

STATISTICAL MECHANICS FOR COMPLEXITY

A CELEBRATION OF THE 80TH BIRTHDAY OF CONSTANTINO TSALLIS

RIO DE JANEIRO, 6 TO 10 NOVEMBER 2023

Memories of a long-standing and fruitful collaboration

Andrea Rapisarda

Dipartimento di Fisica e Astronomia "Ettore Majorana" and INFN

Università di Catania, Italy

Complexity Science Hub Vienna, Austria



UNIVERSITÀ
degli STUDI
di CATANIA



COMPLEXITY
SCIENCE
HUB
VIENNA



Istituto Nazionale di Fisica Nucleare

Prologue

Everything started with the publication of this paper with [Vito Latora](#) and [Stefano Ruffo](#)

VOLUME 80, NUMBER 4

PHYSICAL REVIEW LETTERS

26 JANUARY 1998

Lyapunov Instability and Finite Size Effects in a System with Long-Range Forces

Vito Latora*

*Center for Theoretical Physics, Laboratory for Nuclear Sciences and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

Andrea Rapisarda[†]

*Istituto Nazionale di Fisica Nucleare, Sezione di Catania and Dipartimento di Fisica,
Università di Catania, Corso Italia 57, I-95129 Catania, Italy*

Stefano Ruffo[‡]

Centro Internacional de Ciencias, Cuernavaca, Morelos, Mexico
(Received 29 July 1997; revised manuscript received 29 October 1997)

We study the largest Lyapunov exponent λ and the finite size effects of a system of N fully coupled classical particles, which shows a second order phase transition. Slightly below the critical energy density U_c , λ shows a peak which persists for very large N values ($N = 20\,000$). We show, both numerically and analytically, that chaoticity is strongly related to kinetic energy fluctuations. In the limit of small energy, λ goes to zero with an N -independent power law: $\lambda \sim \sqrt{U}$. In the continuum limit the system is integrable in the whole high temperature phase. More precisely, the behavior $\lambda \sim N^{-1/3}$ is found numerically for $U > U_c$ and justified on the basis of a random matrix approximation.
[S0031-9007(97)05121-1]



The Hamiltonian Mean Field Model

$$H = \sum_{i=1}^N \frac{p_i^2}{2} + \frac{1}{2N} \sum_{i,j=1}^N [1 - \cos(\vartheta_i - \vartheta_j)]$$

Antoni and Ruffo PRE 52 (1995) 2361

- The system has an infinite range force
- It is a **useful paradigmatic model to study Hamiltonian long-range interacting (nonextensive) systems** as for example **astrophysical systems**, but also **fragmenting nuclei and atomic clusters**

The Hamiltonian Mean Field Model

The model can be seen as N classical interacting spins or particles moving on the unit circle. One can define the total magnetization \vec{M} as an **order parameter**

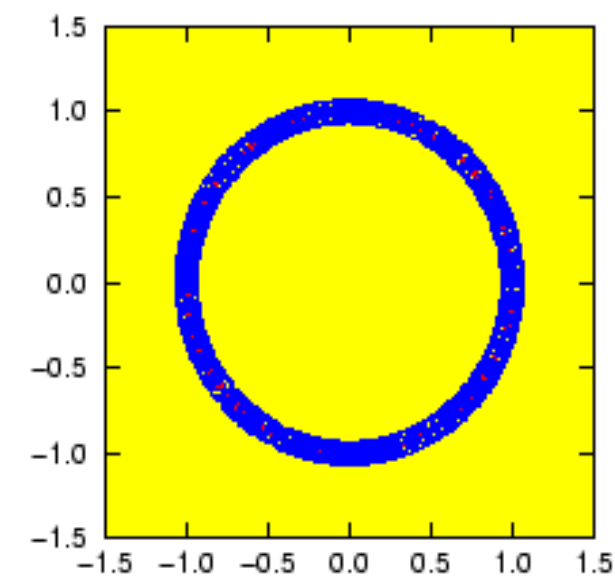
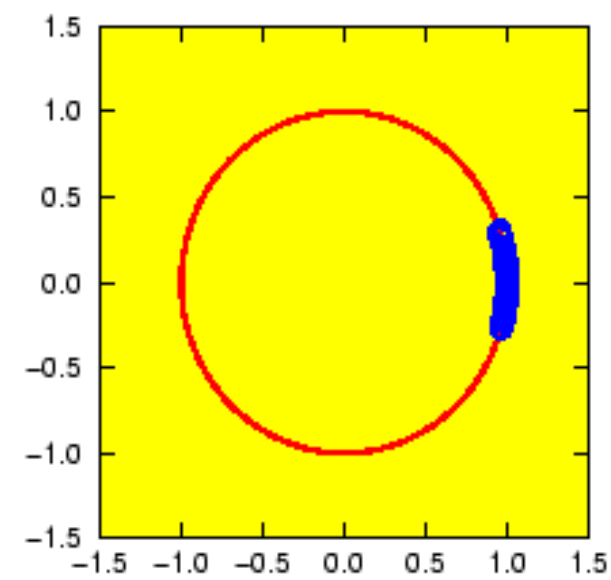
$$\vec{M} = \frac{1}{N} \sum_{i=1}^N \vec{m}_i$$

where the single spin is

$$\vec{m}_i = (\cos\vartheta_i, \sin\vartheta_i)$$

$M=1$ clustered phase
for $U < U_c$

$M=0$ homogeneous
phase for $U > U_c$



The model shows a **second-order phase transition**, passing from a clustered phase to a homogeneous one as a function of energy

The Hamiltonian Mean Field Model

Critical behavior of the model

The model has a second order phase transition.

The critical point is at

$$U_c = \frac{3}{4} \quad \text{and} \quad T_c = \frac{1}{2}$$

Close to the critical point one gets for $\beta \sim \beta_c$

$$M \approx \frac{4}{\beta} \sqrt{\frac{1}{2} - \frac{1}{\beta}} \quad U \approx \frac{1}{2\beta} \left[1 - \frac{8(\beta-2)}{\beta} \right] + \frac{1}{2}$$

Hence M vanishes with the classical critical mean field exponent $1/2$

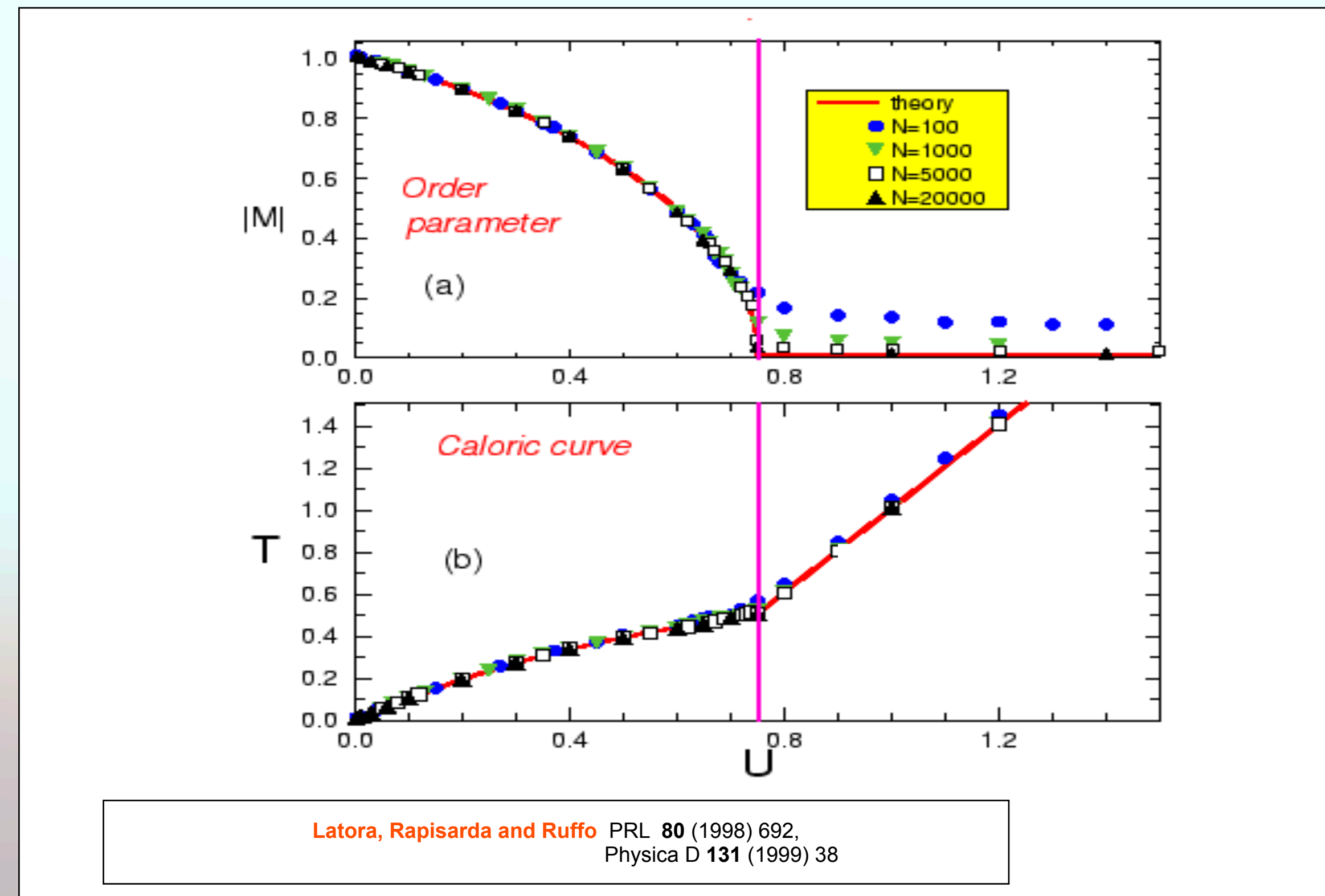
On the other hand, the specific heat $C_V = \frac{\partial U}{\partial T}$ is

$$C_V(T_c) = \frac{5}{2} \quad \text{and} \quad C_V = \frac{1}{2} \quad \text{for} \quad T > T_c$$

Close to the critical point $C_V \approx (T_c - T)^{-\alpha}$ with $\alpha = 0$

Comparison with numerical simulations at equilibrium

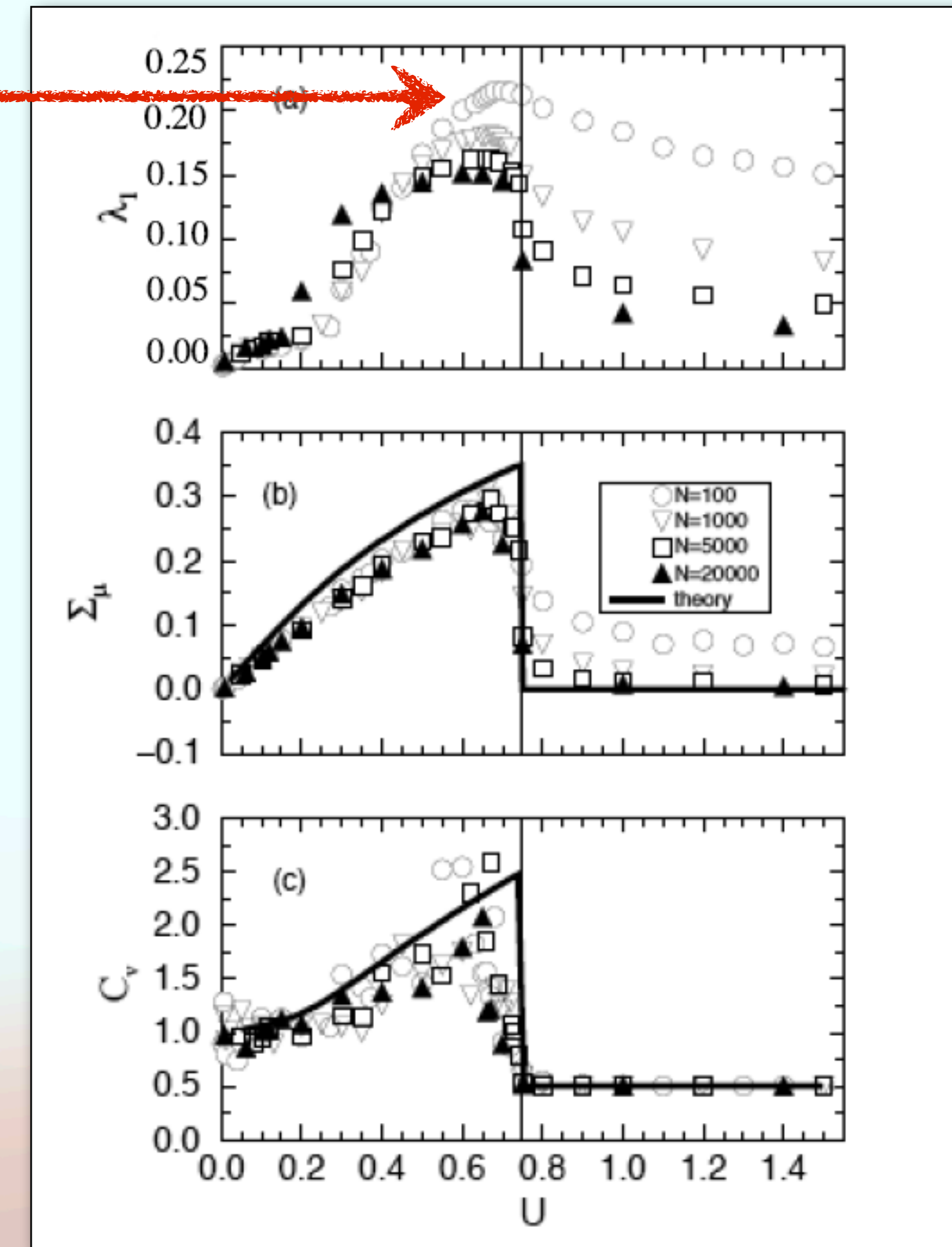
Good agreement between the **exact canonical solution** and **numerical microcanonical simulations at equilibrium** for various sizes N of the system



The Hamiltonian Mean Field Model

One finds a **maximum of the Largest Lyapunov Exponent** in correspondence of the **critical point**, where fluctuations in kinetic energy and the specific heat have also a peak!

Latora, Rapisarda and Ruffo
Physica D **131** (1999) 38

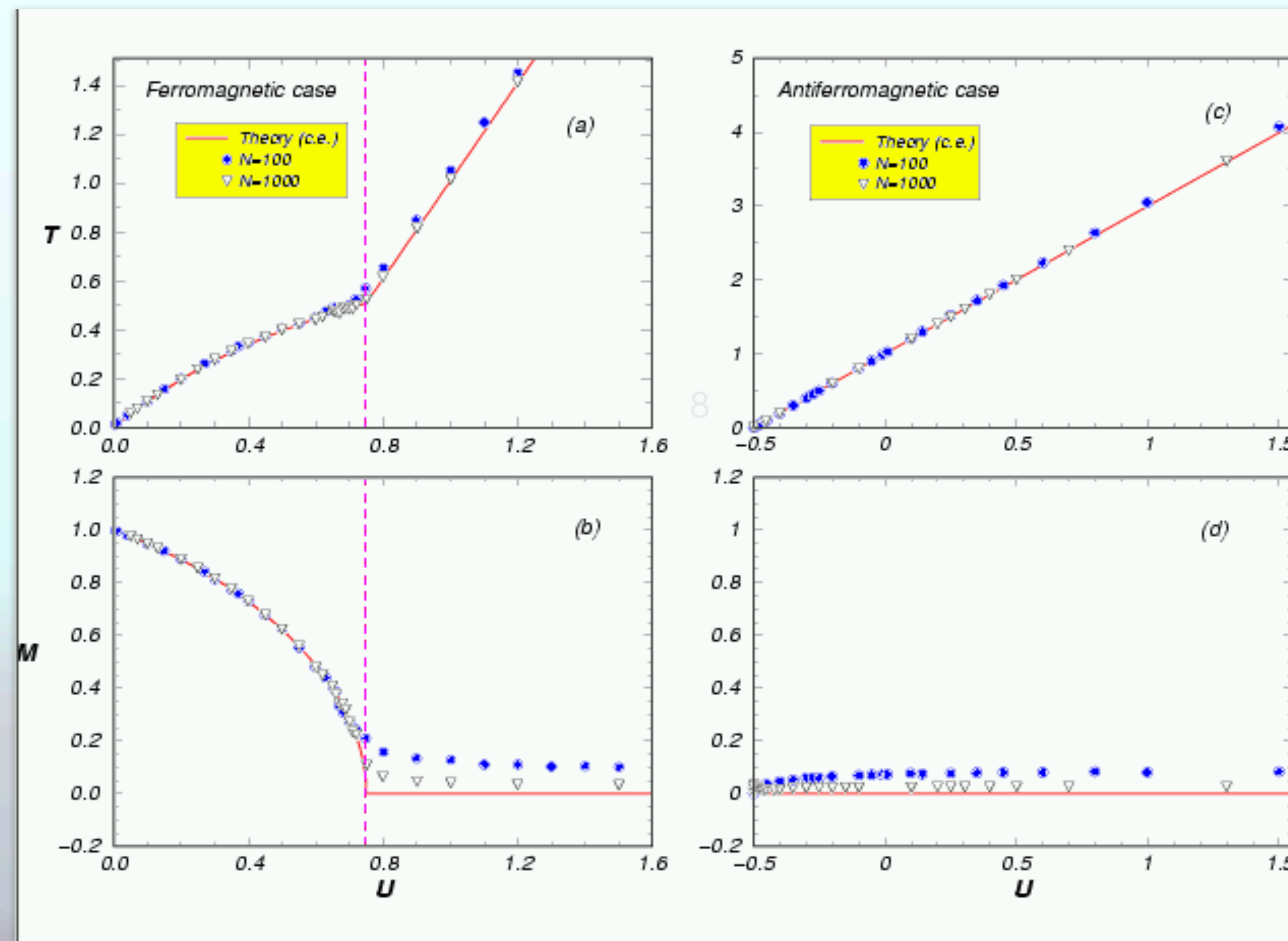


The Hamiltonian Mean Field Model

Antiferromagnetic behavior of HMF

The HMF model can have also an antiferromagnetic behavior if one considers

$$H = K - V$$



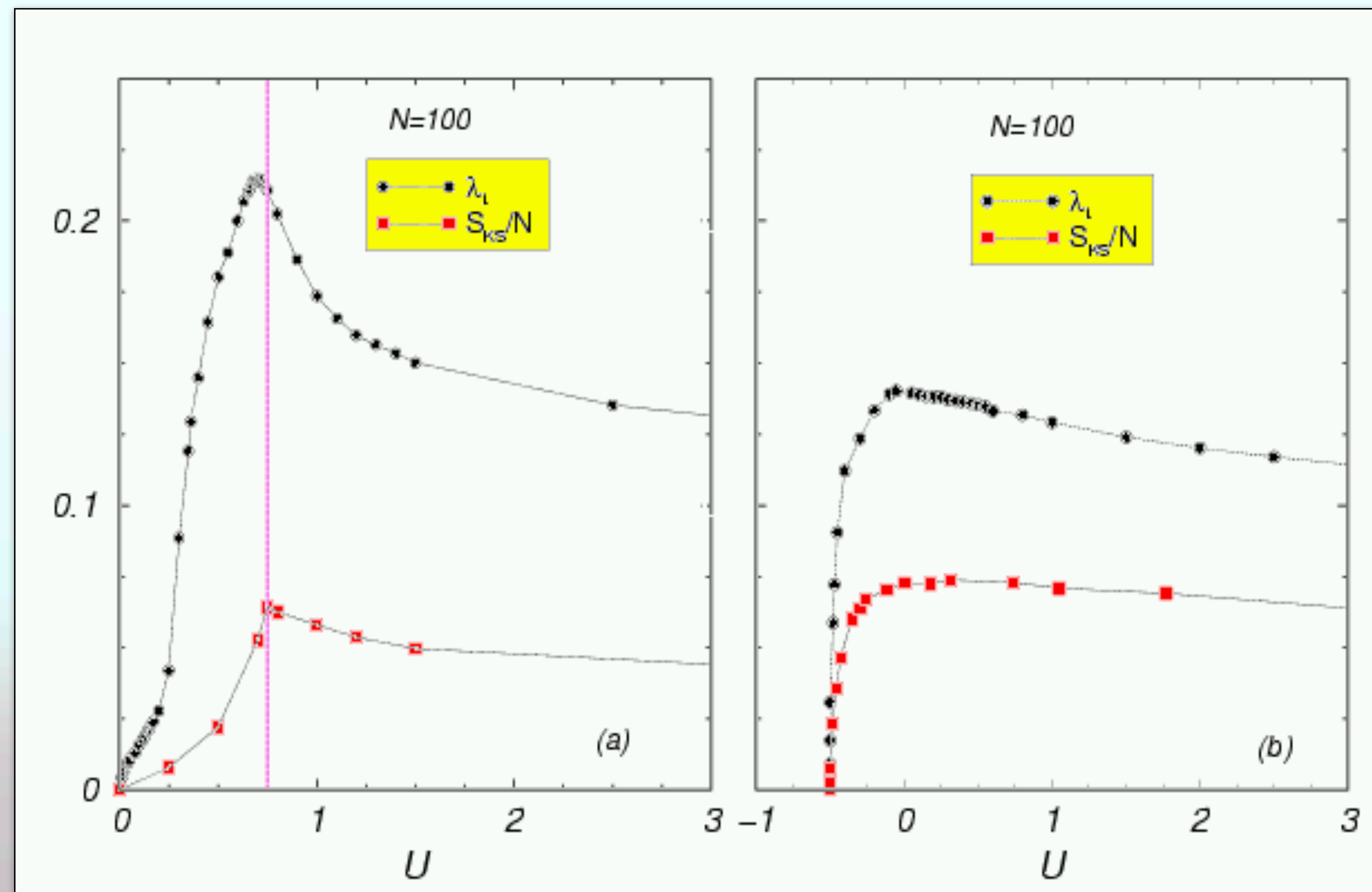
The general canonical solution for $\pm V$ is

$$U = \frac{1}{2\beta} + \frac{\varepsilon}{2} (1 - M^2)$$

with $\varepsilon = \pm 1$

The Hamiltonian Mean Field Model

One has a different behavior of the Largest Lyapunov exponent and the KS entropy in the ferromagnetic and antiferromagnetic case



