Simulating different entropic functionals from a two-level quantum system

Andre M. C. Souza (UFS and INCT-SC)

"CRITICALIDADE DE MODELOS MAGNÉTICOS DISCRETOS EM REDES HIERÁRQUICAS"

ANDRE MAURICIO CONCEIÇÃO DE SOUZA

Tese de Mestrado apresentada ao Centro Brasileiro de Pesquisas Físicas do Conselho Nacional de Desenvolvimento Científico e Tecnológico, perante Banca Examinadora constituída pelos seguintes professores:

Constantino Tsallis - Presidente

Raimundo Rocha dos Santos

Aníbal Omar Caride

Rio de Janeiro, 08 de janeiro de 1993

"SISTEMAS ESTATÍSTICOS COMPLEXOS E MECÂNICA ESTATÍSTICA NÃO EXTENSIVA"

André Maurício Conceição de Souza

Tese de Doutorado apresentada no Centro Brasileiro de Pesquisas Físicas, do Conselho Nacional de Desenvolvimento Científico e Tecnológico, fazendo parte da Banca Examinadora os seguintes professores:

Constantino Tsallis - Presidente

Mário José de Oliveira

Paulo Murilo Castro de Oliveira

Nami Fux Svaiter

Alfredo Miguel Ozorio de Almeida

Rio de Janeiro, 30 de janeiro de 1997

PHYSICAL REVIEW B VOLUME 47, NUMBER 18 1 MAY 1993-II

Quenched bond-mixed cubic ferromagnet in a planar self-dual lattice: Critical behavior

André M. C. de Souza* and Constantino Tsallis
Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, 22290 Rio de Janeiro, Rio de Janeiro, Brazil

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(Received 28 October 1992)

The critical behavior of the quenched bond-mixed ferromagnetic cubic model, on a planar self-dual hierarchical lattice, is investigated within a simple real-space renormalization group. We obtain the complete phase diagram of the system, exhibiting three phases. This phase diagram is believed to be of high precision for the square lattice. The correlation-length critical exponents and the universality classes are determined as well.

PHYSICAL REVIEW E VOLUME 48, NUMBER 2 AUGUST 1993

Prototype for memory effects in the time evolution of damage

Constantino Tsallis, Francisco Tamarit, and André M. C. de Souza*
Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290 Rio de Janeiro, Rio de Janeiro, Brazil
(Received 11 February 1993)

We introduce a one-dimensional cellular automaton as a prototype for memory effects on damage. The associated Hamming distance as a function of time correctly mimics complex dynamical systems and, for different values of the external parameters, gradually varies between a noiselike behavior and a plateaulike one.

PHYSICAL REVIEW LETTERS

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Statistical-Mechanical Foundation of the Ubiquity of Lévy Distributions in Nature

Constantino Tsallis,[1,2] Silvio V. F. Levy,[3] André M. C. Souza,[2,4] and Roger Maynard[5]

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(Received 12 June

We show that the use of the recently proposed therm
form $S_q \equiv k(1 - \sum_i p_i^q)/(q - 1)$ (where $q \in \boldsymbol{R}$, with q
Shannon entropy $-k \sum_i p_i \ln p_i$), together with the Lévy
theorem, provide a basic step towards the understanding
nature. A consistent experimental verification is proposed

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THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Regular Article

Area-law-like systems with entangled states can preserve ergodicity

Andre M.C. Souza[1,2,a], Peter Rapčan[1,3], and Constantino Tsallis[1,4,5]

