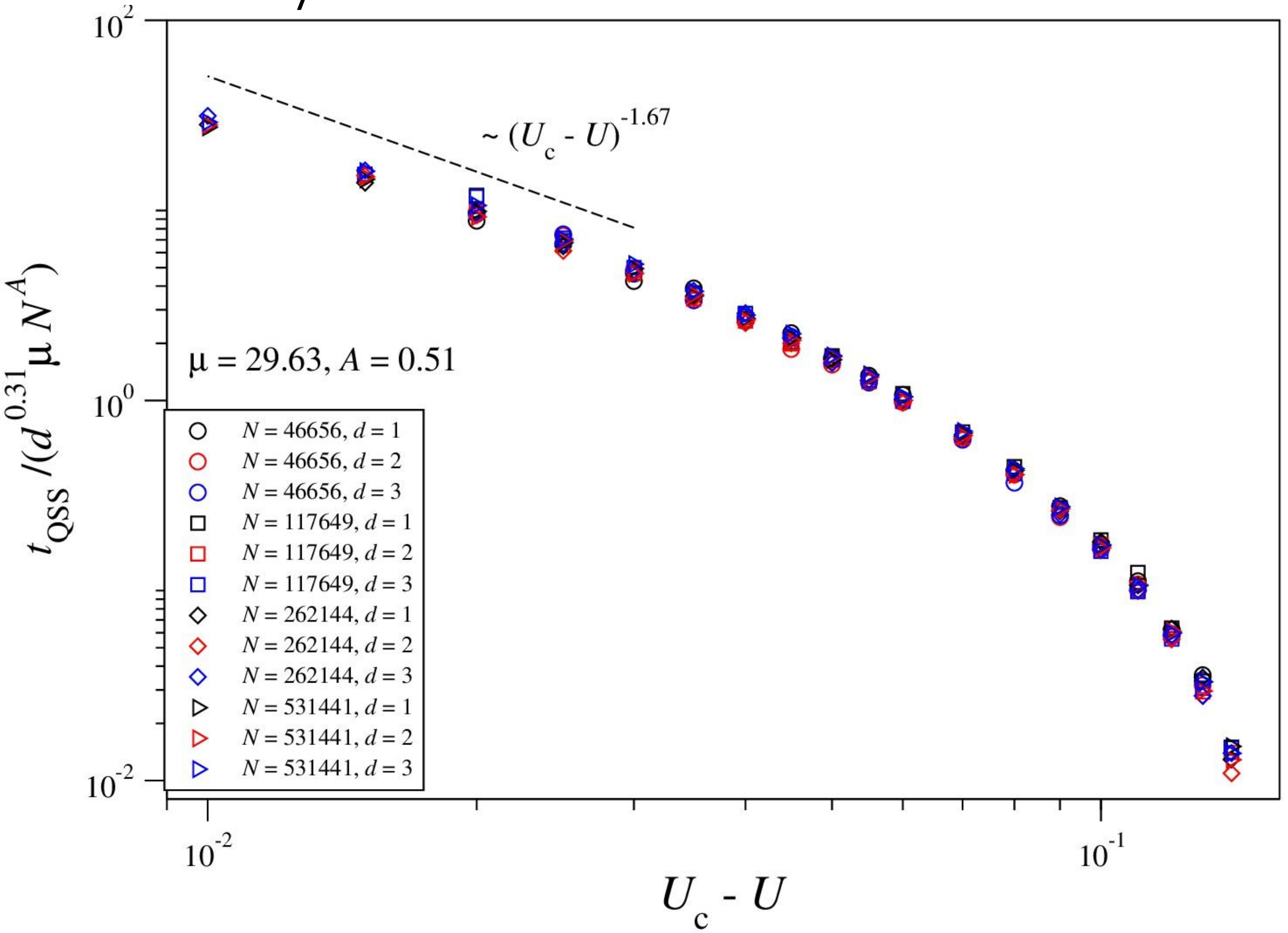
$$\epsilon = \frac{U_c - U}{U_c - 0.69}$$

$\alpha/d = 0.9$



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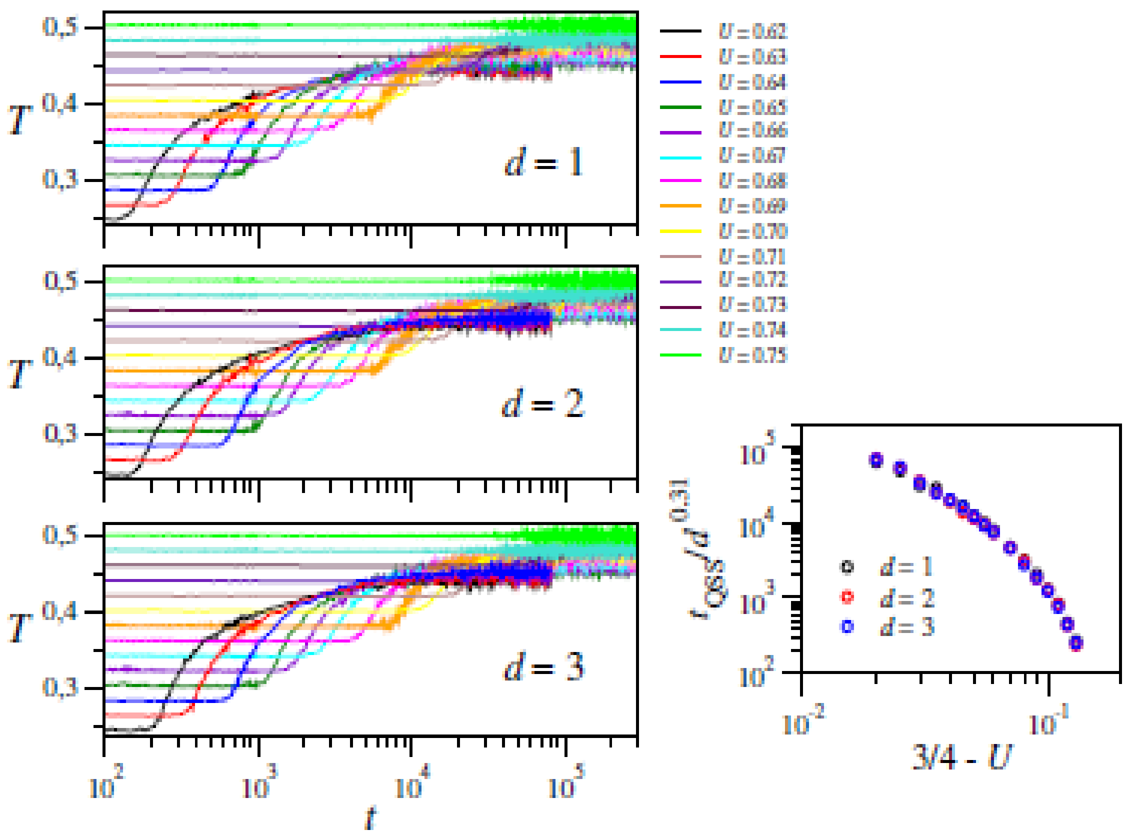