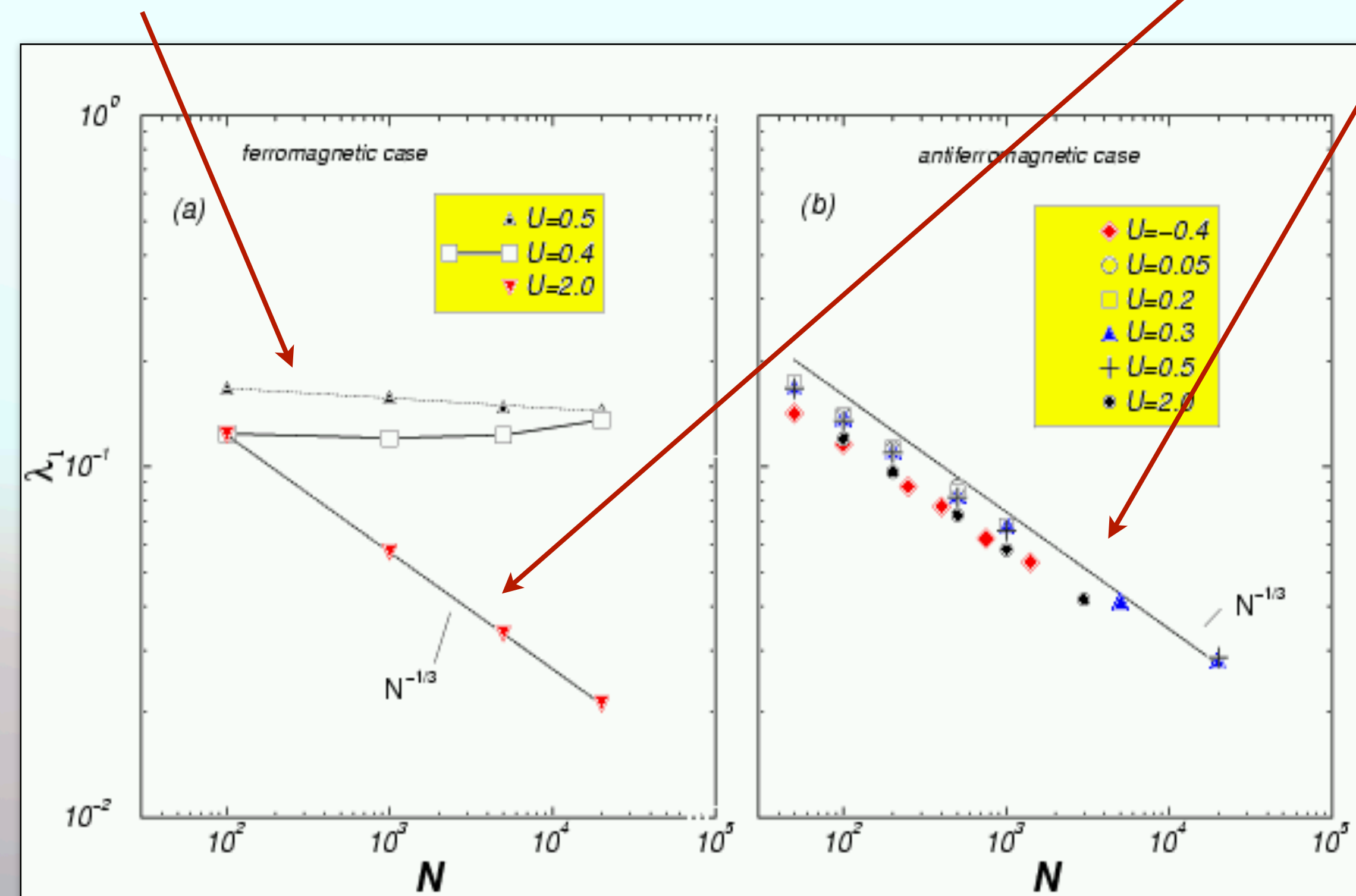
The Hamiltonian Mean Field Model

LLE in the thermodynamical limit

In the thermodynamic limit, the LLE λ_1 goes to zero for the whole energy range in the antiferromagnetic case, while it remains finite, for energies smaller than the critical one ($U_c=0.75$), in the ferromagnetic one. In the latter case it goes to zero for overcritical energies as

$$\lambda_1 = \text{constant}$$

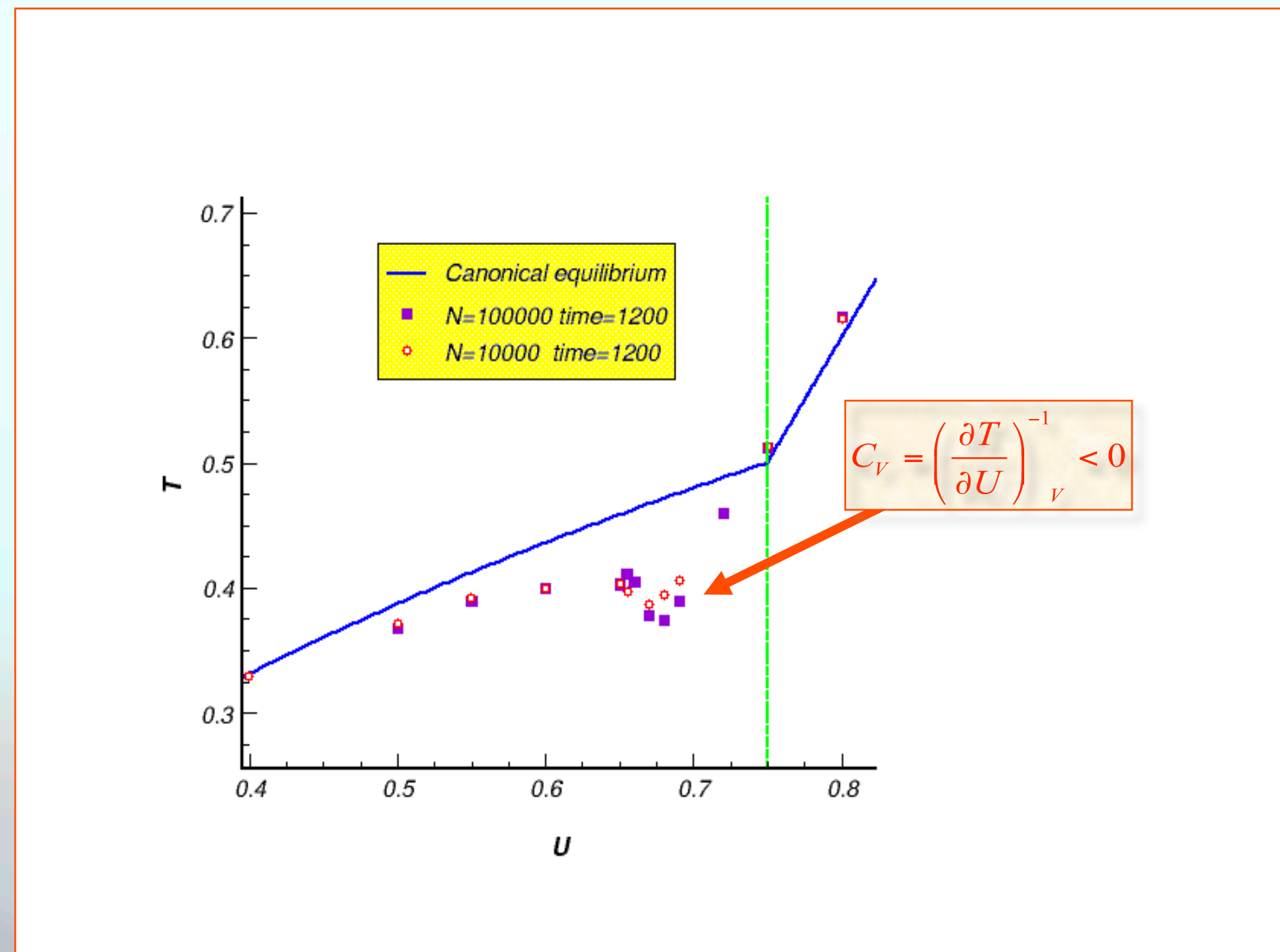
$$\lambda_1 \sim N^{-1/3}$$



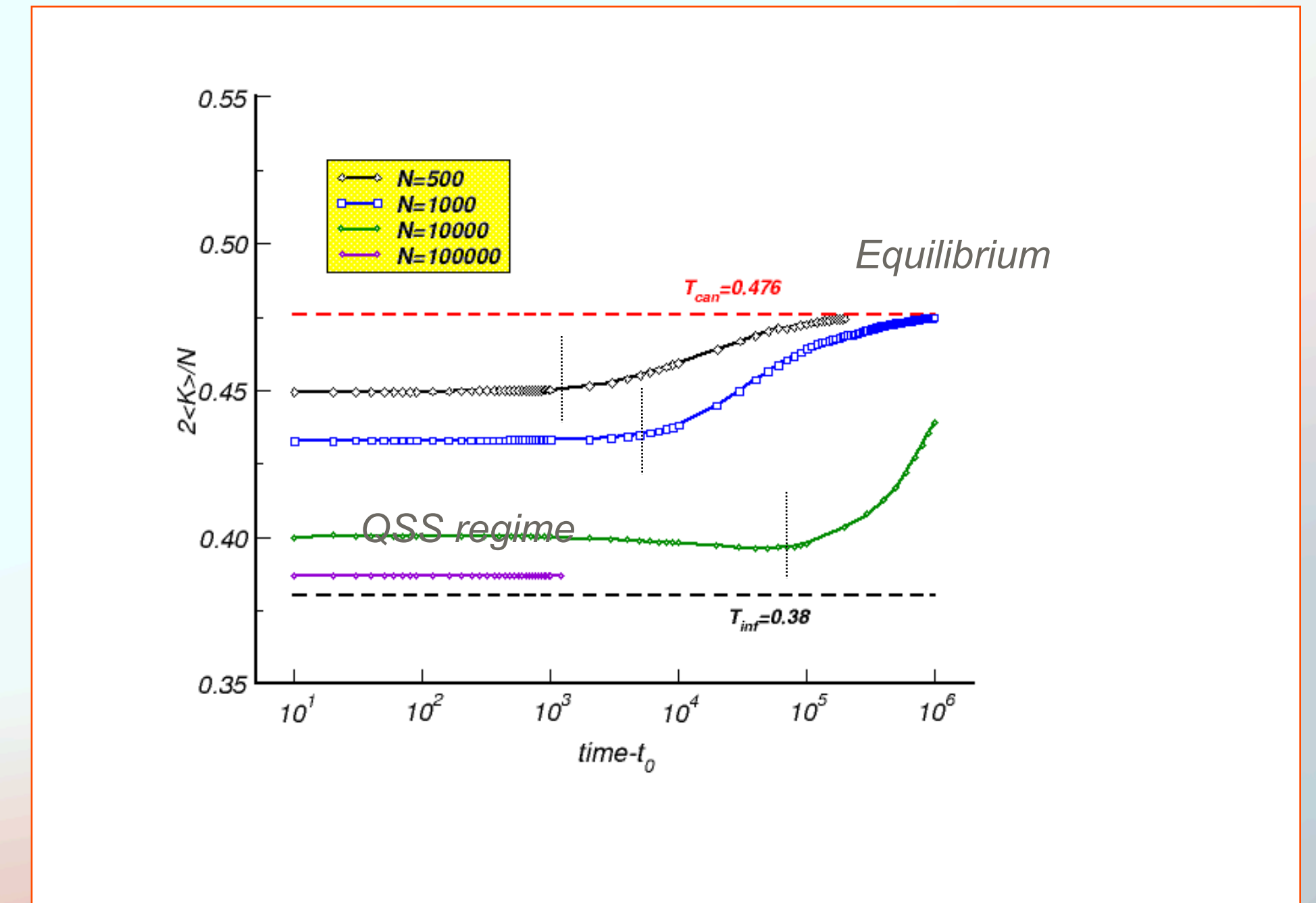
The HMF model shows **very interesting features** also ... **in the out-of-equilibrium regime**

When the system is started with **initial conditions very far from equilibrium**.....

..... one observes **many dynamical anomalies, in particular** in an energy range below the critical point.



Negative specific heat



Long living Quasi-Stationary States

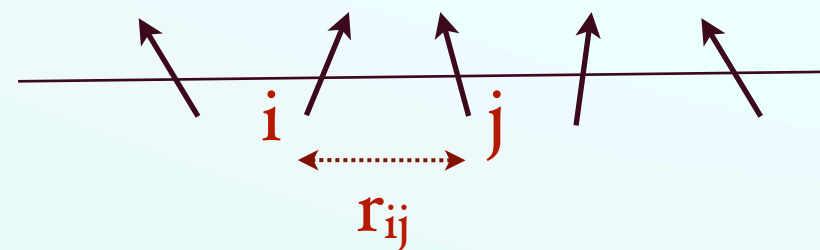
The Generalised Hamiltonian Mean Field Model

α -XY model

$$H = \sum_{i=1}^N \frac{p_i^2}{2} + \frac{1}{2N} \sum_{i \neq j}^N \frac{[1 - \cos(\theta_i - \theta_j)]}{r_{ij}^\alpha}$$

In **1998** the HMF model was generalised by **Celia Anteneodo** and **Constantino** to study the dynamic and thermodynamic behavior as a function of the range of the interaction

spins are put on a lattice



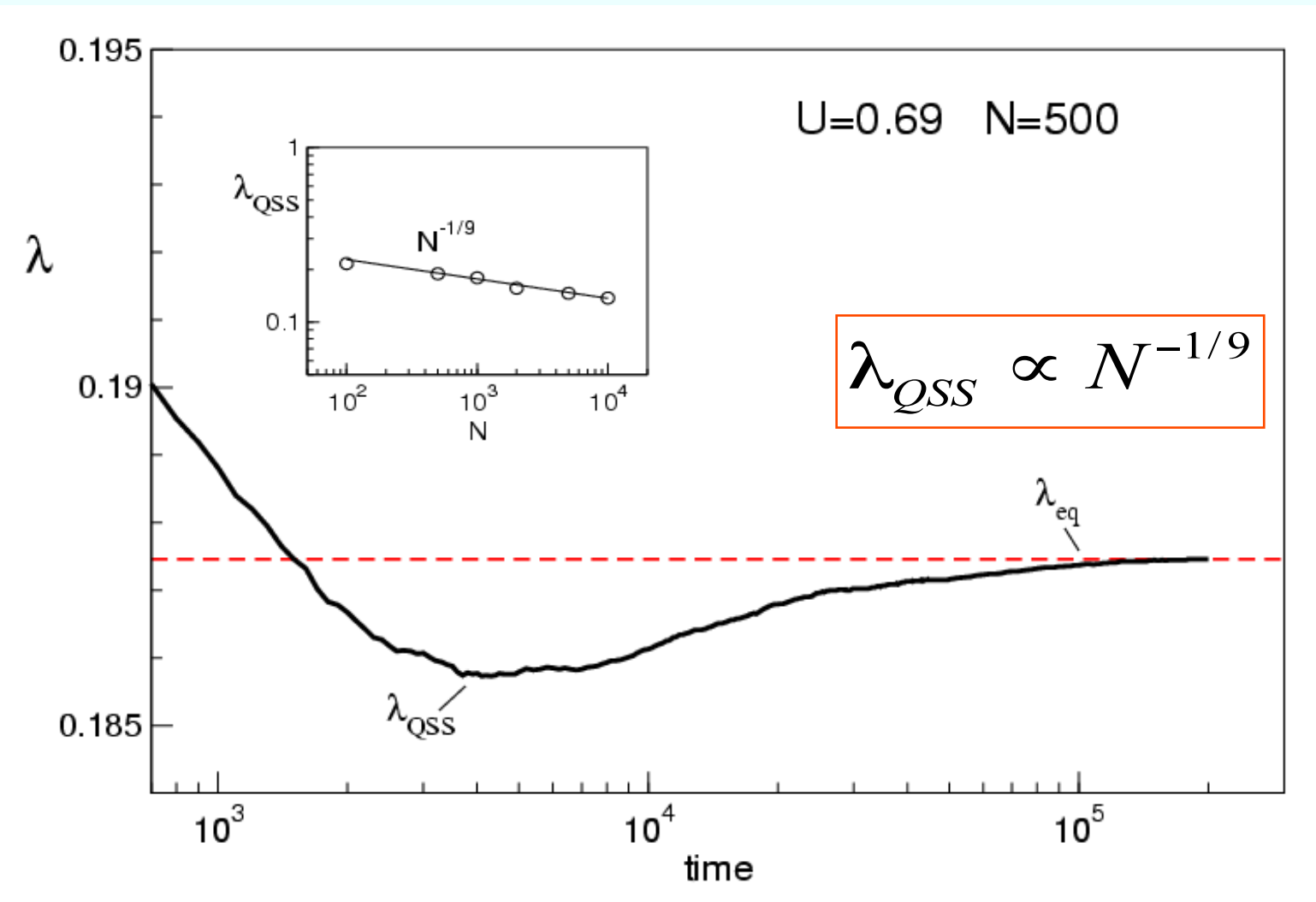
- Anteneodo and Tsallis, PRL 80 (1998) 5313
- Campa, Giansanti and Moroni, PRE 62 (2000) 303
- Tamarit and Anteneodo, PRL 84 (2000) 208
- Campa, Giansanti and Moroni, J. Phys. A 36 (2003) 6897

For $\alpha \leq d$ this generalized model reduces to HMF.

For $\alpha \rightarrow \infty$ one has interaction only among nearest neighbour spins.