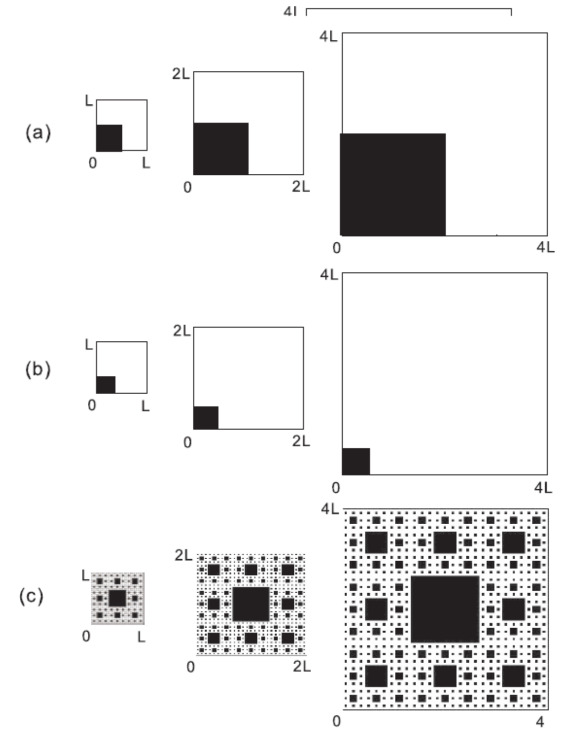


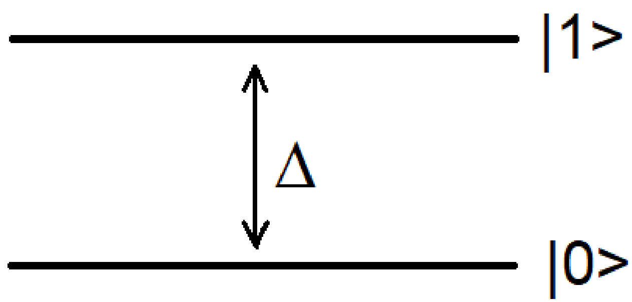
Fig. 5. Classes of the phase-space trajectories covering a (a) *compact* subspace whose corresponding Lebesgue measure remains different from zero in the thermodynamic limit; (b) *compact* subspace whose corresponding Lebesgue measure vanishes in the thermodynamic limit, and; (c) *noncompact* subspace whose corresponding Lebesgue measure vanishes in the thermodynamic limit. Three different size systems are presented.

It was a great honor for me work with Constantino during these years. His intelligence and ability to work as a team are references for the science.

Simulating different entropic functionals from a two-level quantum system

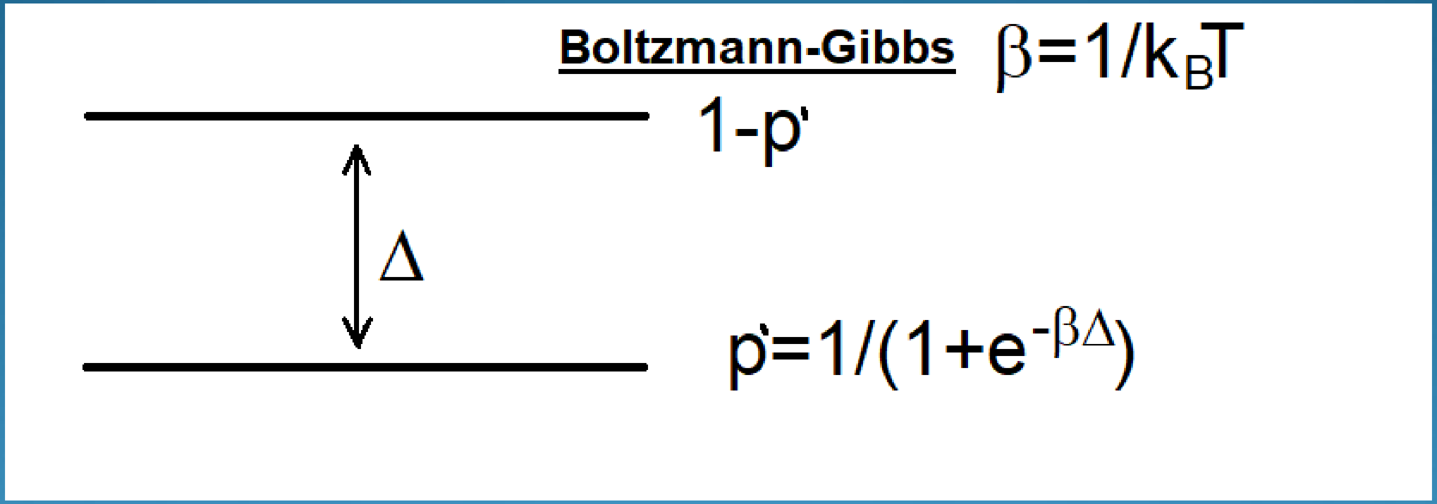
Andre M. C. Souza (UFS and INCT-SC)