$\alpha/d = 0.9, N = 46656$



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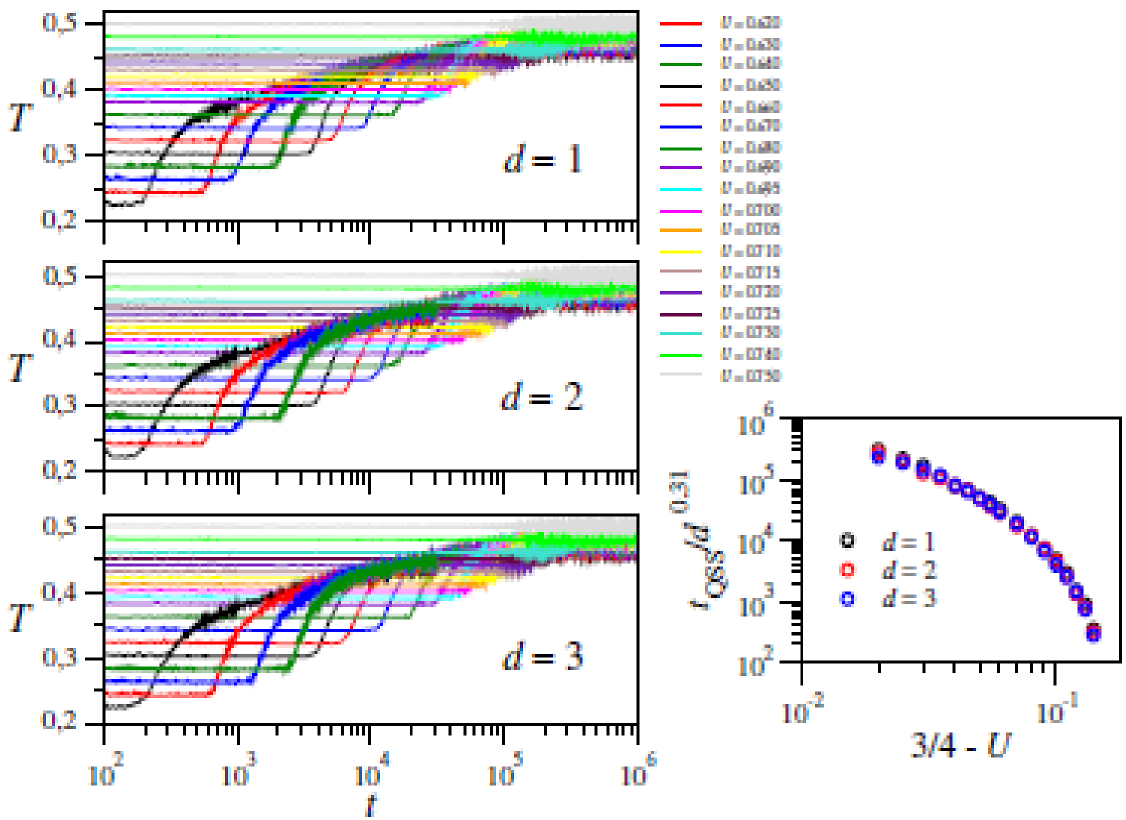
$$\alpha/d = 0.8, N = 46656$$



