Generalized logarithmic and exponential functions

a brighter life

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7 de novembro de 2023

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Universidade de São Paulo (USP)

Influences of Constantino in my scientific life

Constantino influences in my scientific life

PHYSICAL REVIEW A VOLUME 38, NUMBER 8 OCTOBER 15, 1988

Pair-correlation function of the Potts model on the Cayley tree

A. S. Martinez and S. Goulart Rosa, Jr.
Instituto de Física e Química de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13 560 São Carlos, São Paulo, Brazil
(Received 2 February 1988)

A recursive relation to calculate the pair-correlation function of the q-state (ferromagnetic) Potts model on a Cayley tree with a homogeneous magnetic field applied to its surface spins is presented. The results are reduced to the calculation of the pair-correlation function of the model defined on a finite chain.

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Sylvio Goulart Rosa.

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I. INTRODUCTION

The local thermodynamical properties of a statistical mechanics model defined on a Cayley tree with finite branching ratio are known to recover exactly the results of the Bethe-Peierls approximation.[1] This approximation provides an improvement over the mean-field theory since it takes into account the nearest-neighbor correlation of the dynamical variables.

In spite of its importance, and to the best of our knowledge, the first studies of the pair-correlation function were carried out by Falk[2] in the zero-field ferromagnetic Ising model on the open Cayley tree and by Jelitto[3] on a newly defined tree.[4] This lattice, the so-called closed symmetric Cayley tree, is formed by joining by their surface sites two open trees. Besides presenting a study of the free energy[5] and of the specific heat, Jelitto also calculated the pair correlation function where the spins are symmetrically located with respect to the seaming, are in the seaming of the trees, and are in one side of one of the trees. A generalization of this work was carried out by Krizan *et al.*[6] who presented a treatment of the free energy and of the correlation function for the tree with arbitrary branching ratio. (For a recent review of spin systems on the Cayley tree see Ref. 7).

In this paper we shall consider the Potts model defined on the Cayley tree subjected to an external homogeneous magnetic field acting upon the superficial spins. Representing the field by an interaction of the superficial spins with a ghost spin, this model can be related to the closed asymmetric Cayley tree[8] (Fig. 1). The advantage of this equivalence comes from the hierarchical nature of the latter model which allow us to use a simple real-space renormalization procedure[9] to study its pair-correlation function.

Exploring the peculiar topological properties of the Cayley tree, we can reduce the calculation of the correlation function between any pair of spins in the lattice to the case where they are located in special sites in the tree. Proceeding further we show (Sec. III) that the partial summation of the inner spins maps the closed asymmetric tree into an open chain subjected to an effective magnetic field. A recursive relation for the correlation function in the chain is then established (Sec. II) by using the break-collapse method for the thermal transmissivity introduced by Tsallis and Levy.[10]

Break and colapse method.

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Roger Maynard.

Constantino influences in my scientific life

Visit of Constantino to Grenoble in 1994.

Grenoble.

Constantino influences in my scientific life

Asymmetry between gain and loss.

ECONOMETRICA

VOLUME 47 MARCH, 1979 NUMBER 2

PROSPECT THEORY: AN ANALYSIS OF DECISION UNDER RISK

BY DANIEL KAHNEMAN AND AMOS TVERSKY[1]

This paper presents a critique of expected utility theory as a descriptive model of decision making under risk, and develops an alternative model, called prospect theory. Choices among risky prospects exhibit several pervasive effects that are inconsistent with the basic tenets of utility theory. In particular, people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses. In addition, people generally discard components that are shared by all prospects under consideration. This tendency, called the isolation effect, leads to inconsistent preferences when the same choice is presented in different forms. An alternative theory of choice is developed, in which value is assigned to gains and losses rather than to final assets and in which probabilities are replaced by decision weights. The value function is normally concave for gains, commonly convex for losses, and is generally steeper for losses than for gains. Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities. Overweighting of low probabilities may contribute to the attractiveness of both insurance and gambling.

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Clear and neat explanation using q-statistics.

Europhys. Lett., **59** (5), pp. 635–641 (2002)

Risk aversion in economic transactions

C. Anteneodo[1](*), C. Tsallis[1](**) and A. S. Martinez[2](***)
[1] *Centro Brasileiro de Pesquisas Físicas - R. Dr. Xavier Sigaud 150*
22290-180, Rio de Janeiro, RJ, Brazil
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Universidade de São Paulo - Av. Bandeirantes 3900
14040-901 Ribeirão Preto, SP, Brazil

(received 23 January 2002; accepted in final form 5 June 2002)

PACS. `02.50.Le` – Decision theory and game theory.
PACS. `05.45.-a` – Nonlinear dynamics and nonlinear dynamical systems.
PACS. `05.90.+m` – Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems.