Two energy level system (diagonal)



$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \Delta \end{pmatrix}$$

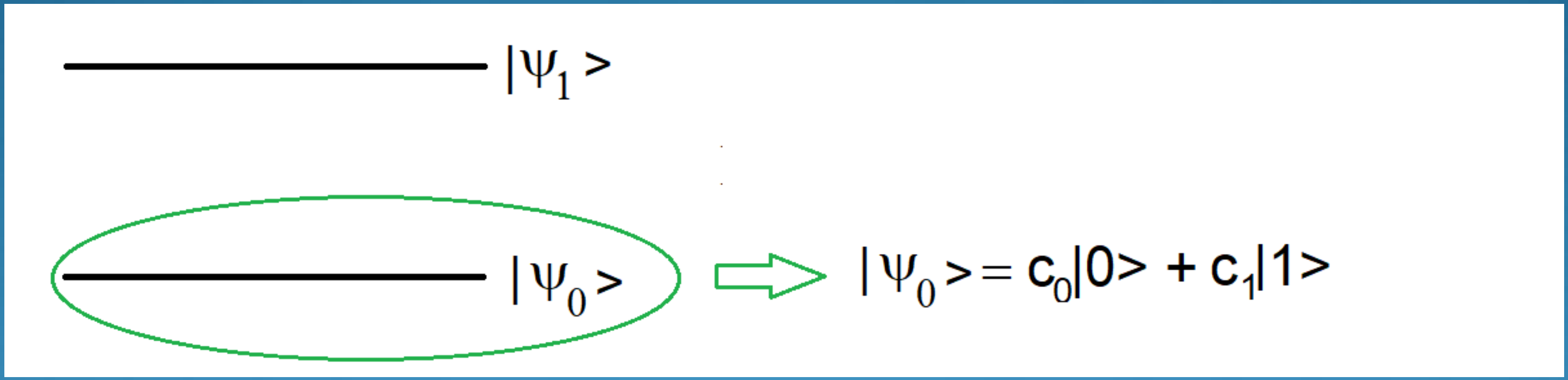
Thermal bath



Let us study the general case of a two energy level system

$$H = \begin{pmatrix} 0 & 0 \\ 0 & \Delta \end{pmatrix} + \lambda \begin{pmatrix} 0 & -z \\ -z & 0 \end{pmatrix}$$

λ (dimensionless), z and Δ are real numbers.



$|\psi_1>$
$|\psi_0>$
$|\psi_0> = c_0|0> + c_1|1>$

Considering

$$\hat{H}|\psi_0(\lambda)\rangle = \varepsilon_0(\lambda)|\psi_0(\lambda)\rangle$$