The Hamiltonian Mean Field Model... out of equilibrium

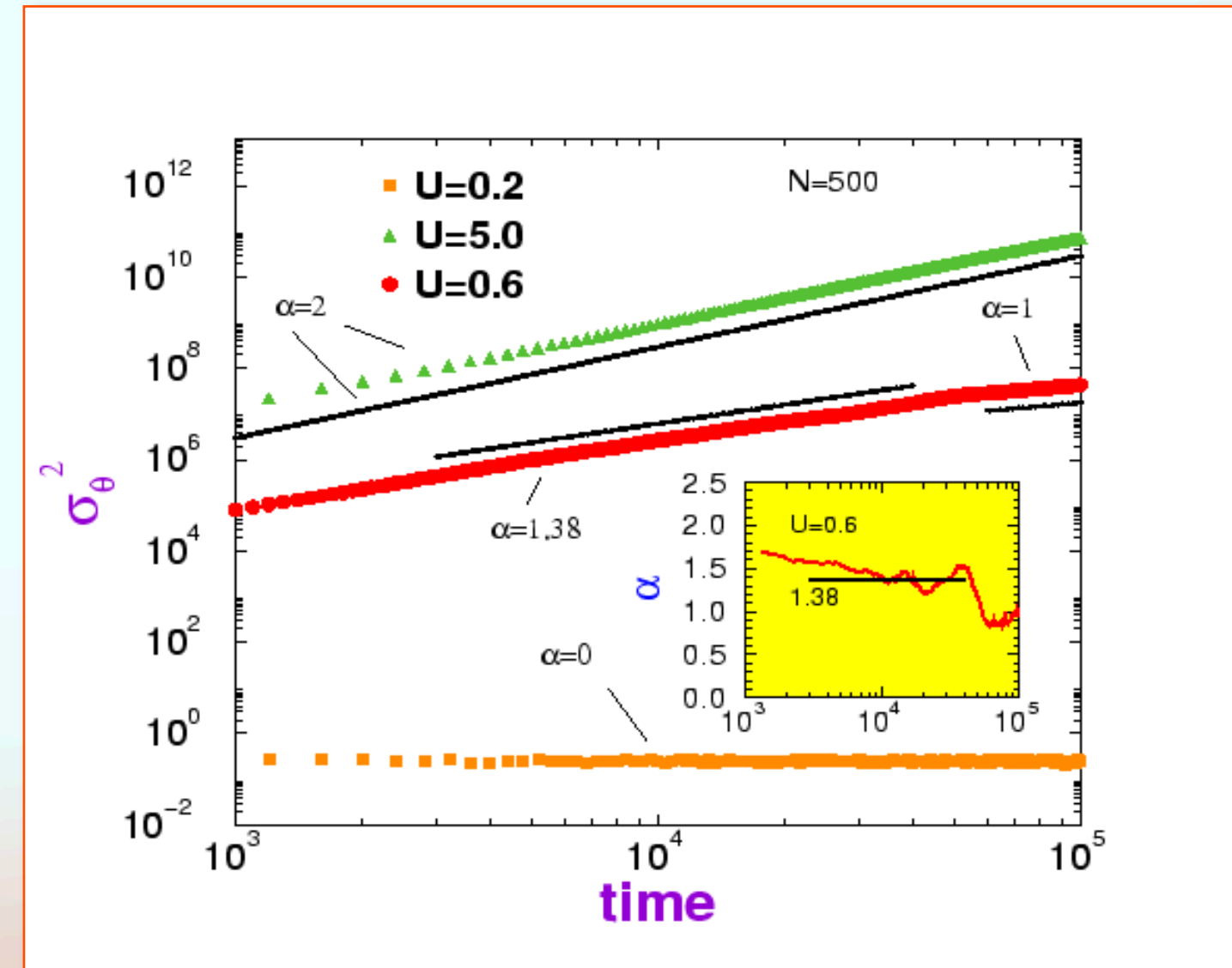
In the QSS regime the largest Lyapunov exponent tends to zero as the size of the system tends to infinity



This scaling can be obtained considering that

$$\lambda \propto M^{2/3} \propto (N^{-1/3})^{1/3} = N^{-1/9}$$

In the QSS regime the system shows superdiffusion



In general one has for the mean square displacement

$$\sigma^2(t) \propto t^\alpha$$

- $\alpha = 1$ Normal diffusion
- $\alpha \neq 1$ Anomalous diffusion

In our case we get **superdiffusion** with an exponent $\alpha=1.38$ in correspondence of the QSS regime.

Latora, Rapisarda, Tsallis Physica A 305 (2002) 129

Latora, Rapisarda, Ruffo, PRL 83 (1999) 2104

How it started...

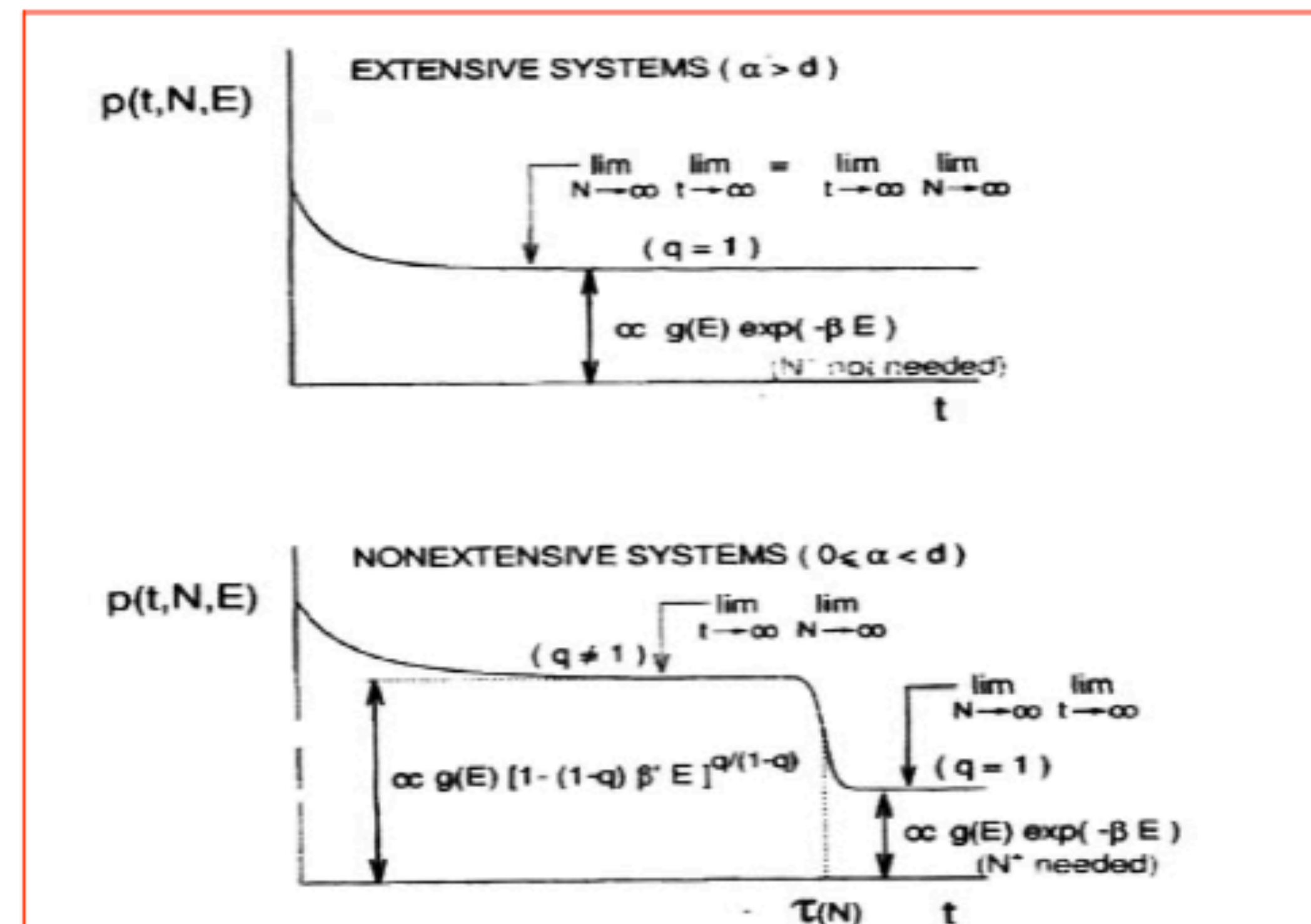
In 1998 Celia and Constantino generalized the HMF model, so Vito Latora and me tried to contact Constantino

In the summertime of 1998 Vito met Constantino in Paris during the Statphys20 conference.

Then, in the fall, we met in Boston all together during a workshop organized by Michel Baranger for the New England Complex Systems Institute.

In a paper published in 1999 in the journal of Brazilian Journal of Physics, Constantino advanced the conjecture that q -statistics could explain the anomalies found in the HMF out-of-equilibrium regime, so we started to work together on this problem.

C.Tsallis, Braz. Jour. of Phys. 29 (1999) 1



$$\text{with } \lim_{N \rightarrow \infty} \tau(N) = \infty \quad \text{and} \quad N^* \equiv \frac{N^{1-\alpha/d} - 1}{1 - \alpha/d}$$

Figure 4. Central conjecture of the present work, assuming a Hamiltonian system which includes two-body (attractive) interactions which, at long distances, decay as $r^{-\alpha}$. The crossover at $t = \tau$ is expected to be slower than indicated in the figure (for space reasons).

First International workshop "HMF meeting", Catania (Italy), september 2000.



First paper with Constantino on the Hamiltonian Mean Field Model and q-statistics in 2001

PHYSICAL REVIEW E, VOLUME 64, 056134

Non-Gaussian equilibrium in a long-range Hamiltonian system

Vito Latora,^{1,2,*} Andrea Rapisarda,^{1,†} and Constantino Tsallis^{3,‡}

¹Dipartimento di Fisica e Astronomia, Università di Catania, and INFN Sezione di Catania, Corso Italia 57, I-95129 Catania, Italy

²Laboratoire de Physique Théorique et Modèles Statistiques, Université Paris-Sud, 91405 Orsay Cedex, France

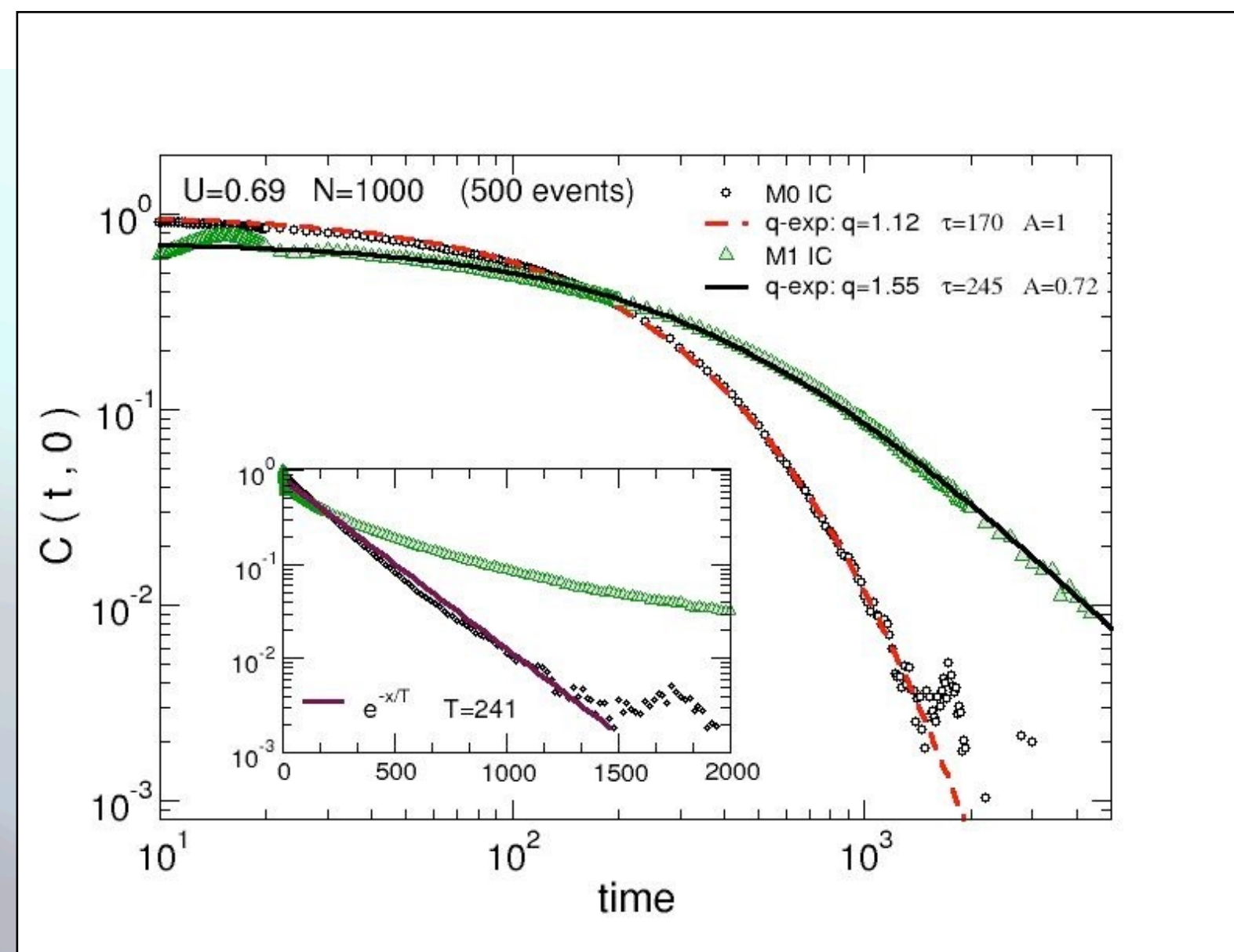
³Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

(Received 26 March 2001; revised manuscript received 22 June 2001; published 30 October 2001)

We study the dynamics of a system of N classical spins with infinite-range interaction. We show that, if the thermodynamic limit is taken before the infinite-time limit, the system does not relax to the Boltzmann-Gibbs equilibrium, but exhibits different equilibrium properties, characterized by stable non-Gaussian velocity distributions, Lévy walks, and dynamical correlation in phase space.

DOI: 10.1103/PhysRevE.64.056134

PACS number(s): 05.70.Fh, 64.60.Fr, 05.50.+q



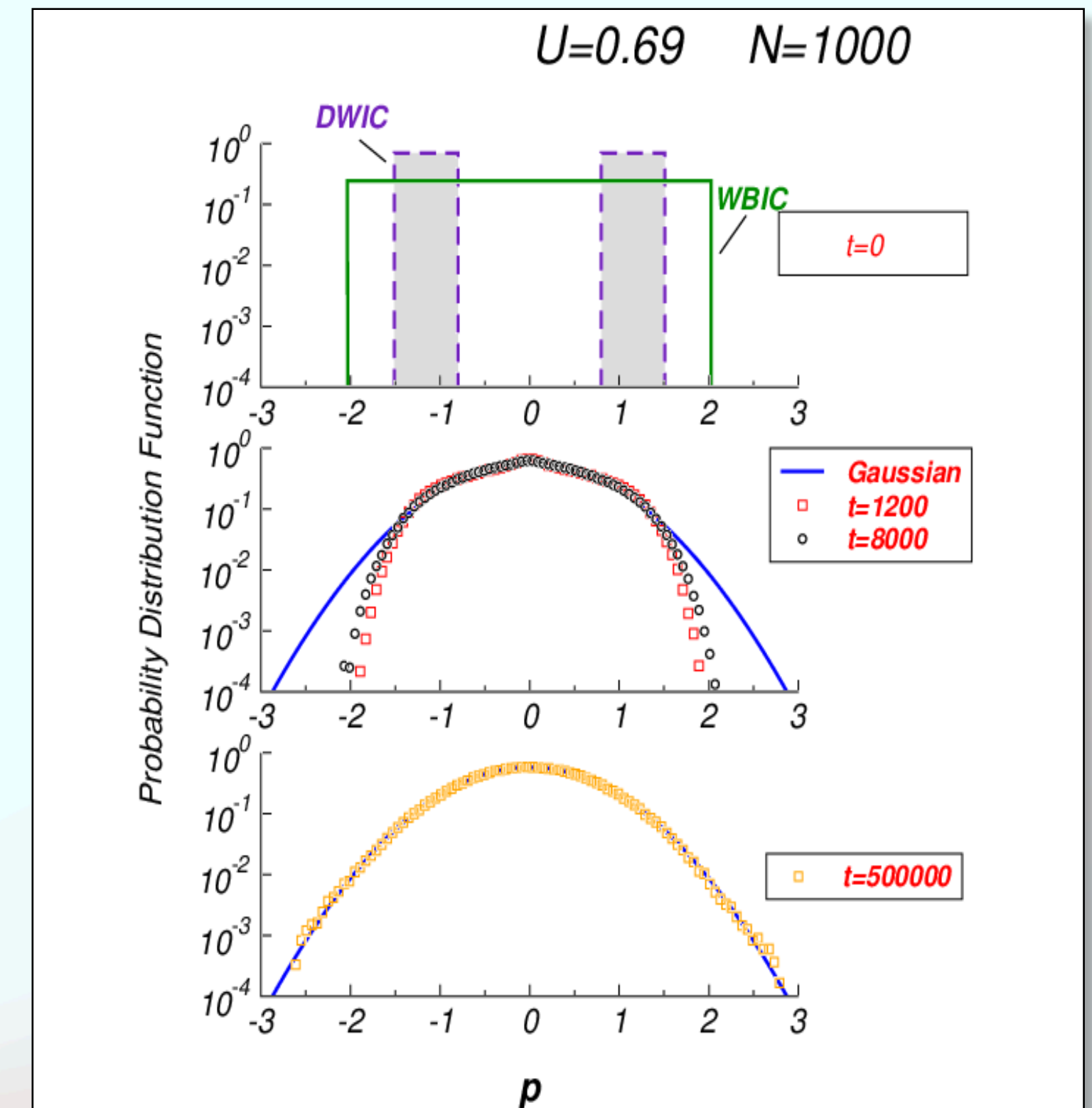
The decay of the velocity correlation function can be reproduced very well by means of the generalized q-exponential

$$Ae_q(x) = A[1 + (1-q)x]^{1/(1-q)}$$

In our case $x=-t/\tau$. Within a generalized Fokker-Plank equation which generates Tsallis q-exponential pdfs [1], one can extract the following relation between the exponent of the anomalous diffusion and q

$$\gamma = \frac{2}{3-q}$$

In our case $\gamma = 1.38-1.4$, thus we expect $q=1.55-1.6$, which is confirmed by the fit in the figure for M1IC. On the other hand, for M0IC the decay is almost exponential.



Not only HMF... q -statistics and the logistic map



14 August 2000

PHYSICS LETTERS A

Physics Letters A 273 (2000) 97–103

www.elsevier.nl/locate/pla

The rate of entropy increase at the edge of chaos

Vito Latora^{a,b,c}, Michel Baranger^a, Andrea Rapisarda^c, Constantino Tsallis^{d,e,f,*}

^a Center for Theoretical Physics, Laboratory for Nuclear Sciences and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

^b Department of Physics, Harvard University, Cambridge, MA 02138, USA

^c Dipartimento di Fisica, Università di Catania, and Istituto Nazionale di Fisica Nucleare, Sezione di Catania Corso Italia 57, I-95129 Catania, Italy

^d Centro Brasileiro de Pesquisas Físicas, Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil

^e Physics Department, University of North Texas, P.O. Box 311427, Denton, TX 76203, USA

^f Department of Mechanical Engineering, Massachusetts Institute of Technology, Rm. 3-164, Cambridge, MA 02139, USA

Received 27 June 2000; accepted 6 July 2000

Communicated by C.R. Doering

V. Latora et al. / Physics Letters A 273 (2000) 97–103

101

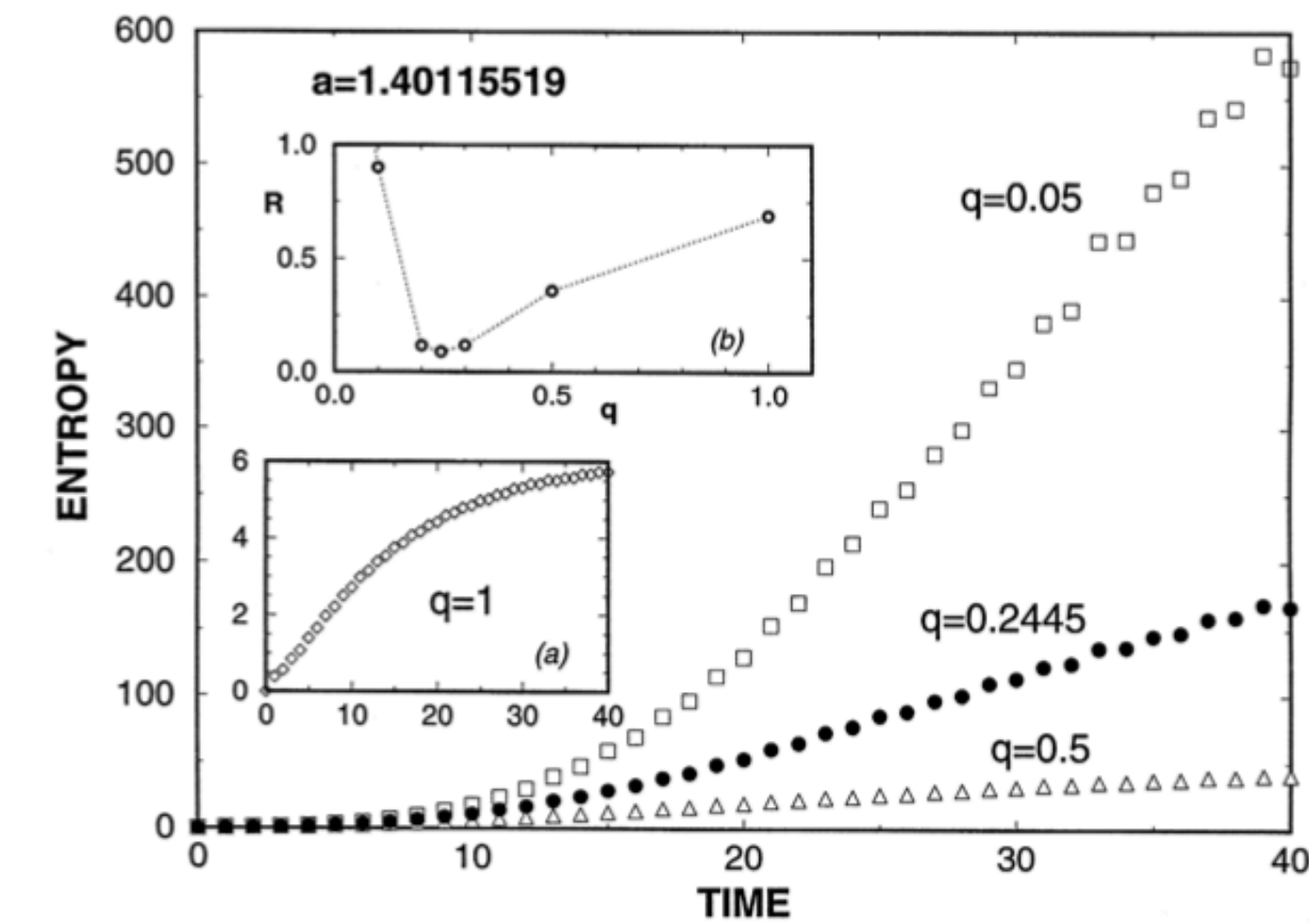


Fig. 4. Time evolution of S_q for $a = a_c$. We consider four different values of q and $W = 10^5$. The case $q = 1$ is reported in the inset (a) with a different scale. Results are averages over 1251 runs. We show the coefficient of nonlinearity R versus q in the inset (b). See text. The 4-digit precision for q^* was not attained through the present numerical procedure, but using the scaling $1/(1-q) = 1/\alpha_{\min} - 1/\alpha_{\max}$. The present procedure does not provide higher precision than $q^* = 0.24\dots$

In collaboration with **Michel Baranger**



Series of Next Conferences



International Conference NEX2001, "Non extensive Thermodynamics and physical applications", Villasimius, Cagliari (Italy), may 2001.