Abstract. – Most people are risk-averse (risk-seeking) when they expect to gain (lose). Based on a generalization of "expected utility theory" which takes this into account, we introduce an automaton mimicking the dynamics of economic operations. Each operator is characterized by a parameter q which gauges people's attitude under risky choices; this index q is in fact the entropic one which plays a central role in nonextensive statistical mechanics. Different long-term patterns of average asset redistribution are observed according to the distribution of parameter q (chosen once forever for each operator) and the rules (*e.g.*, the probabilities involved in the gamble and the indebtedness restrictions) governing the values that are exchanged in the transactions. Analytical and numerical results are discussed in terms of how the sensitivity to risk affects the dynamics of economic transactions.

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Generalization of Cajueiro discount function.

Physica A 390 (2011) 1763–1772

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ELSEVIER

The additive property of the inconsistency degree in intertemporal decision making through the generalization of psychophysical laws

Natália Destefano, Alexandre Souto Martinez *

Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP), Universidade de São Paulo (USP), Av. Bandeirantes, 3900 14040-901 Ribeirão Preto, SP, Brazil
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A R T I C L E I N F O

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Econophysics

A B S T R A C T

Intertemporal decision making involves choices among options whose effects occur at different moments. These choices are influenced not only by the effect of reward value perception at different moments, but also by the time perception effect. One of the main difficulties that affect standard experiments involving intertemporal choices is the simultaneity of both effects on time discounting. In this paper, we unify the psychophysical laws and discount value functions using the one-parameter exponential and logarithmic functions from nonextensive statistical mechanics. Also, we propose to measure the degree of inconsistency. This quantity allow us to discriminate both effects of time and value perception on discounting process and, by integration, obtain other main quantities like impulsivity and discount functions.

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Physica A 490 (2018) 250–259

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

ELSEVIER

Decision making generalized by a cumulative probability weighting function

Lindomar Soares dos Santos [a,*], Natália Destefano [a], Alexandre Souto Martinez [a,b]

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HIGHLIGHTS

- A new cumulative probability weighting function for the discount process is proposed.
- This function is based on q-logarithms and q-exponentials.
- It is assumed individuals behave similarly in the face of probabilities and delays.
- This proposition retrieves consecrated cases and models of psychophysical perception.
- The new functional form is supported by phenomenological models.

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Cajueiro discount function leads to relativistic time perception

frontiers
in Applied Mathematics and Statistics

ORIGINAL RESEARCH
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Check for updates

Inconsistency and Subjective Time Dilation Perception in Intertemporal Decision Making

Lindomar Soares dos Santos[1]* *and Alexandre Souto Martinez*[1,2]

[1] *Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto, Universidade de São Paulo, Ribeirão Preto, Brazil,* [2] *Instituto Nacional de Ciência e Tecnologia em Sistema Complexos, Rio de Janeiro, Brazil*

A large number of studies have demonstrated that intertemporal decision making process usually results in preferences that reverse over time, or choices that are inconsistent over time. Inconsistency can be explained by different discount models by the effect of reward value perception at different moments. Otherwise, one can also understand inconsistency as the result of the time perception effect. Here, we address inconsistency as the result of a subjective time dilation perception effect. We use arguments inspired by the special theory of relativity and focused our study on a generalized model that encompasses psychophysical effects on time perception, where we look for a transformation of the time interval between the pay times of two rewards. Additionally, we present a generalized two-argument hyperbolic utility function for the Bernoulli (logarithmic) one, associating their difference to subjective time intervals.

Keywords: econophysics, psychophysics, intertemporal decision making, inconsistency, time perception, generalized models, utility functions

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Population dynamics

von Foester *et al.* model

Doomsday: Friday, 13 November, A.D. 2026

At this date human population will approach infinity if it grows as it has grown in the last two millenia.