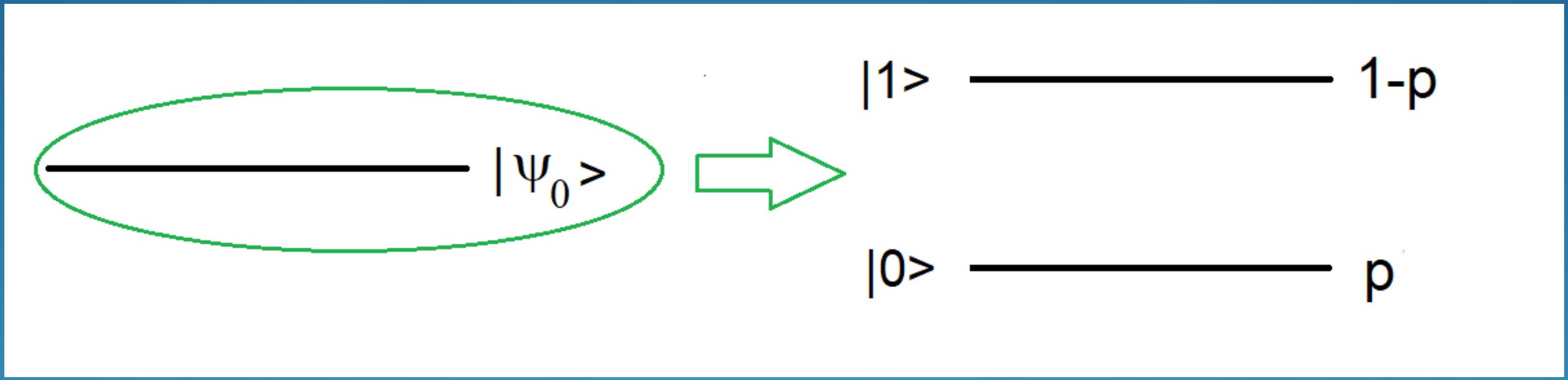
The ground state is

$$\varepsilon_0(\lambda) = \frac{\Delta}{2}\left\{1 - \sqrt{1 + (2z\lambda/\Delta)^2}\right\}$$

The $|\psi_0>$ can be expanded in terms of the two non-interacting states.

$$|\psi_0(\lambda)\rangle = \sqrt{1-p}\,e^{i\beta}|1\rangle + \sqrt{p}\,e^{i\alpha}|0\rangle$$

We assume $\alpha = \beta$



$|\psi_0>$
|1>
1-p
|0>
p

The probability p of occupying the excited state is obtained as

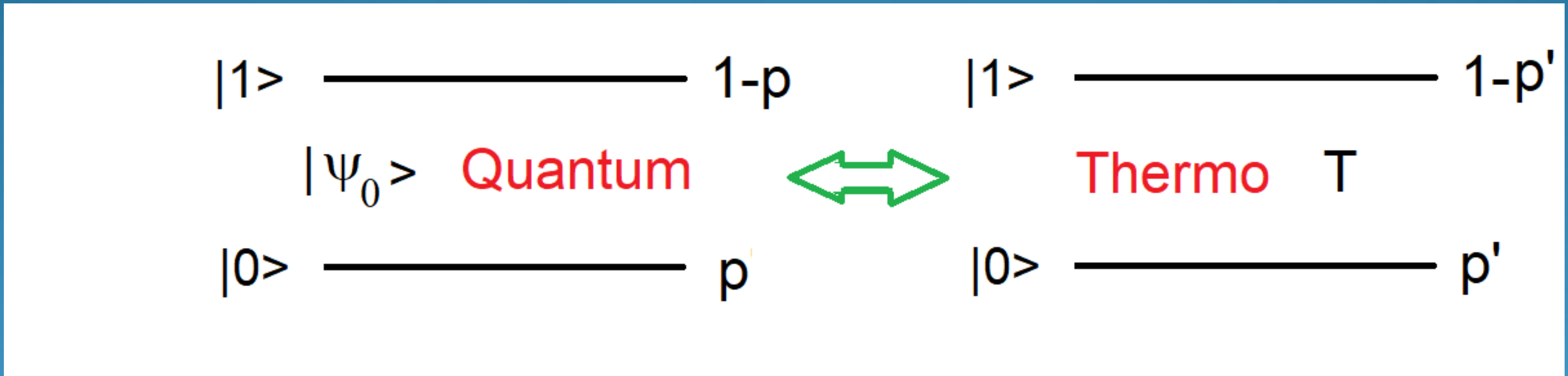
$$p = \frac{\tau^2}{2(1+\tau^2-\sqrt{1+\tau^2})}$$

where $\tau = 2z/\lambda\Delta$.

The inverse relation is given by

$$\tau = \frac{2\sqrt{p(1-p)}}{|2p-1|}$$

We can define an analog of the absolute temperature scale in such a manner that it is possible to make a thermodynamic interpretation for the quantum systems in the ground-state.