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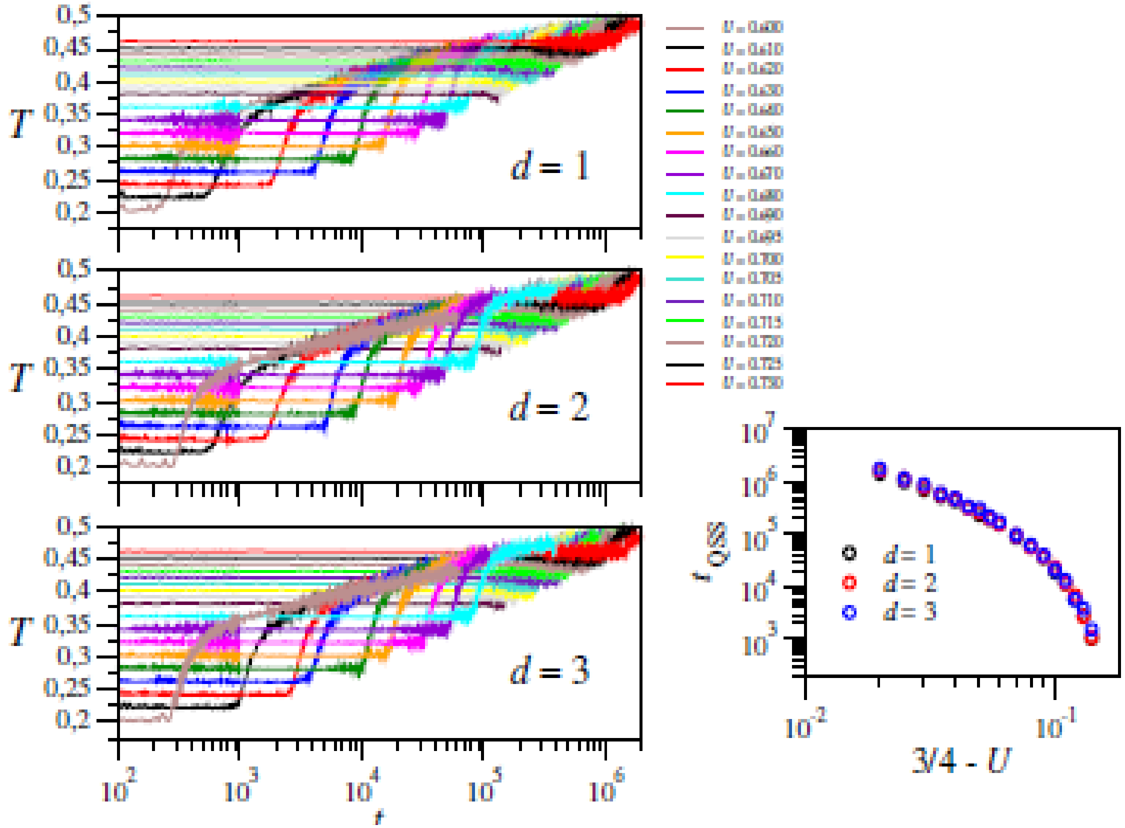
$\alpha/d = 0.7, N = 46656$



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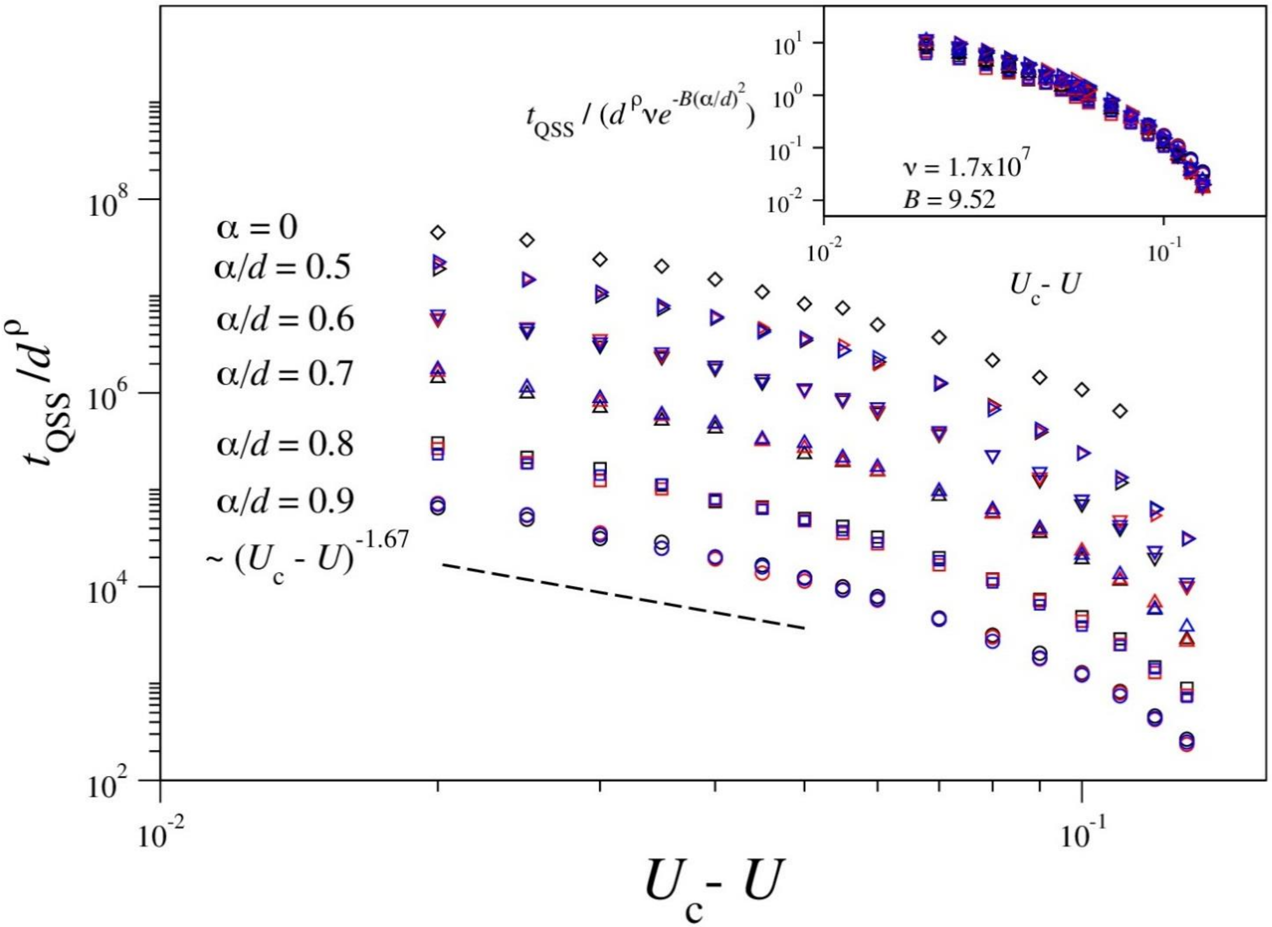