Series of Next Conferences



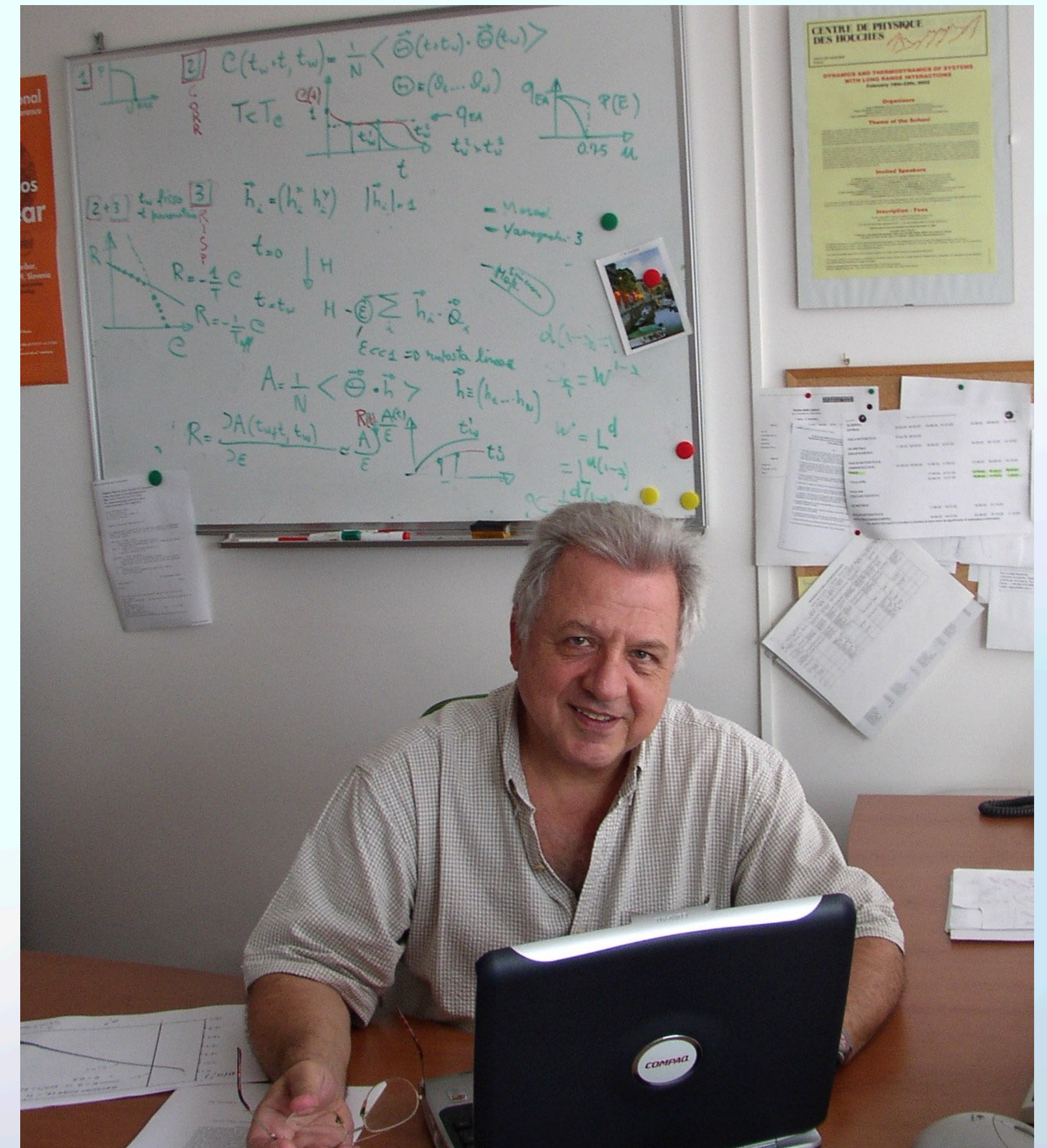
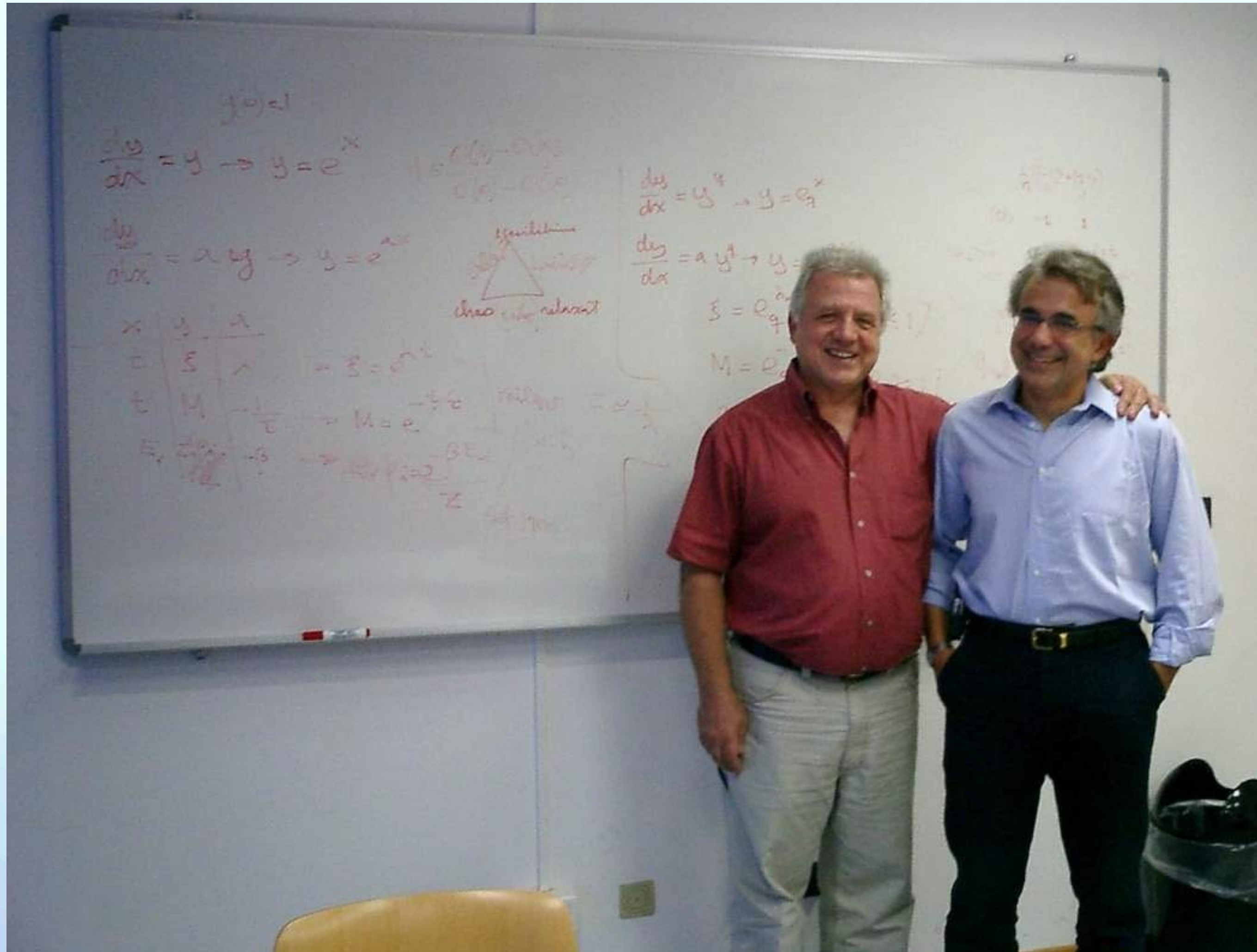
International Conference NEXT2001, "Non extensive Thermodynamics and physical applications", Villasimius, Cagliari (Italy), may 2001.

Series of Next Conferences



International Conference NEXT2003, "News and Expectations in Thermostatistics", Villasimius, Cagliari (Italy), 22-28 september 2003.


Catania October 2003




Conference for Constantino's 60th birthday - Rio de Janeiro, november 2003



The beginning of the **School on Complexity** in Erice, July 2004



«ETTORE MAJORANA» FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE
TO PAY A PERMANENT TRIBUTE TO GALILEO GALILEI, FOUNDER OF MODERN SCIENCE
AND TO ENRICO FERMI, THE "ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES



INTERNATIONAL SCHOOL OF SOLID STATE PHYSICS

31st Course: COMPLEXITY, METASTABILITY AND NONEXTENSIVITY

ERICE-SICILY: 20 - 26 JULY 2004

Sponsored by the: European Science Foundation • INFN • Italian Ministry of Education, University and Scientific Research
• Sicilian Regional Government • University of Catania • University of Palermo

Complexity in internet and earthquakes networks
• S. ABE, University of Tsukuba, J

Complexity in the brain
• F.T. ARECCHI, University of Florence, I

Superstatistics
• C. BECK, University of London, UK

Fractal growth of carbon schwarzites
• G. BENEDEK, University of Milan, I

Complexity in solid state physics
• A.R. BISHOP, Los Alamos Nat. Lab, NM, USA

Lattice-Boltzmann models
• B. M. BOGHOSIAN, Tufts University, Medford, MA, USA

Random geometric graphs
• B. BOLLOBAS, University of Memphis, TN, USA

Non-Gaussian effects in financial markets
• L. BORLAND, San Francisco, CA, USA

Econophysics and glasses
• J.P. BOUCHAUD, Saclay, Gif-Sur-Yvette, F

Dynamical instability, diffusion and Fourier heat law
• G. CASATI, University of Insubria, Como, I

Nonequilibrium statistical mechanics
• E.G.D. COHEN, Rockefeller University, New York, NY, USA

Granular systems
• A. CONRIGLIO, University of Naples, I

Networks and mesoscopic clusters
• J. DOYE, University of Cambridge, UK

Active Brownian particles - stochastic dynamics of swarm systems
• W. EBELING, Humboldt University, Berlin, D

Models of financial markets
• D. FARMER, Santa Fe Institute, NM, USA

Experiments in granular systems
• J.P. GOLLUB, Haverford University, PA, USA

Anomalous diffusion and intermittency
• P. GRIGOLINI, University of North Texas, Denton, TX, USA

Stochastic dynamics
• P. HANGGI, Augsburg University, D

Complexity in biological systems
• K. KANEKO, Tokyo University, J

Architecture of complex systems
• V. LATORA, University of Catania, I

Complexity in financial markets
• R.N. MANTEGNA, University of Palermo, I

Noise and fluctuations
• F. MARCHESONI, University of Camerino, I

SOC models
• M. PACZUSKI, Imperial College, London, UK

Complexity in cosmic structures
• L. PIETRONERO, University of Rome, I

Dynamics in low dimensional lattices
• A. PROVATA, Demokritos Center, Athens, GR

Metastability and glassy dynamics in Hamiltonian systems
• A. RAPISARDA, University of Catania, I

Aging and glassy states in maps
• A. ROBLEDO, UNAM, Mexico City, MEX

Nucleation and critical phenomena in earthquakes and other driven threshold systems
• J. RUNDLE, Center for Comp. Sci. and Engineering, Davis, CA, USA

Models of complex systems
• H.E. STANLEY, Boston University, MA, USA

Aging and spin-glasses
• F.A. TAMARIT, University of Cordoba, RA

Nonextensive statistical mechanics
• C. TSALLIS, CBPF, Rio de Janeiro, Brazil and Santa Fe Institute, NM, USA

Diffusion in complex networks
• A. VESPIGNANI, LPT, Orsay, F

Complexity in collective behavior
• T. VICSÉK, Eotvos University, Budapest, H

Complexity at the elementary level
• A. ZICHICHI, INFN & University, Bologna, I, and CERN, Geneva, CH

PURPOSE OF THE COURSE

There is increasing evidence that a large class of complex, usually nonequilibrium, phenomena in different fields such as for instance physics, chemistry, geophysics, biophysics and econophysics, share similar dynamical and structural properties. Such phenomena include self-organization, metastability, anomalous relaxation, aging, glassy states, amorphous clustering, non-Gaussian distributions, non-Markovian processes, mesoscopic dissipation, scale-invariant growth, among others. The aim of the meeting is to emphasize common features that could reveal unifying concepts within a general theoretical framework.

APPLICATIONS

Persons wishing to attend the Course should apply in writing to:

- Professor Andrea RAPISARDA
Università di Catania
Dipartimento di Fisica e Astronomia
& INFN Sezione di Catania
Via S. Sofia, 64 I-95123 CATANIA - Italy
Tel ++39 095 378 5408 - Fax ++39 095 378 5231
e-mail: andrea.rapisarda@ct.infn.it
www.ct.infn.it/~rapis/ERICE-COMPLEX04

specifying:

- full name(s), address, age, nationality;
- academic qualifications, present position and affiliation and/or a short CV;
- their specific interest in the Course.

• PLEASE NOTE
Participants must arrive in Erice on July 20, not later than 5 pm.

POETIC TOUCH

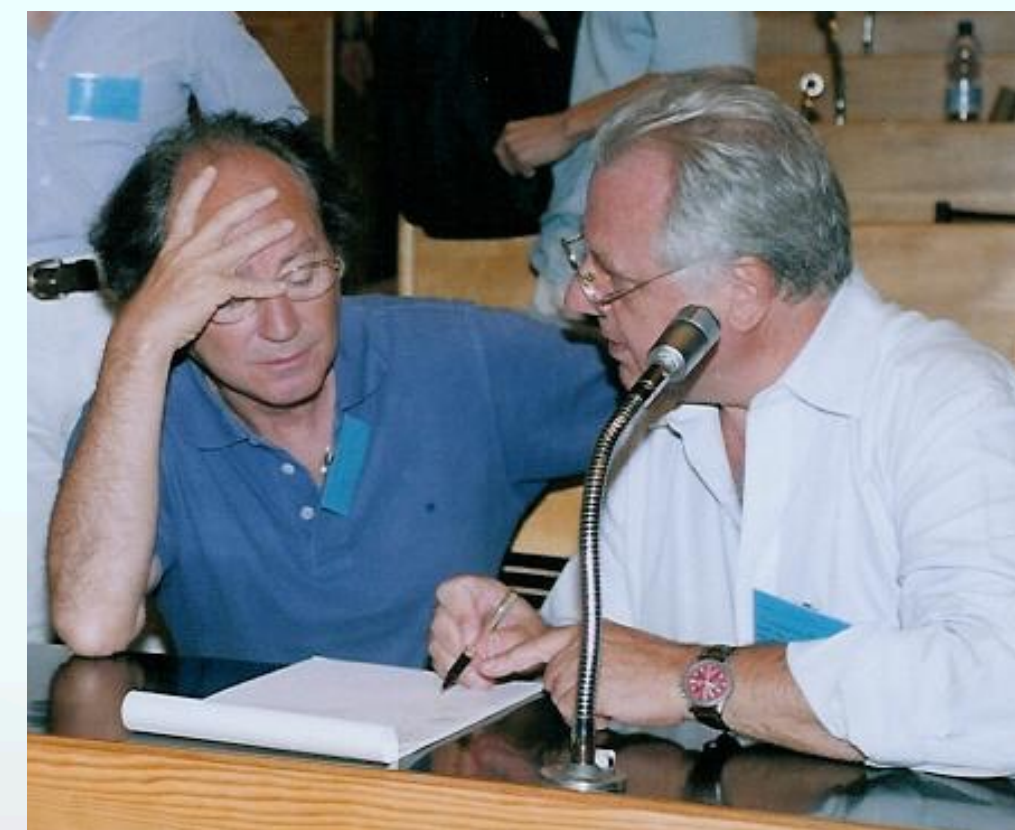
According to legend, Erice, son of Venus and Neptune, founded a small town on top of a mountain (750 metres above sea level) more than three thousand years ago. The founder of modern history — i.e. the recording of events in a methodical and chronological sequence as they really happened without reference to mythical causes — the great Thucydides (~500 B.C.), writing about events connected with the conquest of Troy (1183 B.C.) said: «After the fall of Troy some Trojans on their escape from the Achaei arrived in Sicily by boat and as they settled near the border with the Sicilians all together they were named Elym: their towns were Segesta and Erice». This inspired Virgil to describe the arrival of the Trojan royal family in Erice and the burial of Anchise, by his son Eneas, on the coast below Erice. Homer (~1000 B.C.), Theocritus (~300 B.C.), Polybius (~200 B.C.), Virgil (~50 B.C.), Horace (~20 B.C.), and others have celebrated this magnificent spot in Sicily in their poems. During seven centuries (XIII-XIX) the town of Erice was under the leadership of a local oligarchy, whose wisdom assured a long period of cultural development and economic prosperity which in turn gave rise to the many churches, monasteries and private palaces which you see today.

In Erice you can admire the Castle of Venus, the Cyclopean Walls (~800 B.C.) and the Gothic Cathedral (~1300 A.D.). Erice is at present a mixture of ancient and medieval architecture. Other masterpieces of ancient civilization are to be found in the neighbourhood: at Motya (Phoenician), Segesta (Elymian), and Selinunte (Greek). On the Aegadian Islands — theatre of the decisive naval battle of the first Punic War (264-241 B.C.) — suggestive neolithic and paleolithic vestiges are still visible: the grottoes of Favignana, the carvings and murals of Levanzo.

Splendid beaches are to be found at San Vito Lo Capo, Scopello, and Comino, and a wild and rocky coast around Monte Cofano: all at less than one hour's drive from Erice.

More information about the other activities of the Ettore Majorana Centre can be found on the WWW at the following address:
<http://www.cesem.infn.it>

C. BECK - A. RAPISARDA - C. TSALLIS
DIRECTORS OF THE COURSE
G. BENEDEK
DIRECTOR OF THE SCHOOL
A. ZICHICHI
EMFSC PRESIDENT AND DIRECTOR OF THE CENTRE



The beginning of the School on Complexity in Erice, July 2004

World Scientific
Connecting Great Minds

Sign in | Register | Help | Cart

Search Citation DOI / ISSN / ISBN Advanced Search

Home Books Journals Resources About Us Publish with Us Open Access

Home > All Publications > All Books > The Science and Culture Series — Physics > Complexity, Metastability and Nonextensivity

This Book All Subjects > Physics

FEATURED ARTICLES OF 2015
INTERNATIONAL JOURNAL OF BIFURCATION AND CHAOS
FREE-TO-READ
for registered users

The Science and Culture Series — Physics
Complexity, Metastability and Nonextensivity
Proceedings of the 31st Workshop of the International School of Solid State Physics
Erice, Sicily, Italy, 20 – 26 July 2004

Edited by: **C Beck** (University of London, UK), **G Benedek** (Università di Milano Bicocca, Italy), **A Rapisarda** (Università di Catania, Italy), **C Tsallis** (CBPF, Brazil & Santa Fe Institute for Complex Systems, New Mexico, USA)

About This Book **E-Book**

A broad introduction and overview of current interdisciplinary studies on complexity, this volume is an ideal starting point for scientists and graduate students who wish to enter the field. The book features a diverse collection of the latest research work not found in a single volume elsewhere.