We can introduce a "ground-state thermodynamics" defining the ground-state internal energy, ground-state free energy and ground-state entropy, respectively, as:

[A. M. C. Souza and F. D. Nobre, Phys. Rev. E 95, 012111 (2017)]

$$U(\lambda) = \langle \hat{H}_0(\lambda) \rangle = \sum_i p(\lambda) E_i(0)$$

$$F(\lambda) = \langle \hat{H}(\lambda) \rangle - k\lambda \langle \hat{V}(0) \rangle$$

$$S(\lambda) = k(\langle \hat{V}(0) \rangle - \langle \hat{V}(\lambda) \rangle)$$

k is a constant suitably chosen by dimensional requirements

$\langle \ldots \rangle = \langle \psi_0 | \ldots | \psi_0 \rangle$

Implying that:

$$U(\lambda) = \langle \psi_0(\lambda) | \hat{H}_0 | \psi_0(\lambda) \rangle = (1-p)\Delta$$

We can write two distinct forms of energy exchange

$$dU = -\Delta dp + (1-p)d\Delta$$

One associated with variations in the occupation probabilities and other with variations in the gap energy level

The first form is identified with heat

$$\delta Q = -\Delta dp = \theta dS,$$

whereas the second with work

$$\delta W = (1 - p)d\Delta = \sigma d\Delta$$

The thermodynamics first law is identified as

$$dU = \delta Q + \delta W = \theta dS + \sigma d\Delta$$

The effective temperature is identified directly to the fundamental relation

$$\frac{\partial U}{\partial S} = \theta .$$

We found $\theta = \lambda z / k$ $$S = k\sqrt{p(1-p)}$$

Consistently, analogous to the standard thermodynamics, the potentials thermodynamics and response functions may be also derived.

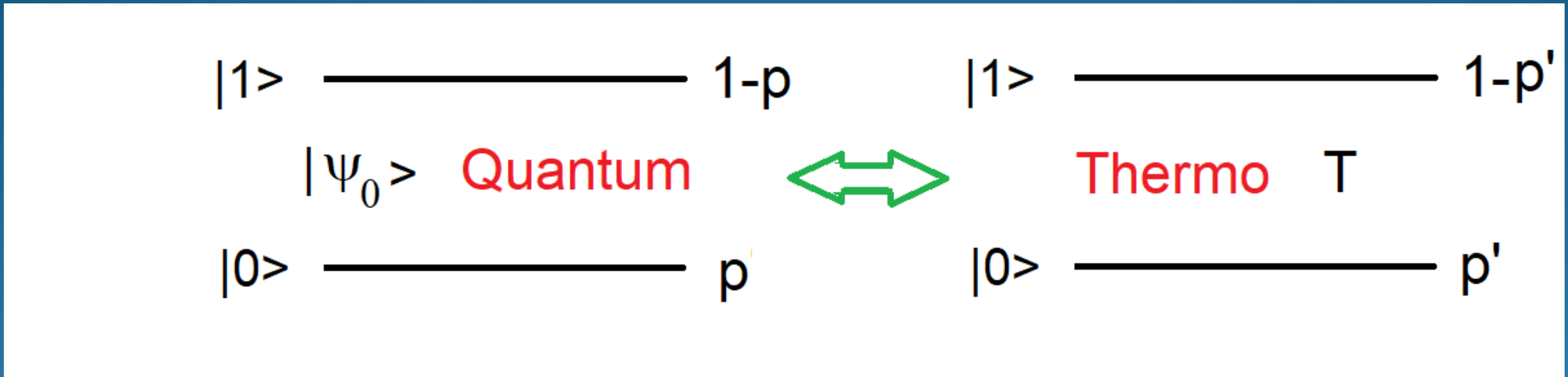
Similar results to the above formalism have been found for a model of interacting vortices for type-II superconductors at low temperature.

A non-additive entropic forms and an appropriate effective temperature emerge as an appropriate framework, exhibiting properties very similar to those of the usual thermodynamic temperature T.