- t_{QSS} increases with U_c-U and α/d: $t_{QSS} = d^{\rho}\nu(N)\exp[-B(N)(\alpha/d)^2]\epsilon^{-\xi}$

$$N = 46656$$



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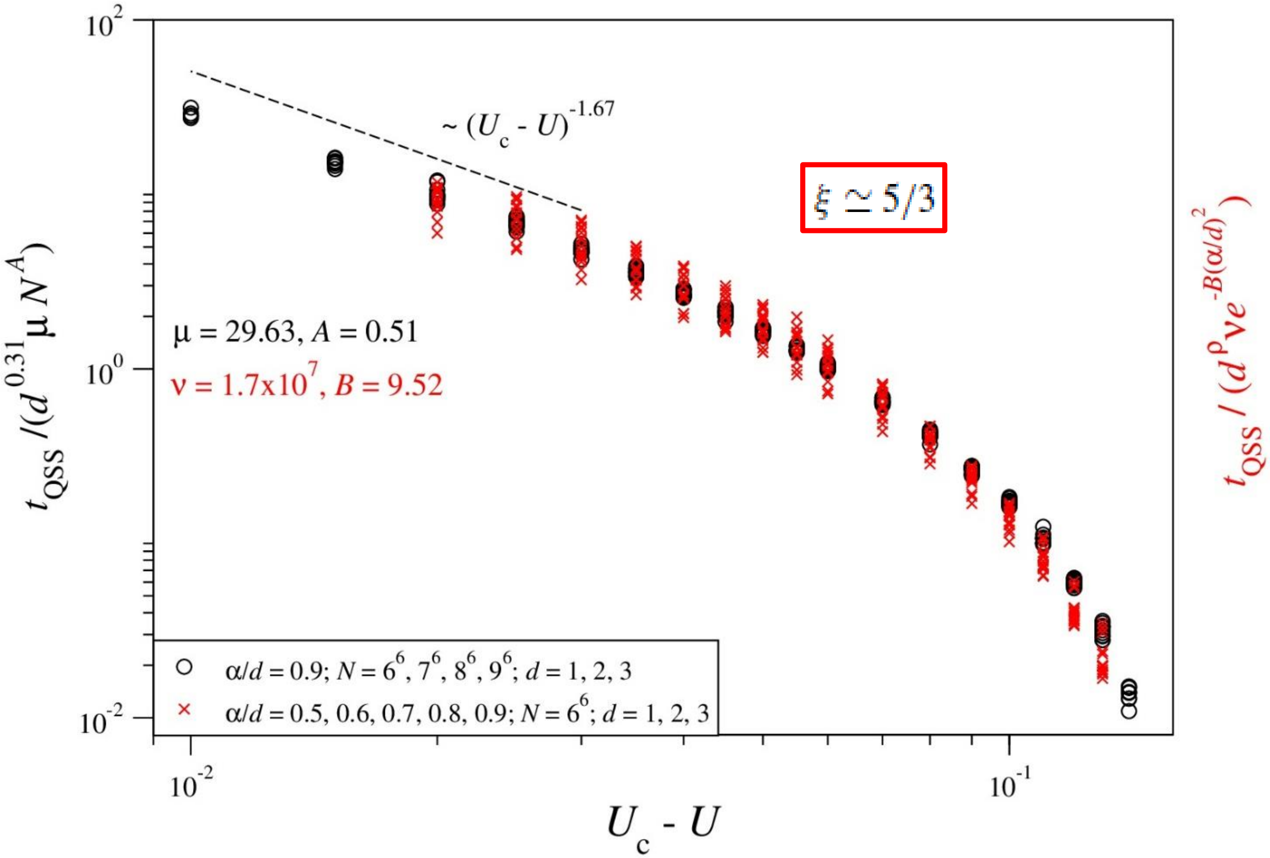
Scaling of t_{QSS} with U_c-U, N and α/d

- t_{QSS} increases with U_c-U and α/d: $t_{QSS} = d^{\rho}\nu(N)\exp[-B(N)(\alpha/d)^2]\epsilon^{-\xi}$