Among the highly regarded contributors to the volume are the 2004 Boltzmann medalists E G D Cohen and H E Stanley; G Parisi, Boltzmann medalist in 1992 and Dirac medalist in 1999; and numerous internationally renowned experts, such as S Abe, F T Arecchi, J-P Bouchaud, A Coniglio, W Ebeling, P Grigolini, R Mantegna, M Paczuski, A Robledo, L Pietronero, A Vespignani, and T Vicsek.

428pp Sep 2005
ISBN: 978-981-256-525-9 (hardcover)
GBP160.00 Buy Now
ISBN: 978-981-4479-18-9 (ebook)
GBP128.00 Buy Now
Add book to favorites

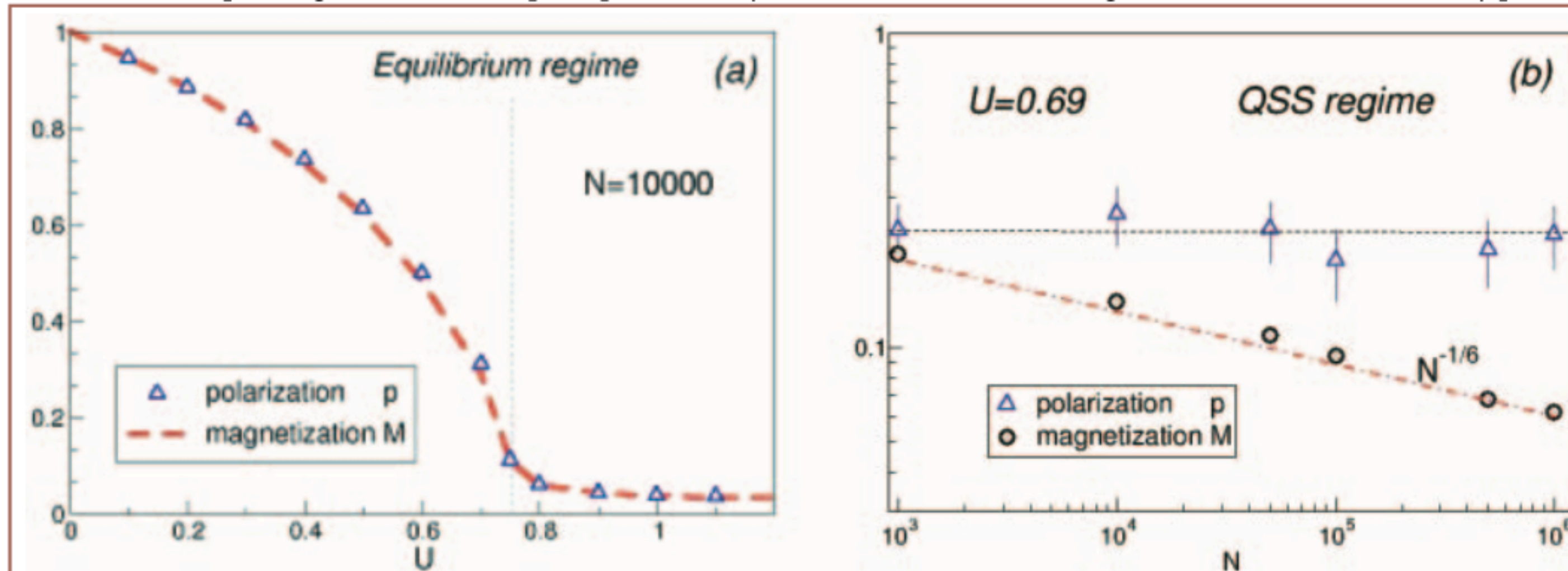
THE SCIENCE AND CULTURE SERIES — PHYSICS

Series Editor: A. Zichichi

COMPLEXITY, METASTABILITY AND NONEXTENSIVITY

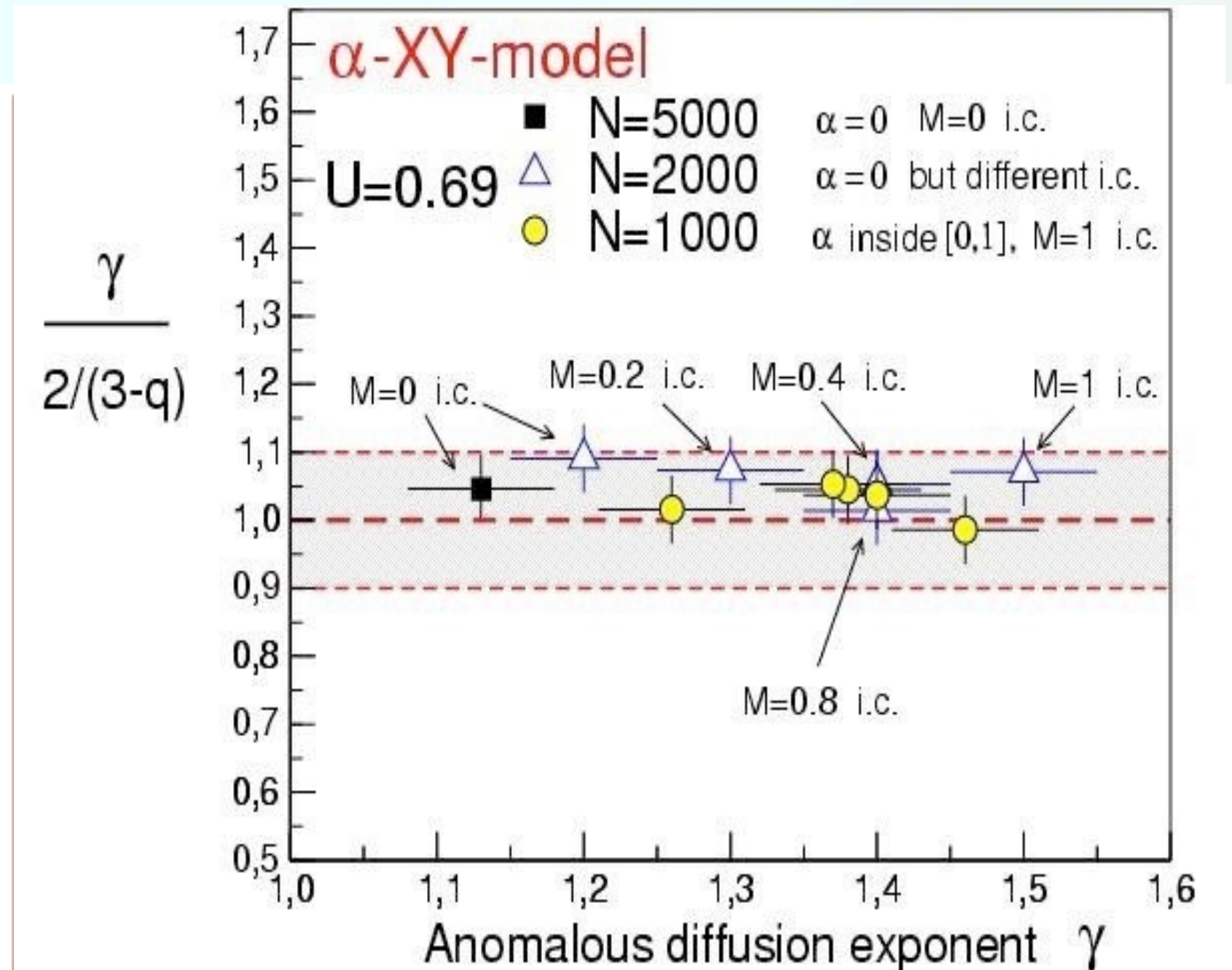
31st Workshop of the International School of Solid State Physics

Editors
C Beck, G Benedek, A Rapisarda and C Tsallis



▲ **Fig. 5:** (a) The magnetization M and the polarization p are plotted vs the energy density for N=10000 at equilibrium: the two order parameters are identical. (b) The same quantities plotted in (a) are here reported vs the size of the system, but in the metastable QSS regime. In this case, increasing the size of the system, the polarization remains constant around a value $p \sim 0.24$ while the magnetization M goes to zero as $N^{-1/6}$.

Glassy behavior and confirmation of q-statistics predictions - december 2005



▲ **Fig. 4:** For different system sizes and initial conditions, and for several values of the parameter α which fixes the range of the interaction of a generalized version of the HMF model [11], the figure illustrates the ratio of the anomalous diffusion exponent γ divided by $2/(3-q)$ vs γ . The entropic index q is extracted from the relaxation of the correlation function (see previous figure). This ratio is always one within the errors of the calculations.

Workshop organized by Sumiyoshi Abe in Kyoto, Japan - march 2005



Crete, august 2005



Dancing sirtaki with Giorgio Parisi



Conference for Alberto Robledo's 60th birthday, Tepoztlan, Mexico 2005



How continued... Durham(UK) - July 2006



This was a **very important workshop** for the many papers that originated after the discussions of some talks on the **Central Limit Theorem at the edge of chaos**



The Abdus Salam
International Centre for Theoretical Physics



SCHOOL & CONFERENCE on COMPLEX SYSTEMS and NONEXTENSIVE STATISTICAL MECHANICS

31 July - 8 August 2006

Miramare, Trieste, Italy

The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, will organize a **School and Conference on Complex Systems and Nonextensive Statistical Mechanics**, to be held from 31 July to 8 August 2006.

The aim of the event is to focus on recent developments in the nonlinear dynamical foundations of nonequilibrium statistical mechanics, more specifically, nonextensive statistical mechanics, its applications to complex systems in physics, economics, geophysics, astrophysics, biology and elsewhere, as well as its connections with the theory of networks, glassy and other metastable systems.

This activity is divided into two parts, the first six days being a School in which mathematical and theoretical aspects of the general theory as well as important applications from diverse fields will be presented by the Lecturers. During the last two days, a Conference will be held to discuss recent developments and open questions in this field. Participants are encouraged to present their recent works as a poster or oral contribution. Please send the title and abstract of the proposed contribution to U. Tirnakli (tirnakli@sci.egi.edu.tr) no later than **1 March 2006**.

International Scientific Committee:

S. Abe (Japan), C. Beck (England), B.M. Boghosian (USA), J.P. Boon (Belgium), E.G.D. Cohen (USA), J.D. Farmer (USA), M. Gell-Mann (USA), H.J. Haubold (Austria), L.P. Kadanoff (USA), G. Kaniadakis (Italy), M. Lissia (Italy), G. Parisi (Italy), A. Plastino (Argentina), P. Quarati (Italy), A. Rapisarda (Italy), A. Robledo (Mexico), K.R. Sreenivasan (Italy).

PARTICIPATION

Scientists and students from all countries which are members of the United Nations, UNESCO or IAEA may attend the activity. As it will be conducted in English, participants should have an adequate working knowledge of this language. Although the main purpose of the Centre is to help research workers from developing countries, through a programme of training activities within a framework of international cooperation, a limited number of students and post-doctoral scientists from developed countries are also welcome to attend.

As a rule, travel and subsistence expenses of the participants should be borne by the home institution. Every effort should be made by candidates to secure support for their fare (or at least half-fare). However, limited funds are available for some participants who are nationals of, and working in, a developing country, and who are not more than 45 years old. Such support is available only for those who attend the entire activity. There is no registration fee.

The **Application Form** is obtainable from the ICTP WWW server: <http://agenda.ictp.it/smr.php?1763> (which will be constantly up-dated) or from the activity Secretariat. It should be completed and returned before **31 March 2006** to:

School and Conference on Complex Systems and Nonextensive Statistical Mechanics
(smr 1763 - c/o Ms. Nadia van Buuren)
the Abdus Salam International Centre for Theoretical Physics
Strada Costiera 11, 34014 Trieste, Italy.

or

smr1763@ictp.it (please save and send file attachments in RTF format)

Telephone: +39-040-2240-1111 Telefax: +39-040-2240-304 E-mail: smr1763@ictp.it
ICTP Home Page: <http://www.ictp.it/>

DIRECTORS:

Constantino TSALLIS
(CBPF, Brazil and SFI, USA)

Ugur TIRNAKLI
(Ege University, Turkey)

LOCAL ORGANIZER:

Matteo MARSILI
(ICTP, Italy)

Lecturers

S. Abe	Japan
I. Andricioael	USA
C. Beck	UK
E.G.D. Cohen	USA
G. Kaniadakis	Italy
A. Plastino	Argentina
A.R. Plastino	S. Africa
S. P. Quarati	Italy
A.K. Rajagopal	USA
A. Rapisarda	Italy
A. Robledo	Mexico
C. Tsallis	Brazil & USA

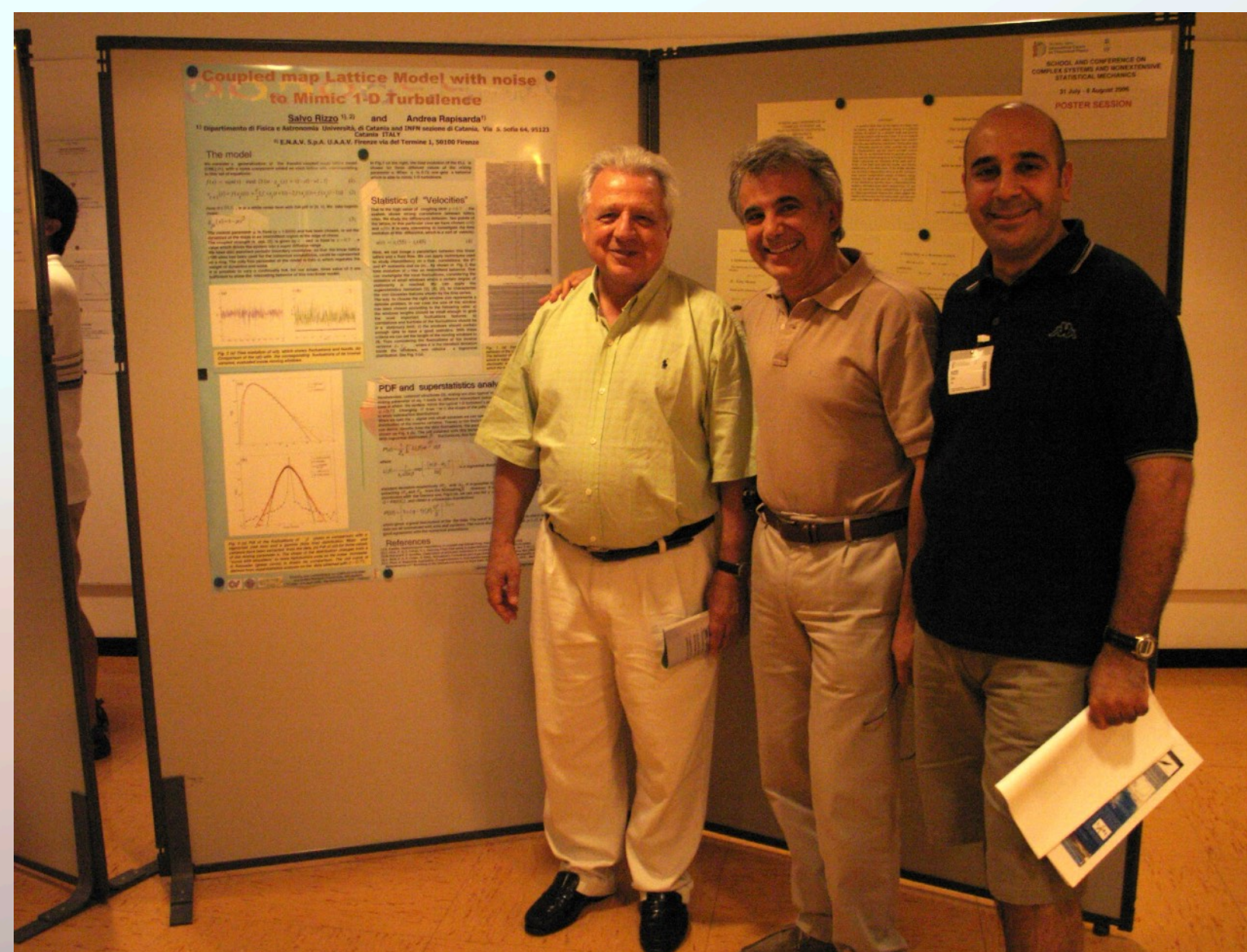
Invited Speakers

C. Anteneodo	Brazil
F. Baldovin	Italy
B.M. Boghosian	USA
J.P. Boon	Belgium
L. Borland	USA
A. Carati	Italy
J. Marsh	USA
J. Naudts	Belgium
H. Suyari	Japan
F.A. Tamarit	Argentina
S. Thurner	Austria
U. Tirnakli	Turkey
P.A. Varotsos	Greece
G. Wilk	Poland
H.O. Wio	Spain

DEADLINE

for requesting participation

31 March 2006



How continued - Rio November 2006



How continued - Catania Next 2007 and Statphys23 in Genoa

AIP | Scitation

Publishers ▾ Publications Topics Collections | Librarians Authors

Home > Publishers > AIP Publishing > AIP Conference Proceedings > Conference Proceeding

AIP | Conference Proceedings

COMPLEXITY, METASTABILITY, AND NONEXTENSIVITY: AN INTERNATIONAL CONFERENCE

Conference date: 1-5 July 2007
 Location: Catania (Italy)
 ISBN: 978-0-7354-0481-6
 Editors: Sumiyoshi Abe, Hans Herrmann, Piero Quarati, Andrea Rapisarda and Constantino Tsallis
 Volume number: 965



XXIII IUPAP International Conference on Statistical Physics

Statphys23 Genova ITALY

IUPAP July 9-13, 2007

WELCOME TO STATPHYS 23

Satellite conference of STATPHYS 23

International Conference on Complexity, Metastability and Nonextensivity
 Dipartimento di Fisica e Astronomia - Università di Catania
 1-5 July 2007

Main topics of the conference:

- Models and dynamics of complex systems
- Nonextensive statistical mechanics
- Glassy dynamics and metastability
- Networks and synchronization
- Interdisciplinary applications

Main speakers

S. Abe (Japan)	C. Beck (UK)	J.-P. Bouchaud (Belgium)	T. Boettcher (Greece)
L. Burlaga (NASA, USA)	F. Caruso (Italy)	G. Casati (Italy)	A. Cerati (Italy)
P.H. Chavanis (France)	E.G.D. Cohen (USA)	K. Dawson (Ireland)	J. Soares Andrade (Brazil)
L. de Arcangelis (Italy)	J.D. Farmer (USA)	A. Giannantoni (Italy)	I. Giardinà (Italy)
G. Kaniadakis (Italy)	H.J. Herrmann (Switzerland)	V. Latora (Italy)	R.N. Mantegna (Italy)
M. Marsili (Italy)	M. Paczuski (Canada)	A.R. Plastino (South Africa)	A. Politi (Italy)
A. Pluchino (Italy)	P. Quarati (Italy)	A. Robledo (Mexico)	S. Ruffo (Italy)
B. Spagnolo (Italy)	H.E. Stanley (USA)	F. Tamarit (Argentina)	U. Tirnakli (Turkey)
S. Thurner (Austria)	C. Tsallis (Brazil)	S. Umarov (USA)	C. Vignat (France)

International Organizing Committee

S. Abe, H.J. Herrmann, P. Quarati, C. Tsallis, A. Rapisarda (Chairman)

www.ct.infn.it/ctnext07

For more information write to: catania-next07@ct.infn.it



Complexity, Metastability and Nonextensivity 1-5 July 2007 Catania

Nonergodicity and central-limit behavior for long-range Hamiltonians

A. PLUCHINO¹, A. RAPISARDA¹ and C. TSALLIS^{2,3}

¹ *Dipartimento di Fisica e Astronomia, Università di Catania, and INFN sezione di Catania - Via S. Sofia 64, I-95123 Catania, Italy*

² *Centro Brasileiro de Pesquisas Físicas - Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil*

³ *Santa Fe Institute - 1399 Hyde Park Road, Santa Fe, NM 87501, USA*

received 27 June 2007; accepted in final form 1 September 2007
published online 21 September 2007

PACS 64.60.My – Metastable phases

PACS 89.75.-k – Complex systems

Abstract – We present a molecular dynamics test of the Central-Limit Theorem (CLT) in a paradigmatic long-range-interacting many-body classical Hamiltonian system, the HMF model. We calculate sums of velocities at equidistant times along deterministic trajectories for different sizes and energy densities. We show that, when the system is in a chaotic regime (specifically, at thermal equilibrium), ergodicity is essentially verified, and the Pdfs of the sums appear to be Gaussians, consistently with the standard CLT. When the system is, instead, only weakly chaotic (specifically, along longstanding metastable Quasi-Stationary States), nonergodicity (*i.e.*, discrepant ensemble and time averages) is observed, and robust q -Gaussian attractors emerge, consistently with recently proved generalizations of the CLT.