In particular, for the vortex model, the entropy $S_{q=2}$ of nonextensive statistical mechanics appears as the conjugated to the effective temperature of the system.

[J. S. Andrade, Jr., G. F. T. da Silva, A. A. Moreira, F. D. Nobre, and E. M. F. Curado, Phys. Rev. Lett. 105, 260601 (2010).

F. D. Nobre, E. M. F. Curado, A. M. C. Souza, and R. F. S. Andrade, Phys. Rev. E 91, 022135 (2015).]



Given an expression for p' from the thermostatistics, is it possible to obtain a quantum equivalent?

YES

Following the Clausius approach, it is possible to define the entropy S.

$$S = S_0 + \int \frac{\delta Q}{\theta} = S_0 - \Delta \int \frac{dp}{\theta}$$

$$\delta Q = -\Delta dp = \theta dS,$$

Remember

It is also possible to express p as a function of θ, i.e., there is a function g that $p = g(\theta)$. Assuming the condition that g is inversive.

$$S = S_0 - \Delta \int \frac{dp}{g^{-1}(p)}$$

And the form of $f(\theta)$ as a function of p is given by

$$\boxed{\lambda = f(\theta)} = \frac{\Delta\sqrt{p(1-p)}}{z|2p-1|}.$$

Remember

$$\hat{H} = \hat{H}_0 + \lambda\hat{V}$$

Remember

$$p = \frac{\tau^2}{2(1+\tau^2-\sqrt{1+\tau^2})}$$

$\tau = 2z/\lambda\Delta$

The Protocol

The protocol starts by choosing the functional form of p as a function of θ. To know the form of the interaction function $\lambda = f(\theta)$ just replace p in previous equation.

$$\lambda = f(\theta) = \frac{\Delta\sqrt{p(1-p)}}{z|2p-1|}.$$

Next, the inverse function $\theta = g^{-1}(p)$ may be found and thus the functional form of entropy is obtained from

$$S = S_0 - \Delta \int \frac{dp}{g^{-1}(p)}$$

Applications

1. BG thermostatistics

The usual BG equilibrium probability (finding a physical configuration in the ground state of a two -level system for a certain temperature θ)

$$p_{eq}^{(BG)} = \frac{1}{1 + e^{-\Delta/k\theta}}$$

Following the protocol:

$$p = p_{eq}^{(BG)} = g(\theta)$$

It is straight to find that

$$g^{-1}(\theta) = -k/\ln((1-p)/p)$$

Hence, we obtain the usual BG entropy

$$S[p] = S_0 - k(p\ln p + (1-p)\ln(1-p))$$

$$S = S_0 - \Delta \int \frac{dp}{g^{-1}(p)}$$

Remember

and, $\lambda = f(\theta) = \dfrac{\Delta\sqrt{p(1-p)}}{z|2p-1|}$

Remember

$$f(\theta) = \frac{\Delta}{2z} cosh\left(\frac{\Delta}{2k\theta}\right)$$

Remember

$$\hat{H} = \hat{H}_0 + \lambda\hat{V}$$

$$S(\lambda) = k(\langle\hat{V}(0)\rangle - \langle\hat{V}(\lambda)\rangle)$$

Applications

2. Tsallis q-thermostatistics

The Tsallis q-thermostatistics equilibrium probability

$$p_{eq}^{(q)} = \frac{1}{1 + \left[\exp_q\left(\frac{-\Delta}{k\theta}\right)\right]^q}$$