- t_{QSS} increases with U_c-U and N: $t_{QSS} = d^{\rho}\mu(\alpha/d)N^{A(\alpha/d)}\,\epsilon^{-\xi}$



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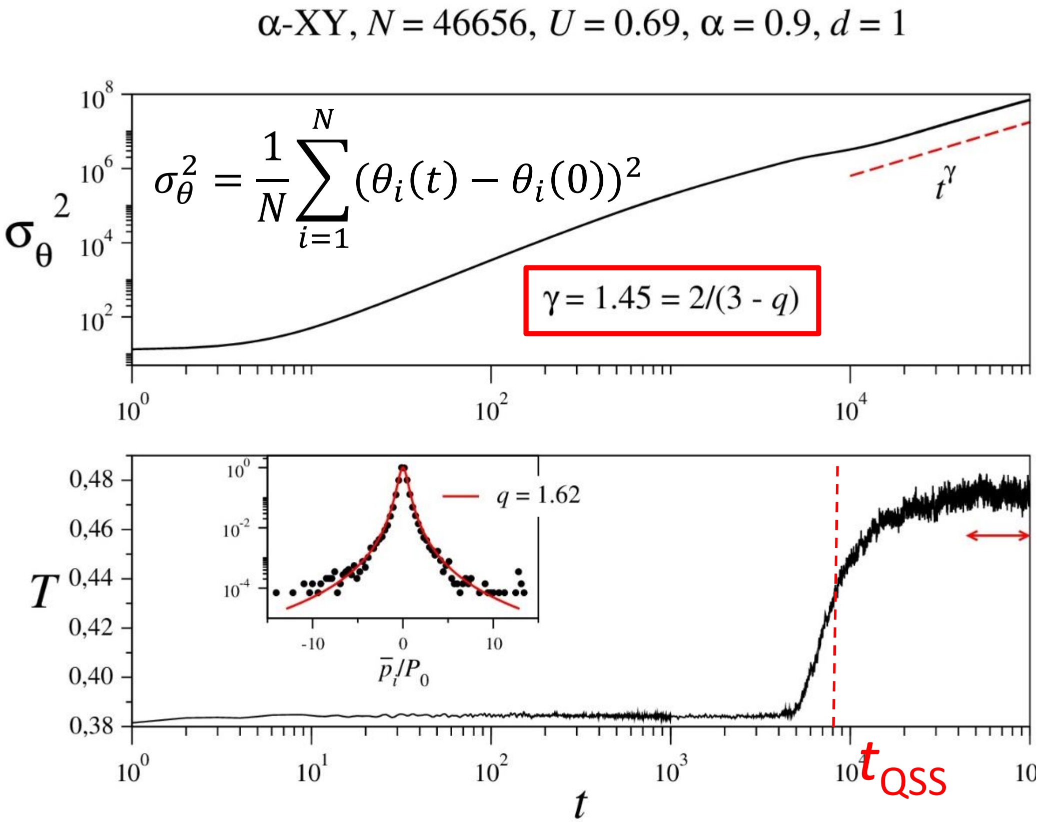