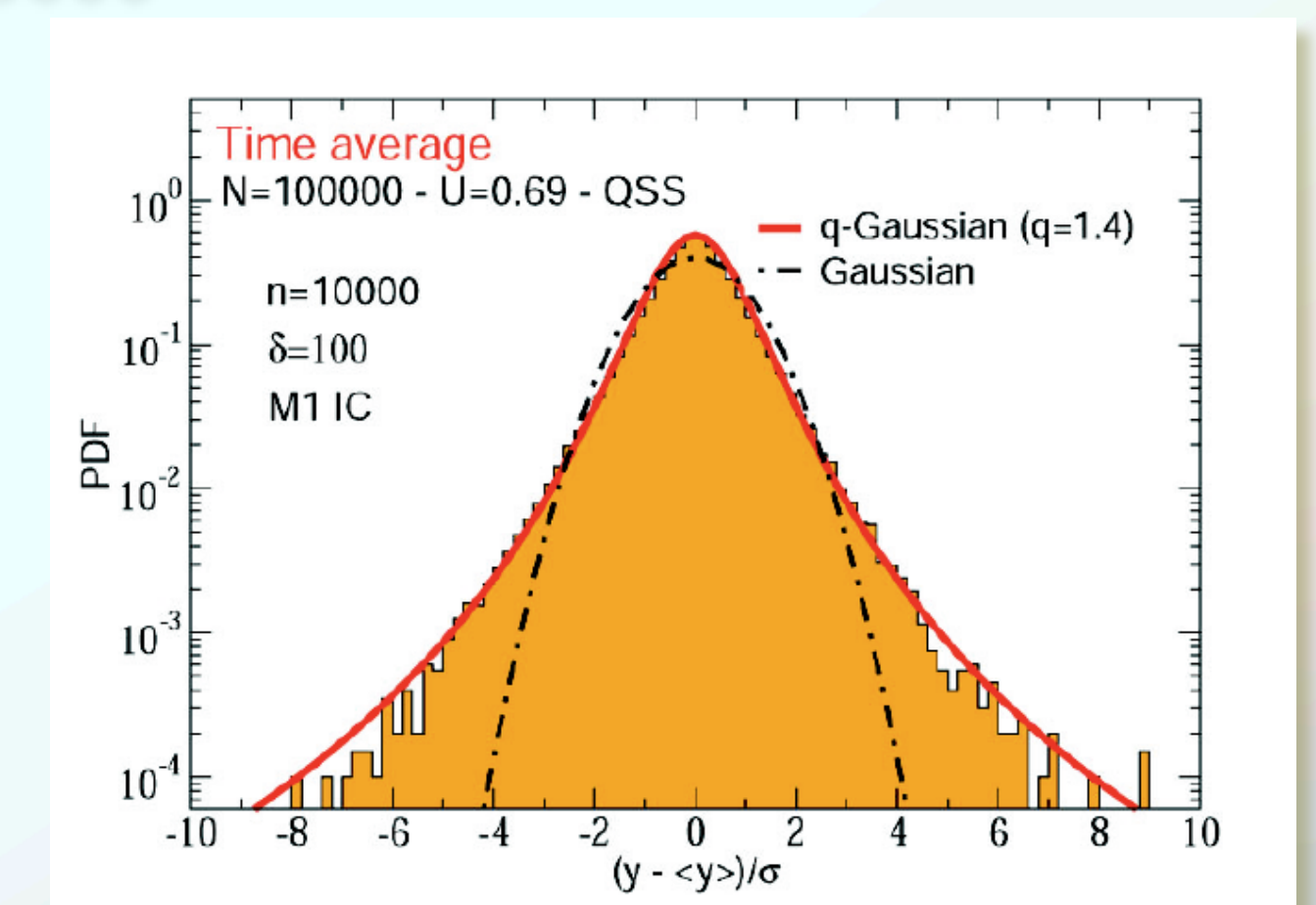
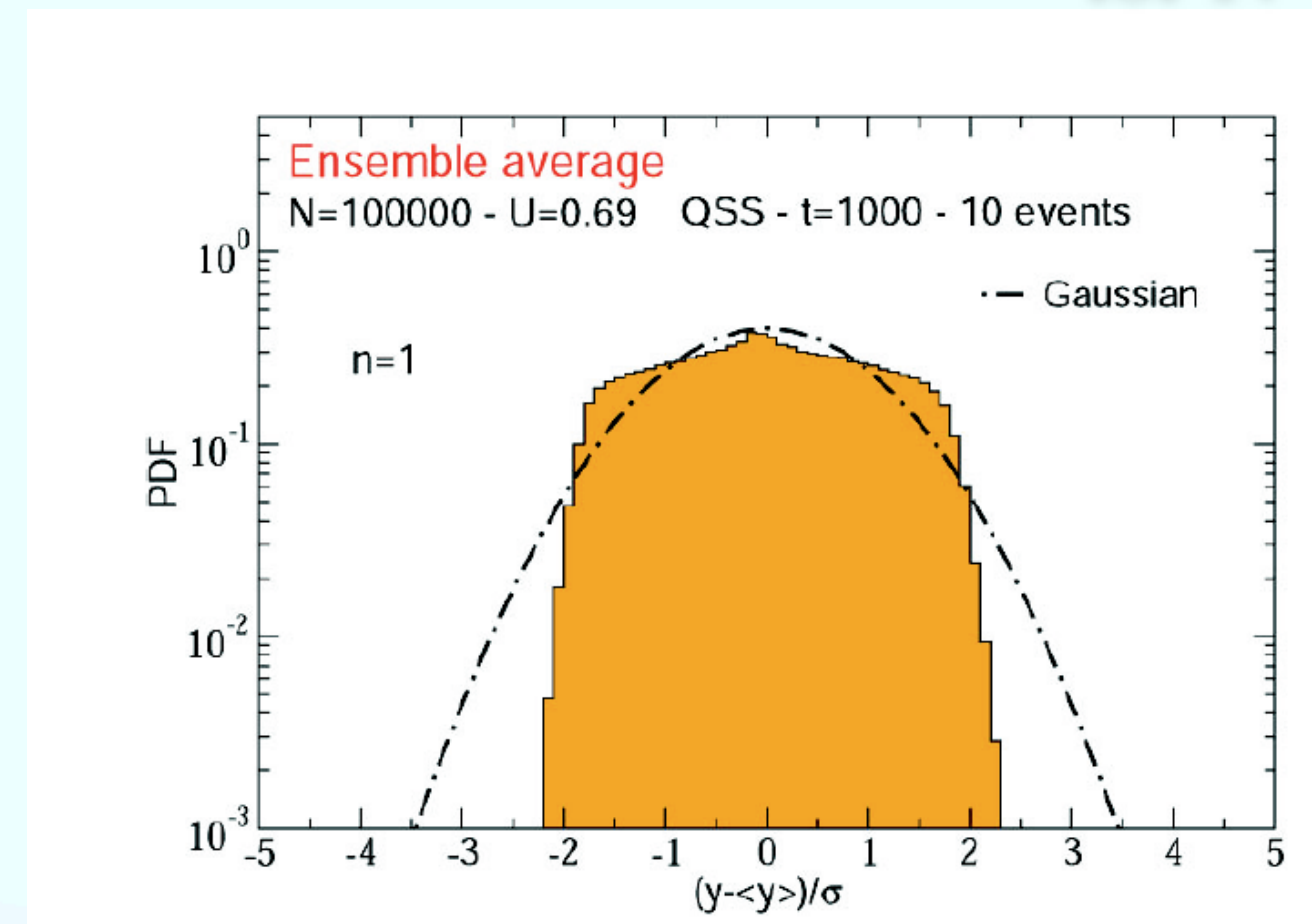
How continued - Central limit behavior for HMF - EPL 2007

We investigated the behavior of PDFs obtained considering time averages of the variables y so defined (along deterministic trajectories in the QSS regime)

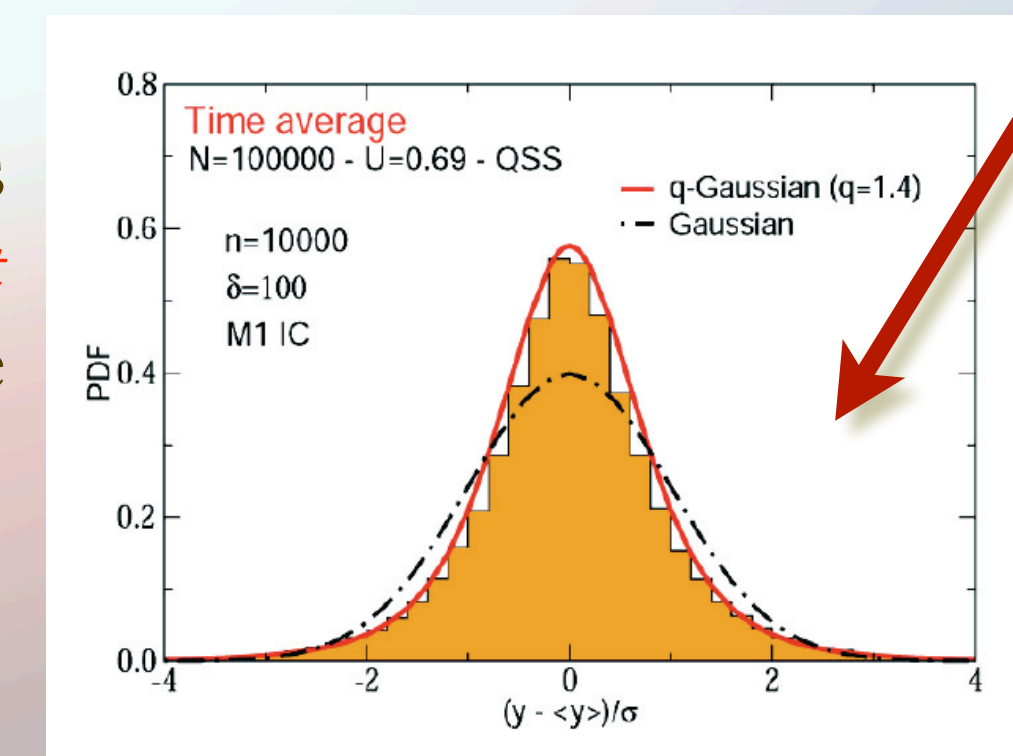
$$y_i = \frac{1}{\sqrt{n}} \sum_i^n p_j(i\delta) \quad j = 1, 2, \dots, N$$

where p_j are the velocities of the j -th rotor taken at fixed intervals of time along the same trajectory

Inequivalence between ensemble average and time average for $N=100000$



The q-Gaussian curve is able to reproduce well *not only the tail*, but also the *central part* of the PDF



Linear scale

How continued - Central limit behavior for HMF - 2008



Available online at www.sciencedirect.com



Physica A 387 (2008) 3121–3128

PHYSICA A

www.elsevier.com/locate/physa

A closer look at the indications of q -generalized Central Limit Theorem behavior in quasi-stationary states of the HMF model

Alessandro Pluchino^a, Andrea Rapisarda^{a,*}, Constantino Tsallis^{b,c}

^a Dipartimento di Fisica e Astronomia, Università di Catania, and INFN sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy

^b Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil

^c Santa Fe Institute, 1399 Hyde Park Road, NM 87501, USA

Received 18 January 2008

Available online 9 February 2008

3126

A. Pluchino et al. / Physica A 387 (2008) 3121–3128

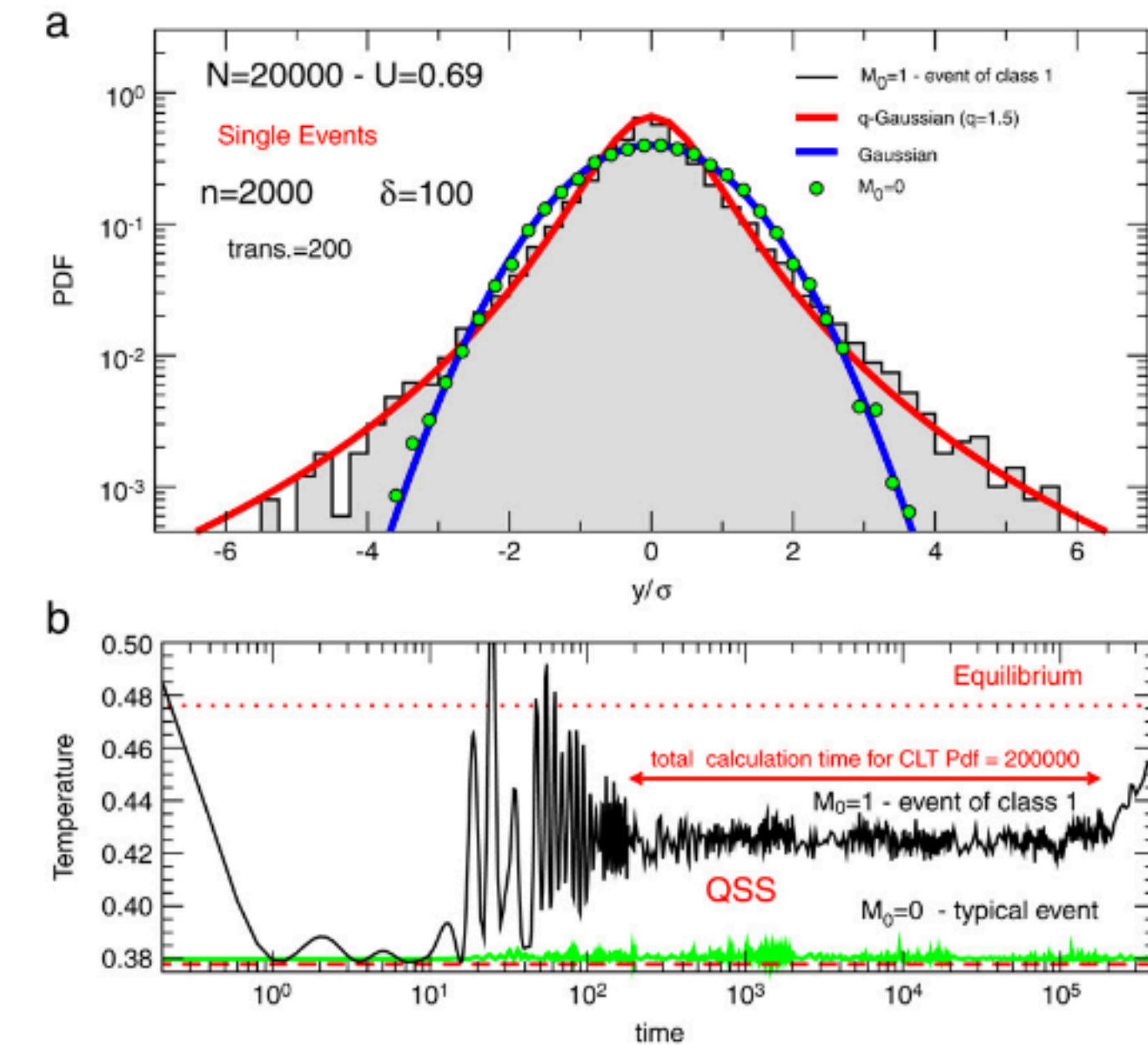


Fig. 6. (a) Comparison of the CLT behavior for the case $U = 0.69$ initial magnetization $M_0 = 1$ (class 1) vs $M_0 = 0$. The size of the system is $N = 20\,000$. A Gaussian (dashed curve) with unitary variance and a q -Gaussian with $A = 0.66$, $q = 1.5$ and $\beta = 1.8$ (full curve) are also reported for comparison. (b) Temperature–time evolutions of the same events shown in panel (a).

Importance of the initial conditions

Iguaçu Next Conference in 2008



Econophysics Colloquium 2009 - Erice, Sicily



In Rio for a long visit - february 2012

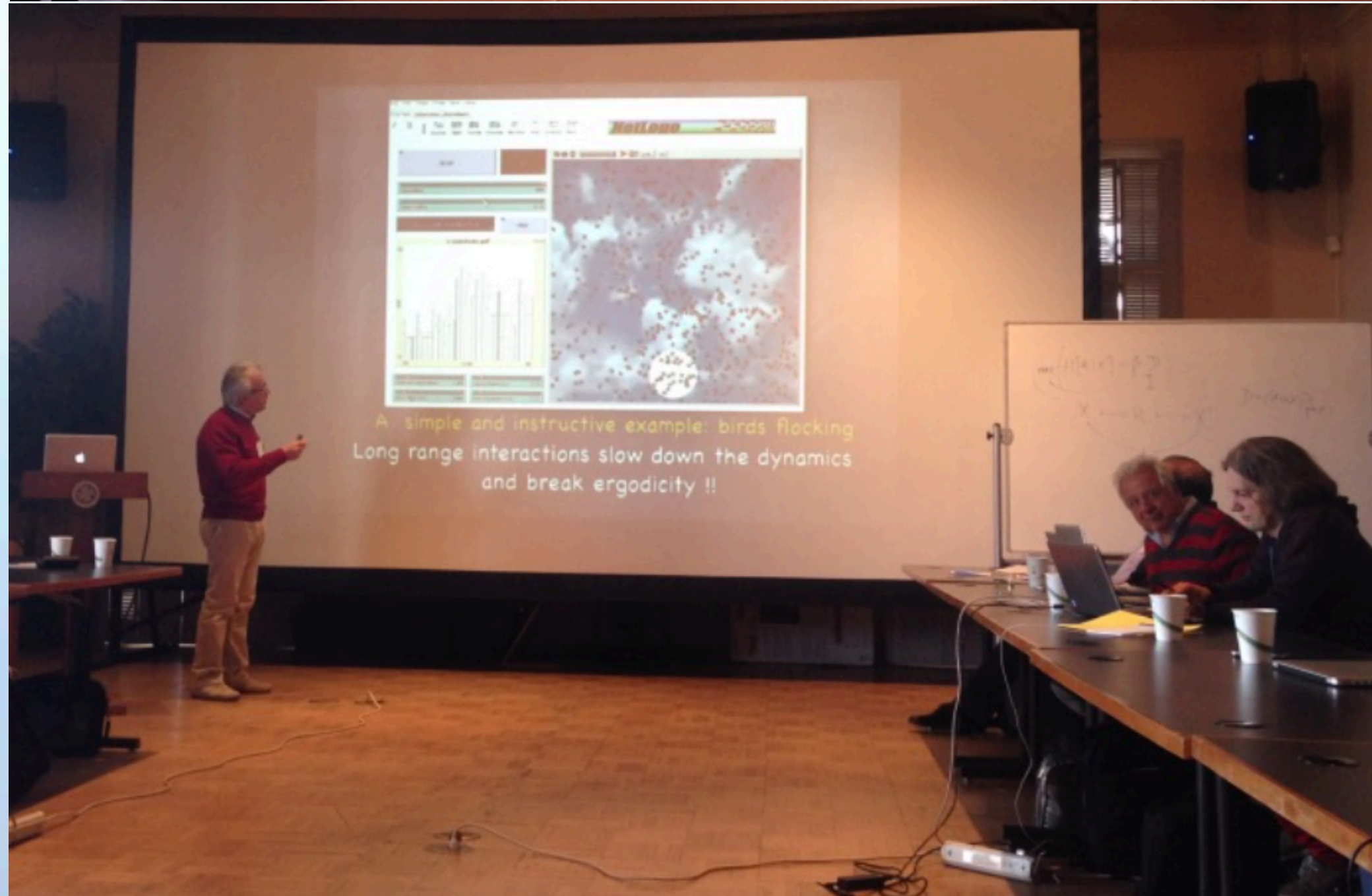
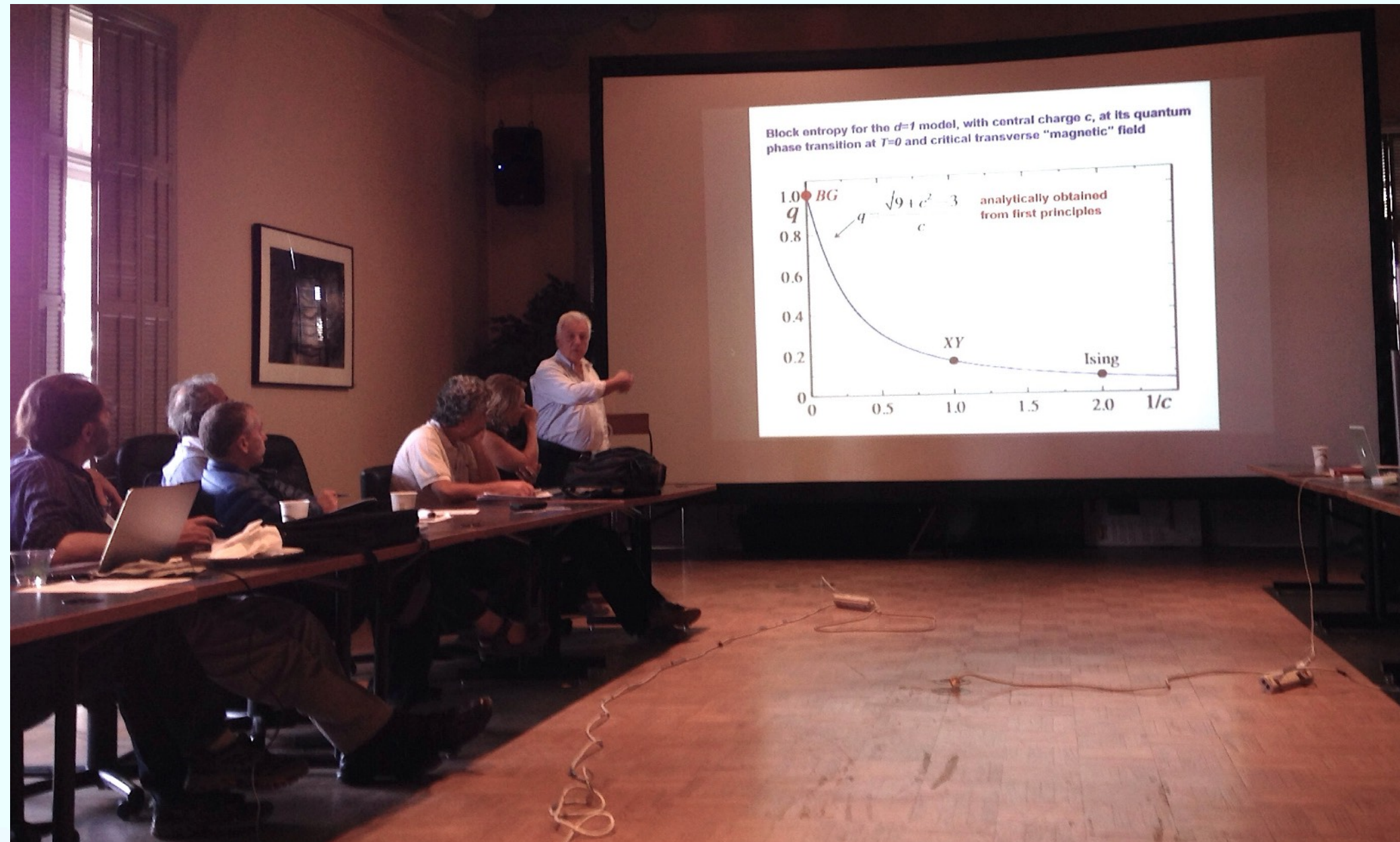