Following the protocol, we find that

$$S[p] = S_0 - k(p^q \ln_q p + (1-p)^q \ln_q (1-p))$$

Exactly the Tsallis q-entropy

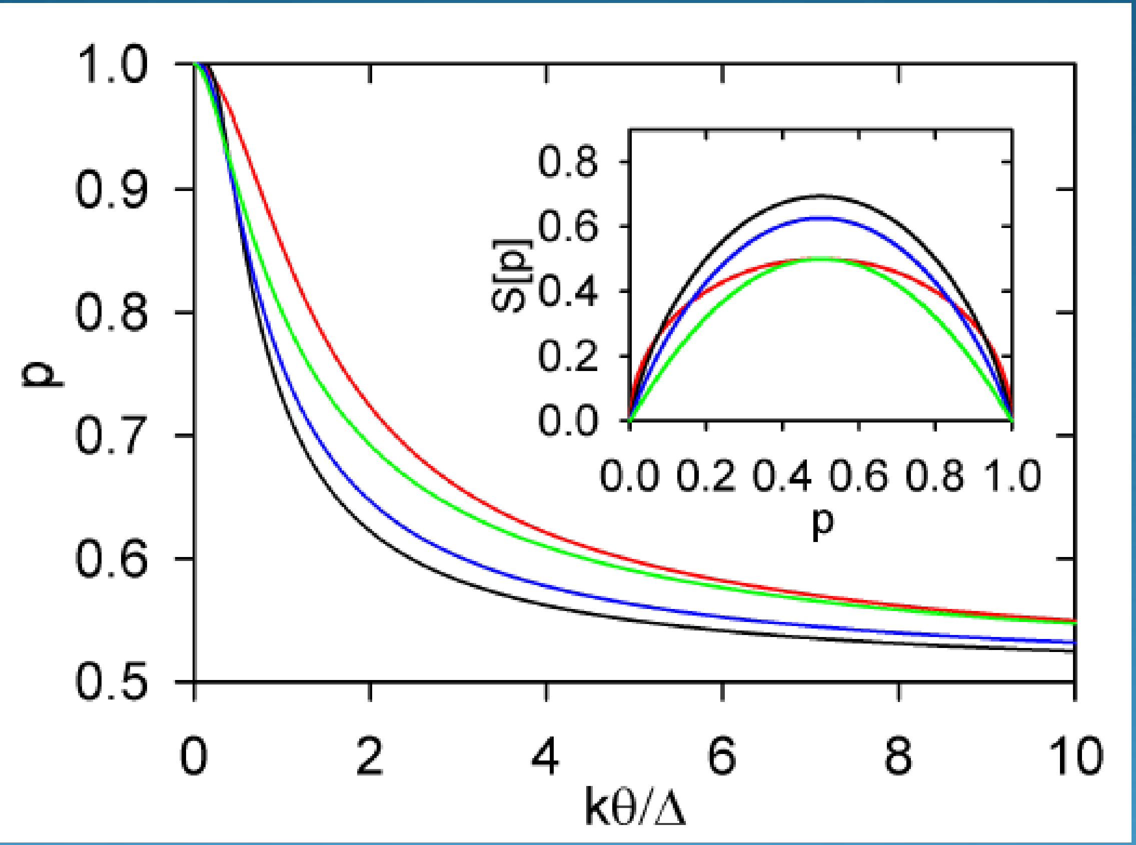
$$S_q = -k \sum_{j=1}^{W} p_i^q \ln_q p_i$$