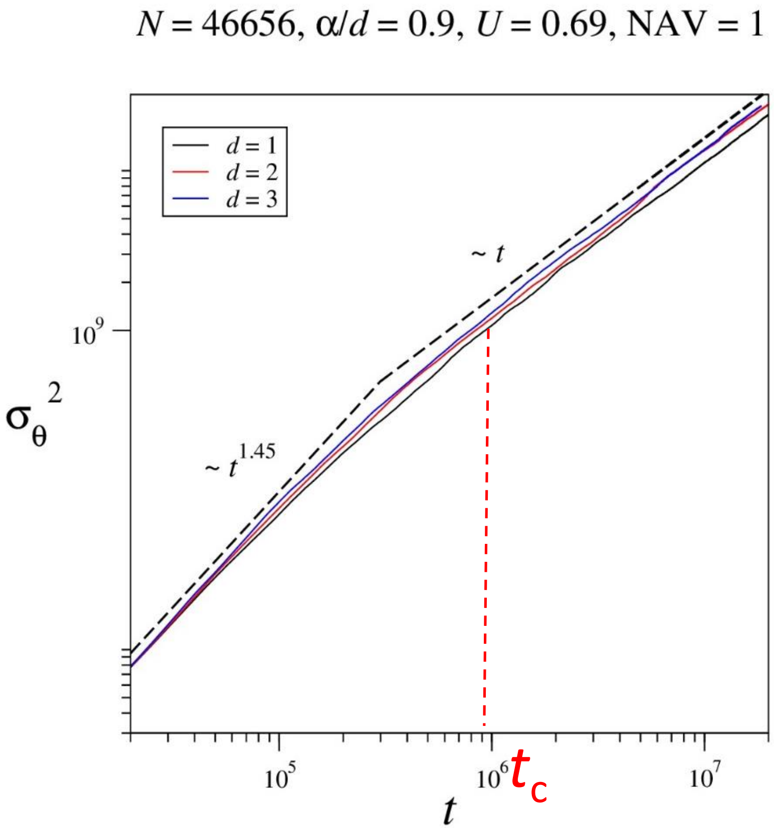
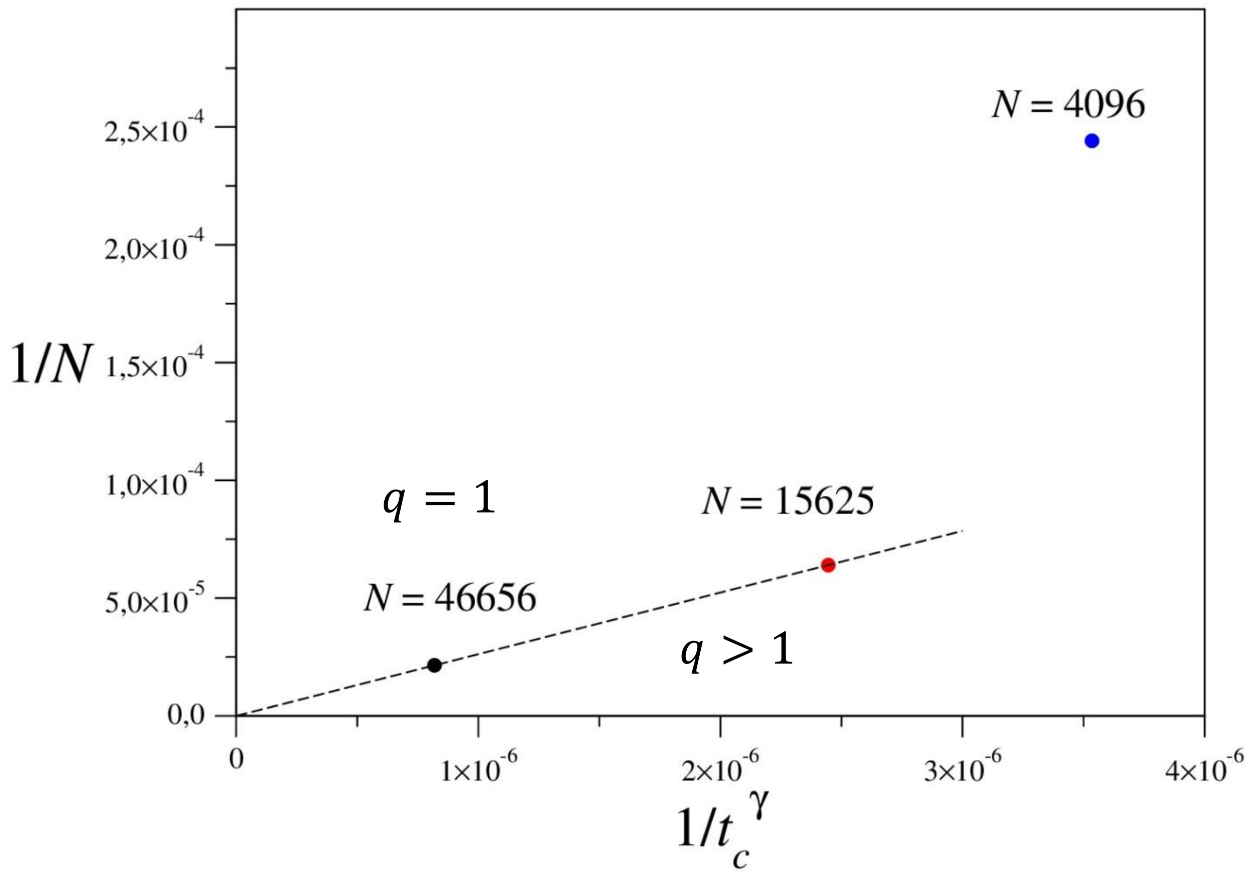
Anomalous diffusion





Anomalous diffusion

$\alpha/d = 0.9,\ U = 0.69,\ \gamma = 0.909$




Thank You.