Wuhan and Peking, October 2012





70th birthday of Constantino - Rio 2013 -





Erice International School of Complexity 2015


«ETTORE MAJORANA» FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE


TO PAY A PERMANENT TRIBUTE TO ARCHIMEDES AND GALILEO GALILEI, FOUNDERS OF MODERN SCIENCE
 AND TO ENRICO FERMI, THE "ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES

INTERNATIONAL SCHOOL OF COMPLEXITY

15th Course: *NEW TRENDS IN STATISTICAL MECHANICAL FOUNDATION OF COMPLEXITY - APPLICATIONS IN HIGH ENERGY AND PLASMA PHYSICS, LONG-RANGE INTERACTIONS, EDGE-OF-CHAOS, AND ELSEWHERE*

ERICE-SICILY: 27 JULY – 3 AUGUST 2015

Sponsored by the: • Italian Ministry of Education, University and Scientific Research • CNPQ • FAPERJ • Sicilian Regional Government •

PROGRAMME AND LECTURERS

Nonextensive statistical mechanics
Nonadditive entropies
Generalized central limit theorems
Superstatistics
Generalized large deviation theories
Nonlinear dynamical systems
Chaotic maps at the Edge-of-Chaos
Long-range interacting systems
Plasma physics
High energy physics
Econophysics
Networks

- G. BENEDEK, University of Milan-Bicocca, IT
- T. BIRO, Academy of Sciences, Budapest, HU

- J. CLEYMANS, University of Cape Town, ZA
- E.M.F. CURADO, CBPF, Rio de Janeiro, BR
- D. DEL-CASTILLO-NEGRETTE, Oak Ridge National Laboratory, TN, US
- A. DEPPMAN, University of Sao Paulo, BR
- R. GALVAO, University of Sao Paulo, BR
- J.A. GOREE, University of Iowa, IA, US
- F.D. NOBRE, CBPF, Rio de Janeiro, BR
- A.R. PLASTINO, Universidad de Buenos Aires Noroeste, Junin, AR
- O. SOTOLONGO COSTA, Universidad de la Habana, CU and UALM, Cuernavaca, MX
- A. RAPISARDA, University of Catania, IT
- A. ROBLEDO, UNAM, Mexico City, MX
- P. TEMPESTA, Universidad Complutense de Madrid, ES
- S. THURNER, Medical University of Vienna, AT
- U. TIRNAKLI, Egee University, Izmir, TR
- C. TSALLIS, CBPF, Rio de Janeiro, BR
- G. WILK, National Centre for Nuclear Research, Warsaw, PL

PURPOSE OF THE COURSE

This Course addresses some new aspects of complex systems, mainly concerning its statistical-mechanical foundations and various applications. Although no general first-principle proof yet exists for Hamiltonian systems, there remains, after 140 years of impressive success, no reasonable doubt that the Boltzmann-Gibbs entropy is the correct one for wide and important classes of physical systems, basically the ergodic ones. Among the very many that violate this hypothesis, there is an important class, namely those that are weakly chaotic, with sub-exponential sensitivity to the initial conditions, typically a power-law time-dependence from the initial conditions. It was proposed in 1988 that the current statistical-mechanical methods can be extended to many other physical systems by generalizing the Boltzmann-Gibbs entropy into nonadditive forms. The aim of the present event is to cover a wide class of such systems. It is now known that nonadditive entropies have large applicability. Typical predictions, verifications and applications of these concepts will be addressed in natural, artificial, and social systems, as shown through theoretical, experimental, observational and computational results, in high energy and plasma physics, nonlinear dynamical systems, free-scale networks, econophysics, granular matter, geophysics, astrophysics, among others.

APPLICATIONS

Persons wishing to attend the Course should apply in writing to:

- Professor Andrea RAPISARDA
 Dipartimento di Fisica e Astronomia, Università di Catania, IT
 e-mail: newtrendserice2015@gmail.com
 web page: <https://sites.google.com/site/ericecomplexity2015>

specifying: i) full name, address, age, nationality, ii) academic qualification, present position and affiliation, iii) their specific interest in the workshop. Students should include a short C.V. in addition to a letter of recommendation from the head of their research group or from a senior scientist active in the field.

PLEASE NOTE

Participants are expected to arrive in Erice on July 26, no later than 5 p.m.

POETIC TOUCH

According to legend, Erice, son of Venus and Neptune, founded a small town on top of a mountain (750 metres above sea level) more than three thousand years ago. The founder of modern history — i.e. the recording of events in a methodic and chronological sequence as they really happened without reference to mythical causes — the great Thucydides (~500 B.C.), writing about events connected with the conquest of Troy (1183 B.C.) said: «After the fall of Troy some Trojans on their escape from the Achaei arrived in Sicily by boat and as they settled near the border with the Sicilians all together they were named Elymi: their towns were Segesta and Erice.» This inspired Virgil to describe the arrival of the Trojan royal family in Erice and the burial of Anchise, by his son Enea, on the coast below Erice. Homer (~1000 B.C.), Theocritus (~300 B.C.), Polybius (~200 B.C.), Virgil (~50 B.C.), Horace (~20 B.C.), and others have celebrated this magnificent spot in Sicily in their poems. During seven centuries (XIII-XIX) the town of Erice was under the leadership of a local oligarchy, whose wisdom assured a long period of cultural development and economic prosperity which in turn gave rise to the many churches, monasteries and private palaces which you see today. In Erice you can admire the Castle of Venus, the Cyclopean Walls (~800 B.C.) and the Gothic Cathedral (~1300 A.D.). Erice is at present a mixture of ancient and medieval architecture. Other masterpieces of ancient civilization are to be found in the neighbourhood: at Motya (Phoenician), Segesta (Elymian), and Selinunte (Greek). On the Aegadian Islands — theatre of the decisive naval battle of the first Punic War (264-241 B.C.) — suggestive neolithic and paleolithic vestiges are still visible: the grottoes of Favignana, the carvings and murals of Levanzo.

Splendid beaches are to be found at San Vito Lo Capo, Scopello, and Corino, and a wild and rocky coast around Monte Cofano: all at less than one hour's drive from Erice.

More information about the other activities of the «ETTORE MAJORANA» FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE can be found on the WWW at the following address: <http://www.esem.infn.it>

A. ZICHICHI
DIRECTOR OF THE SCHOOL

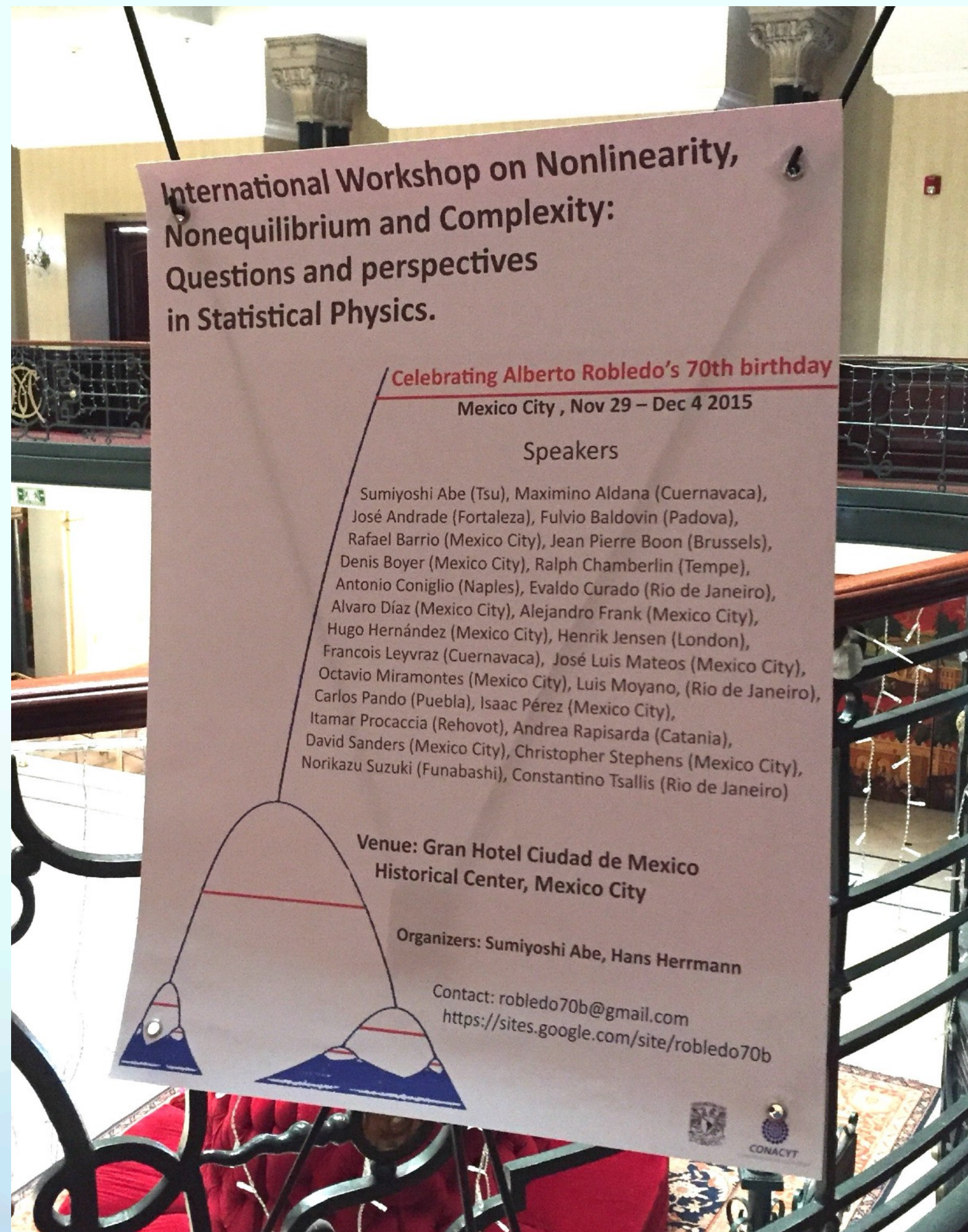
G. BENEDEK – R. GALVAO – A. RAPISARDA – C. TSALLIS
CO-DIRECTORS OF THE COURSE

G. BENEDEK – M. GELL-MANN – C. TSALLIS
CO-DIRECTORS OF THE SCHOOL

A. ZICHICHI
PRESIDENT OF THE EMFCSC



Alberto Robledo's 70th birthday - Mexico City 2015



Econophysics colloquium - ICTP San Paolo - july 2016



At Complexity Science Hub Vienna



2018



2022

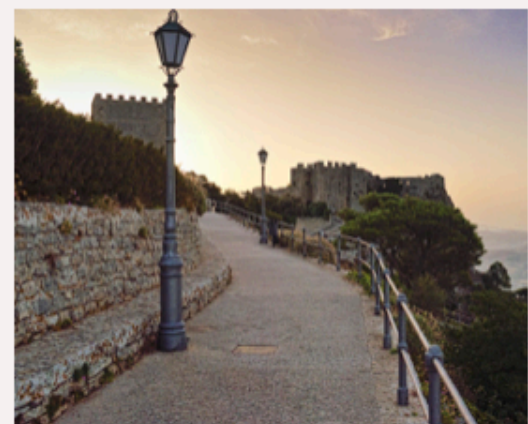


Erice International School on Complexity 2019

Ettore Majorana Foundation and Centre for Scientific Culture
International School on Complexity - XVI Course

Nonextensive Statistical Mechanics, Superstatistics and Beyond: Theory and Applications in Astrophysical and other Complex Systems

July 2-8, 2019 Erice, Italy



This course belongs to the series **International School on Complexity**, directed by A. Zichichi, and co-directed by G. Benedek, M. Gell-Mann, A. Rapisarda and C. Tsallis. It will be held at the **Ettore Majorana Foundation and Centre for Scientific Culture** in Erice, Italy during the period July 2-8, 2019.

Purpose of the Course

After more than 140 years of impressive success there is no reasonable doubt that the Boltzmann-Gibbs (BG) entropy is the correct one to be used for a wide and important class of physical systems, basically those whose (nonlinear) dynamics is strongly chaotic i.e., for classical systems with positive maximal Lyapunov exponent, which are mixing and ergodic. However, a plethora of physical complex systems exists for which such simplifying dynamical hypotheses are violated; typical examples are those for which the maximal Lyapunov exponent vanishes, leading to sub-exponential sensitivity to the initial conditions, which can of course occur in a variety of mathematical ways. Corresponding anomalies are found in a variety of quantum systems as well. In order to statistically describe the dynamics of such systems, various generalised forms of statistical mechanics have been proposed such as those using the nonadditive entropies S_q (where q is a real number which, for $q=1$, recovers the BG entropy), kappa distributions (also known as q -Gaussians, where kappa is simply related to q), superstatistical approaches, among various others. In the last decades, these new generalised statistical mechanical formalisms have found a large variety of very successful applications, even beyond the realm of physics. This course aims to cover the most recent analytical, experimental, observational and computational aspects and examples where these new extended formalisms have found fruitful applications.

Topics include, but are not limited to:

- Generalised Central Limit theorems;
- Generalised Large Deviation theory;
- Low-dimensional nonlinear conservative and dissipative dynamical systems near the edge of chaos;
- Long-range-interacting many-body classical Hamiltonian systems;
- Complex networks;
- Area-law-like quantum systems;
- Applications in astrophysics, space and other plasma physics, geophysics, high energy physics, cosmology, granular matter, cold atoms, econophysics, theoretical and structural chemistry, biophysics, social systems, power grids, image and time series processing, among others.

Invited speakers



Evaldo M.F. Curado
CBPF, Rio de Janeiro, Brazil



Henrik J. Jensen
Imperial College, London, UK
and CSH, Vienna Austria



David J. McComas
Princeton University, USA



Ralph Metzler
University of Potsdam,
Germany



Fernando D. Nobre
CBPF, Rio de Janeiro, Brazil



Viviane Pierrard
Université Catholique de
Louvain, Belgium



Angel R. Plastino
Univ. Nacional de la Provincia
de Buenos Aires, Argentina



Benjamin Schaefer
Max Planck Institute,
Goettingen, Germany



Piergiulio Tempesta
Universidad Complutense,
Madrid, Spain



Stefan Thurner
Medical University of Wien and
CSH Vienna, Austria



Panayiotis Varotsos
University of Athens, Greece



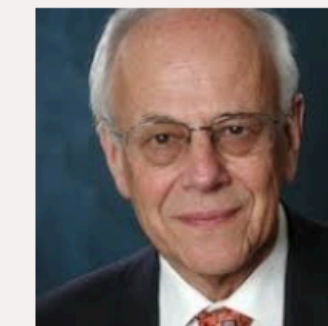
Peter Yoon
University of Maryland, USA



Course Directors



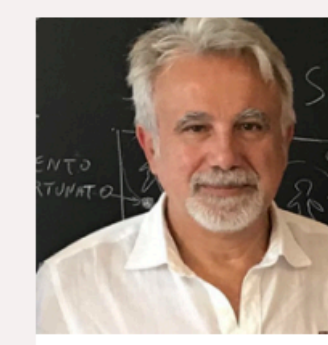
Christian Beck
Queen Mary,
University of London, UK



Giorgio Benedek
Università di Milano Bicocca,
Italy



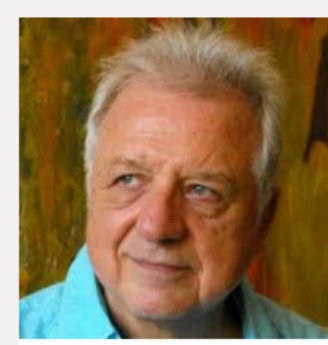
George Livadiotis
Southwest Research Institute,
USA



Andrea Rapisarda
University of Catania and Infn,
Italy, CSH Vienna, Austria

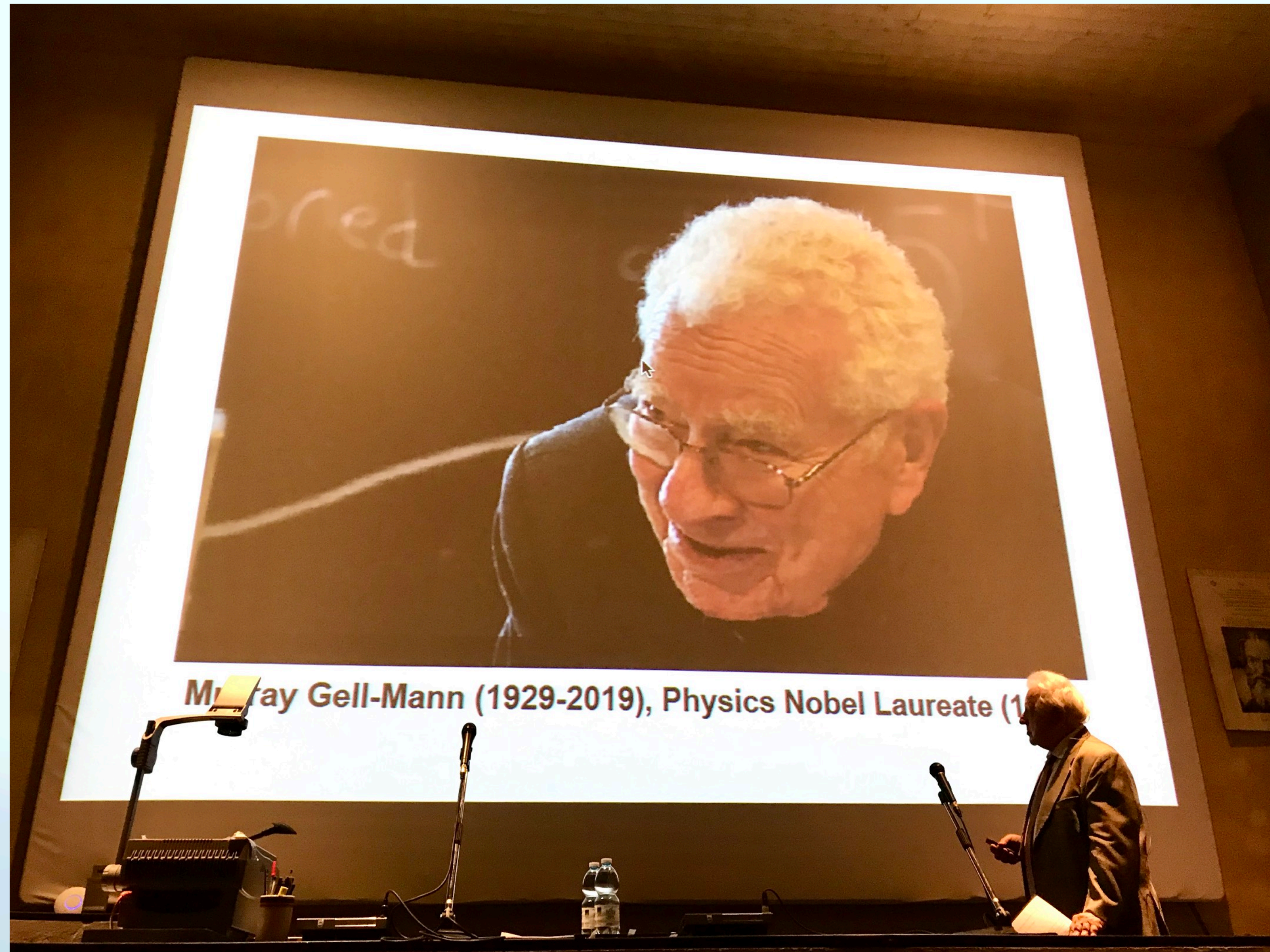


Ugur Tirnakli
Ege University, Turkey



Constantino Tsallis
CBPF Brazil, Santa Fe Institute
USA, CSH Vienna, Austria

Commemorating M. Gell-Mann



Erice International School on Complexity 2022

Ettore Majorana Foundation and Centre for Scientific Culture
 International School on Complexity - XVII Course
Stochastic Forecasting in Complex Systems
 August 25-31, 2022 Erice, Italy



This course belongs to the series International School on Complexity, directed by A. Zichichi, and co-directed by G. Benedek, A. Rapisarda and C. Tsallis. It will be held at the Ettore Majorana Foundation and Centre for Scientific Culture in Erice, Italy during the period August 25-31, 2022.