$$\ln_q x = (x^{1-q} - 1)/(1-q)$$

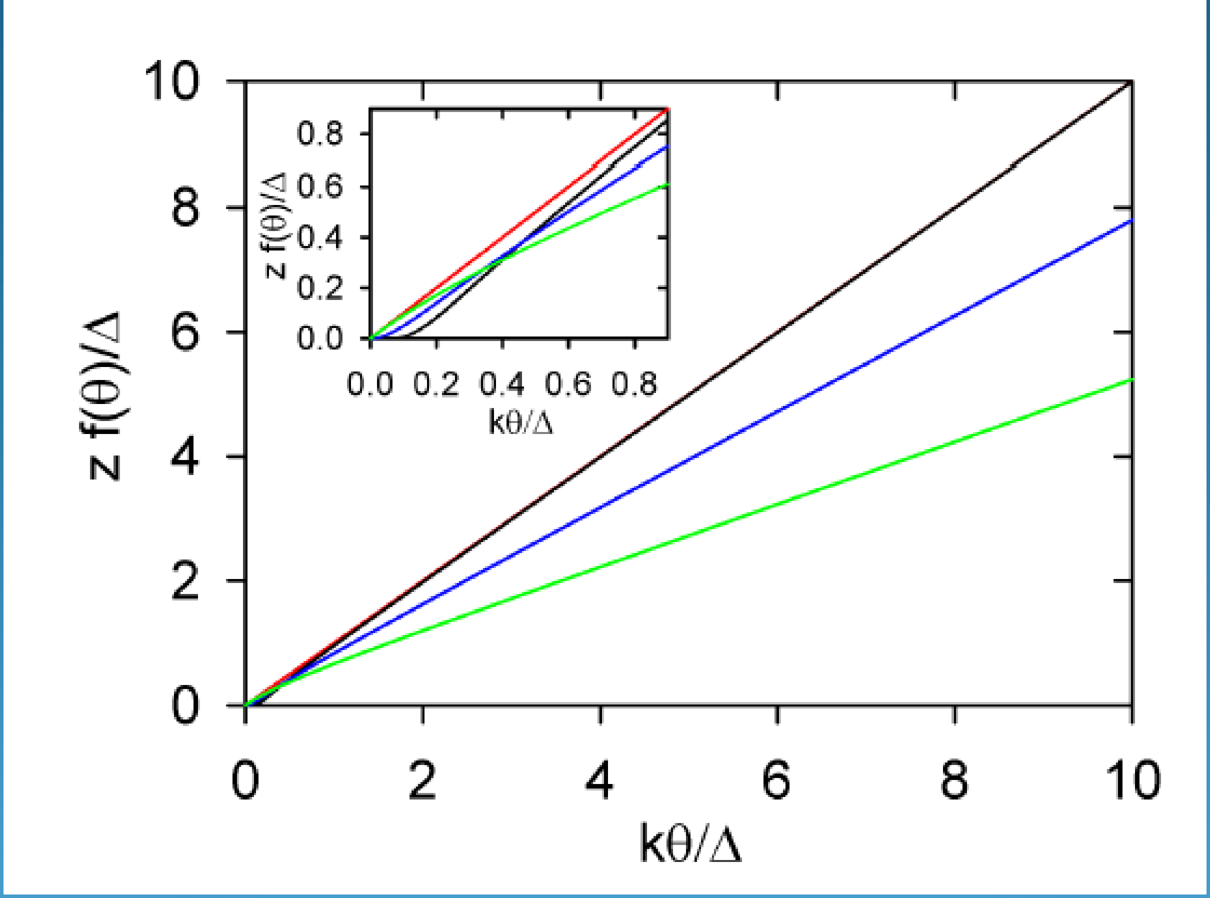
We found

$$f(\theta) = \frac{1}{z} \frac{\Delta}{\left| \left[\exp_q\left(\frac{-\Delta}{k\theta}\right)\right]^{-q/2} - \left[\exp_q\left(\frac{-\Delta}{k\theta}\right)\right]^{q/2} \right|}.$$

Naturally, the limit q→ 1 leads to BG thermostatistics.



Bolzmann-Gibbs (black curve), q = 1.3 (Blue curve), q = 2 Tsallis (green curve) and Souza & Nobre (red curve) thermostatistics.



Bolzmann-Gibbs (black curve), q = 1.3 (Blue curve), q = 2 Tsallis (green curve) and Souza & Nobre (red curve) thermostatistics.

Conclusions

Different thermostatistics can be extracted from two-level quantum systems

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The extraction protocol is based on a two-level Hamiltonian in which the non-diagonal term is associated with an effective temperature

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The extraction protocol is based on a two-level Hamiltonian in which the non-diagonal term is associated with an effective temperature

The functional form of this term defines the functional form of its conjugated parameter, the entropy S

Conclusions

Consistent with thermodynamic framework, the present approach establishes heat- and work-like quantities from the thermodynamics first law, similar results to the second and third laws of thermodynamics and that the efficiency of the proposed Carnot Cycle is independent of thermostatistics.

I can say that I have a very happy life, and part of this happiness was made possible by meeting Tsallis and as a consequence many of you here.

Thanks Tsallis, happy birthday
Thanks my friends

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