Purpose of the Course

This course aims to cover the most recent analytical, experimental, observational and computational aspects about stochastic forecasting of complex systems

Topics include, but are not limited to:

- Brain Dynamics;
- Earthquake dynamics;
- Financial markets dynamics;
- Complex networks;
- Self-Organized criticality;
- Agent-based models applied to complex socio-economical systems;
- Epidemic models;
- Probabilistic prediction and scenario evaluation;
- Statistical mechanics of complex systems.

The Course is supported by the PRIN project 2017WZFTZP **STOFOCS**



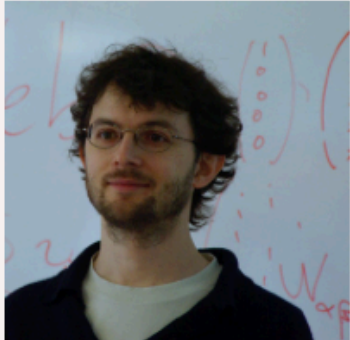
Invited speakers



Jean-Philippe Bouchaud
 CFM & Econophysics Chair,
 Ecole Polytechnique Paris,
 France



Raffaella Burioni
 University of Parma, Italy



Damien Challet
 Centrale Supélec and Université
 Paris-Saclay, Gif-sur-Yvette
 France



Tiziana Di Matteo
 King's College London, UK



Sebastian Hainzl
 GFZ Potsdam, Germany



Hans Herrmann
 ESPCI Paris, France



Eugenio Lippiello
 Campania University, Italy



Daniele Marinazzo
 Ghent University, Belgium



Warner Marzocchi
 University of Naples Federico II,
 Italy



Oren Shriki
 Ben Gurion University, Israel



Stefan Thurner
 Medical University of Wien and
 CSH Vienna, Austria



Constantino Tsallis
 CBPF Brazil, Santa Fe Institute
 USA, CSH Vienna, Austria

Course Directors



Lucilla De Arcangelis
 Campania University, Italy



Rosario Nunzio Mantegna
 University of Palermo, Italy and
 CSH Vienna, Austria



Andrea Rapisarda
 University of Catania and Infn.,
 Italy, CSH Vienna, Austria

Teaching in Catania - september 2022



Econophysics Colloquium - Lipari, Sicily - august 2023



Review

Nonextensive Footprints in Dissipative and Conservative Dynamical Systems

Antonio Rodríguez ^{1,*}, Alessandro Pluchino ^{2,3,†}, Ugur Timakli ^{4,†}, Andrea Rapisarda ^{3,5,†} and Constantino Tsallis ^{5,6,7,†}

- ¹ GISC, Departamento de Matemática Aplicada a la Ingeniería Aeroespacial, Universidad Politécnica de Madrid, Plaza Cardenal Cisneros s/n, 28040 Madrid, Spain
 - ² Istituto Nazionale di Fisica Nucleare, Sezione di Catania, 95123 Catania, Italy
 - ³ Dipartimento di Fisica e Astronomia “E. Majorana”, University of Catania, 95123 Catania, Italy
 - ⁴ Department of Physics, Faculty of Arts and Sciences, Izmir University of Economics, Izmir 35330, Turkey
 - ⁵ Complexity Science Hub Vienna, Josefstadtterstrasse 39, A-1090 Vienna, Austria
 - ⁶ Centro Brasileiro de Pesquisas Físicas and National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, Rio de Janeiro 22290-180, RJ, Brazil
 - ⁷ Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA
- * Correspondence: antonio.rodriguez@upm.es
 † These authors contributed equally to this work.

Abstract: Despite its centennial successes in describing physical systems at thermal equilibrium, Boltzmann–Gibbs (BG) statistical mechanics have exhibited, in the last several decades, several flaws in addressing out-of-equilibrium dynamics of many nonlinear complex systems. In such circumstances, it has been shown that an appropriate generalization of the BG theory, known as nonextensive statistical mechanics and based on nonadditive entropies, is able to satisfactorily handle wide classes of anomalous emerging features and violations of standard equilibrium prescriptions, such as ergodicity, mixing, breakdown of the symmetry of homogeneous occupancy of phase space, and related features. In the present study, we review various important results of nonextensive statistical mechanics for dissipative and conservative dynamical systems. In particular, we discuss applications to both discrete-time systems with a few degrees of freedom and continuous-time ones with many degrees of freedom, as well as to asymptotically scale-free networks and systems with diverse dimensionalities and ranges of interactions, of either classical or quantum nature.

Keywords: nonextensive statistical mechanics; long-range dynamical systems; entropy; complex systems

1. Introduction

Statistical mechanics constitutes one of the pillars of contemporary theoretical physics. It was introduced in the 19th century by L. Boltzmann and J.W. Gibbs, and the name was coined by Gibbs himself. It is based on mechanics (classical, quantum, relativistic), electromagnetism, and theory of probabilities. Probabilities enter through the so-called entropic functional S , whose generic form for discrete stochastic variables is given by

$$S(\{p_i\}) = kF(\{p_i\}) \left(\sum_{i=1}^W p_i = 1 \right), \quad (1)$$

where $F(\{p_i\})$ is an appropriate generic functional, k being typically equal either to unity or to the Boltzmann constant k_B . Historically, Boltzmann and Gibbs used continuous variables ($p(x)$ instead of p_i). The corresponding discrete form is given by



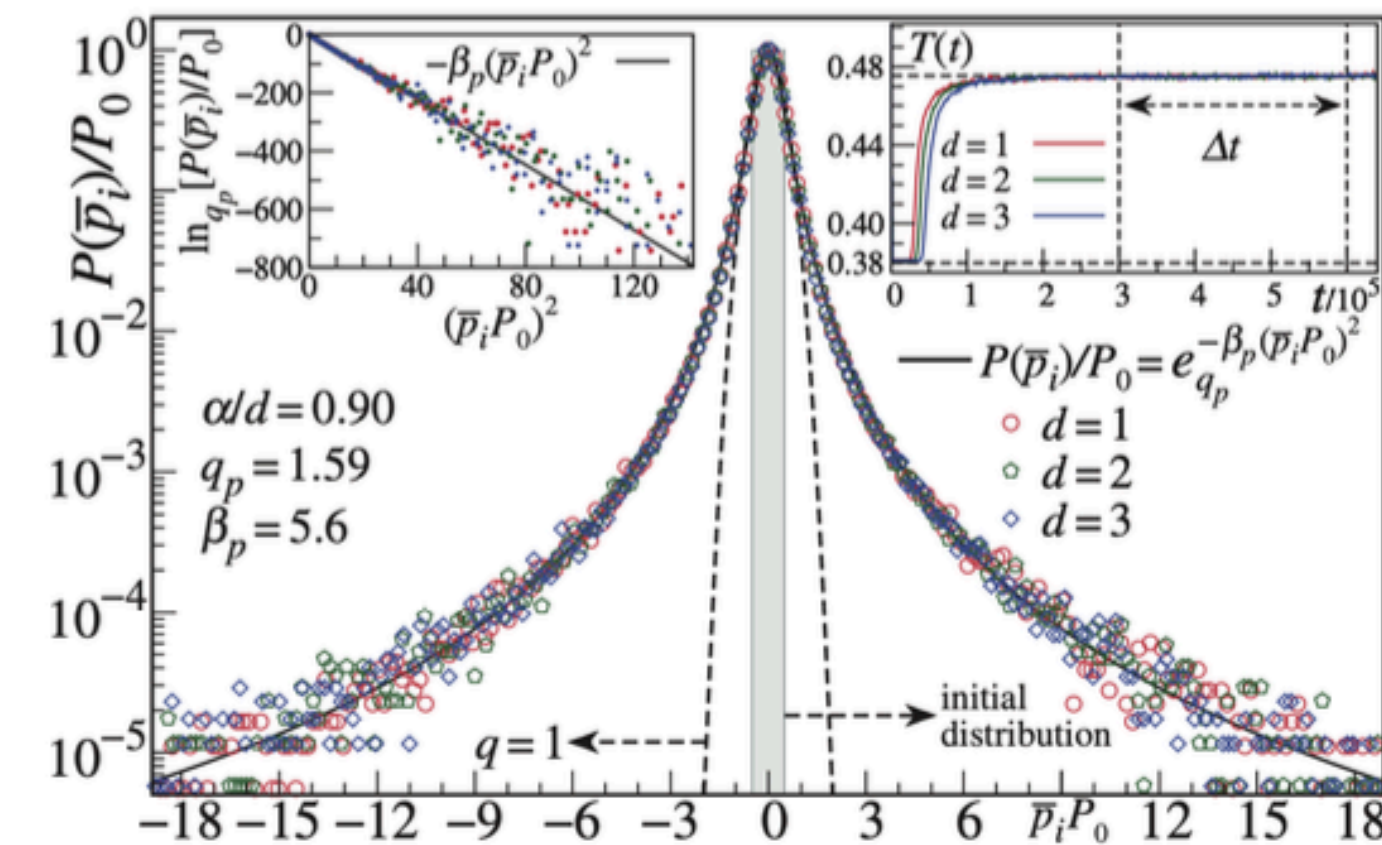
Citation: Rodríguez, A.; Pluchino, A.; Timakli, U.; Rapisarda, A.; Tsallis, C. Nonextensive Footprints in Dissipative and Conservative Dynamical Systems. *Symmetry* **2023**, *15*, 444. <https://doi.org/10.3390/sym15020444>

Academic Editor: Christos Volos

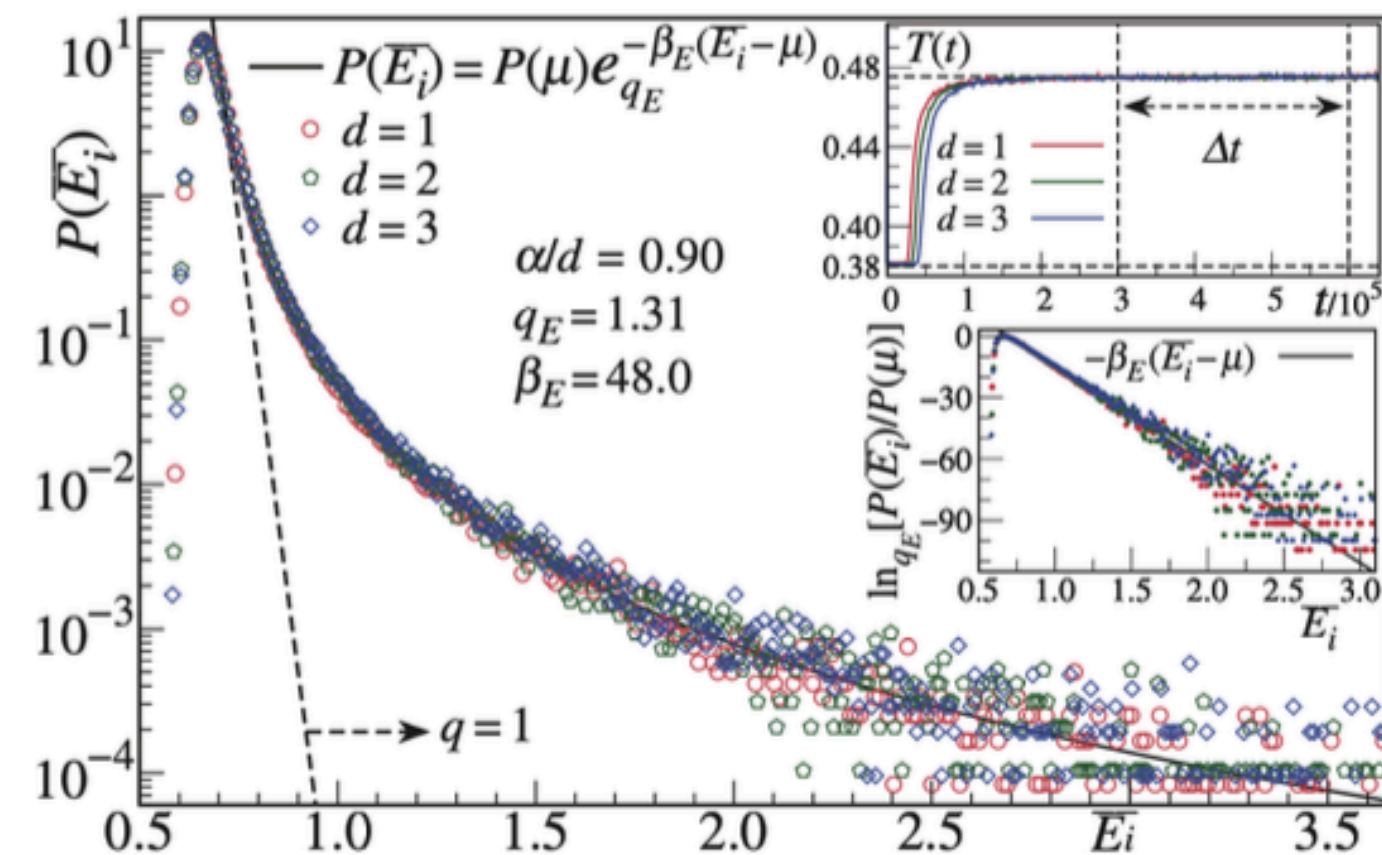
Received: 10 January 2023
 Revised: 23 January 2023
 Accepted: 2 February 2023
 Published: 7 February 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).



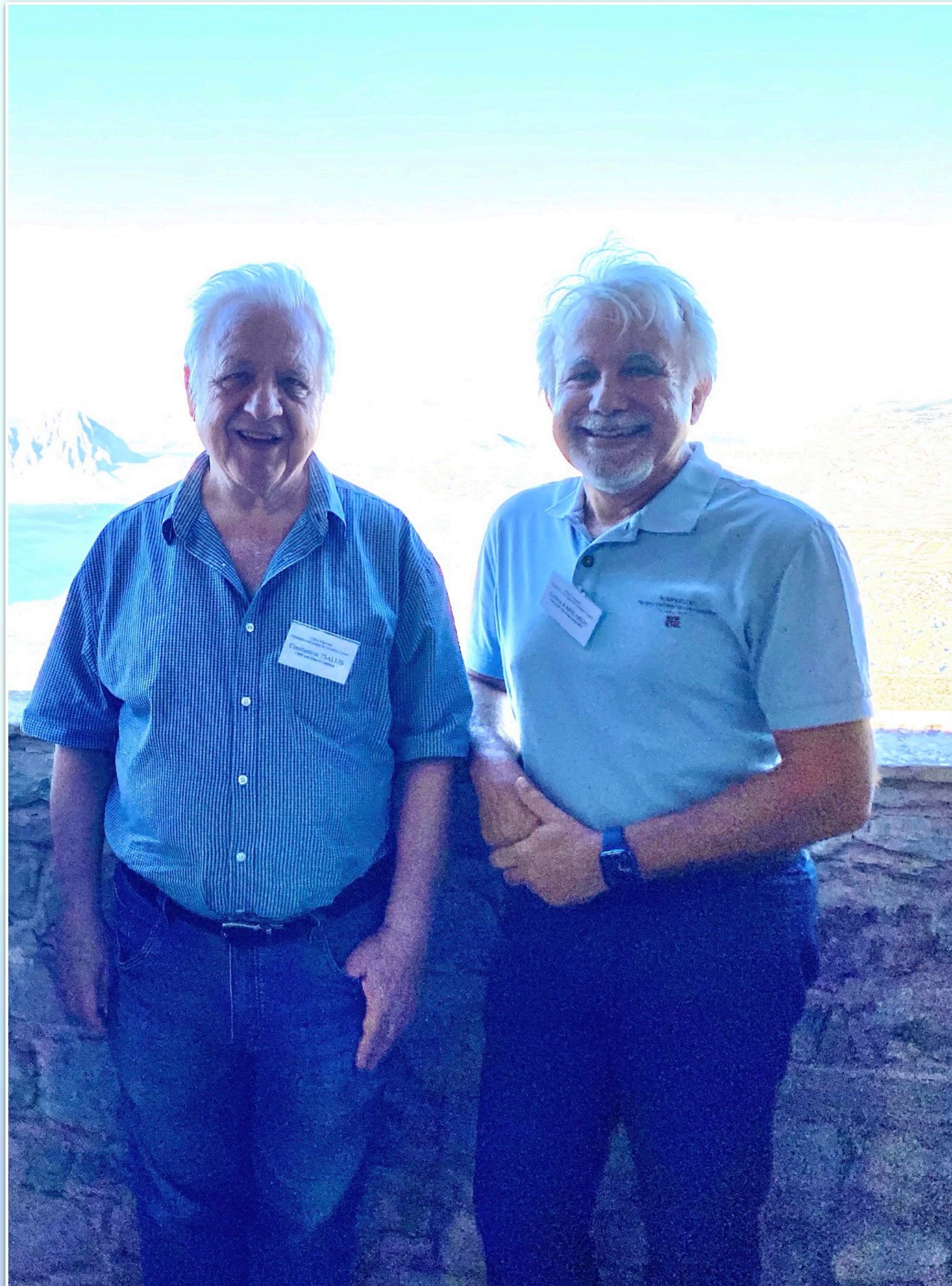
(a) Momentum



(b) Energy

Figure 13. Inertial α -XY d -dimensional model (for $d = 1, 2, 3$) for $\alpha/d = 0.9$. **Left:** q_p -Gaussian distribution of momenta (for comparison, a Maxwellian distribution is indicated in dashed line). **Right:** q_E -exponential distribution of energies (for comparison, a BG distribution is indicated in dashed line). Both distributions are averaged along the very long-time interval indicated in the insets. Figure reproduced from Ref. [72].

Summarising these last 25 years



- 15 coauthored papers
- several international conferences/schools organized together
- infinite number of interesting discussions, *not only about physics !*
- plenty of wonderful moments around the world

Many thanks Constantino for your friendship

It has been a great privilege and honor to have met you and shared so many wonderful moments with you



Happy Birthday

Constantino

Best wishes for your

first 80 years!!