Heinz von Foerster, Patricia M. Mora, Lawrence W. Amiot

Science **132**, 1291–1295 (1960).

von Foester *et al.* model

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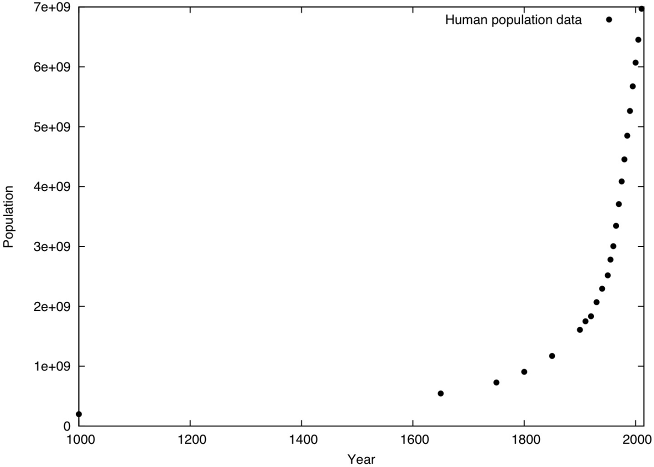
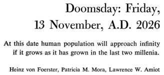


von Foester *et al.* model



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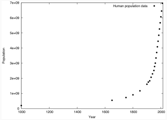


$N_0 = N(0)$

$$
\begin{aligned}
e_{\tilde{q}}(x) &= [1 + \tilde{q}x]^{1/\tilde{q}} \\
\tilde{q}x &\geq -1
\end{aligned}
$$

$$
\begin{aligned}
\frac{dN}{dt} &= kN^{1+\tilde{q}} \\
N(t) &= N_0\ e_{-\tilde{q}}(N_0^{\tilde{q}} kt) = N_0\ e_{-\tilde{q}}\left(-\frac{t}{t_c}\right) \\
t_c &= \frac{1}{N_0^{\tilde{q}} k}
\end{aligned}
$$

Scale function

$$
\tilde{s}_{\tilde{q}}(kt) = \frac{N_0^{-\tilde{q}} - N^{-\tilde{q}}(kt)}{\tilde{q}} = N_0^{-\tilde{q}}\ \ln_{-\tilde{q}}\left(\frac{N(kt)}{N_0}\right)
$$

Problem:

We know N_0, but not $\tilde{q}$.

Carrying capacity

$$p(t) = \frac{N(t)}{N(\infty)} = \frac{N(t)}{K}$$

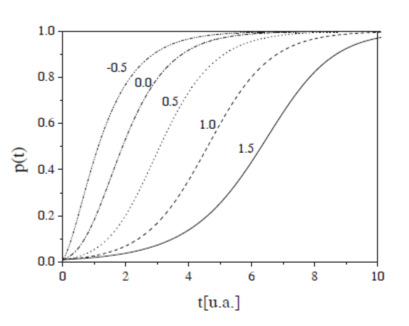
Capacidade de suporte do meio.

Richards' model

$$\frac{d \ln p}{dt} = -k \frac{p^{\tilde{q}} - 1}{\tilde{q}} = -k \ln_{\tilde{q}} p$$

$$p(t) = \frac{1}{e_{\tilde{q}}[\ln_{\tilde{q}}(p_0^{-1}) e^{-kt}]}$$

$$= e_{-\tilde{q}}[-\ln_{\tilde{q}}(p_0^{-1}) e^{-kt}]$$



$$p(t) = e_{-\tilde{q}}[-\ln_{\tilde{q}}(p_0^{-1}) e^{-kt}]$$

$$\ln_{-\tilde{q}} p(t) = \ln_{-\tilde{q}}(p_0) e^{-kt}$$

$$\frac{\ln_{-\tilde{q}} p(t)}{\ln_{-\tilde{q}} p(0)} = e^{-kt}$$

Richards' model

$$\frac{d\ln p}{dt} = -k\,\frac{p^{\tilde{q}}-1}{\tilde{q}} = -k\ln_{\tilde{q}} p$$

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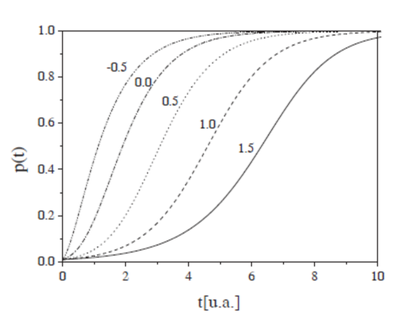

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$$\ln_{-\tilde{q}} p(t) = \ln_{-\tilde{q}}(p_0)e^{-kt}$$

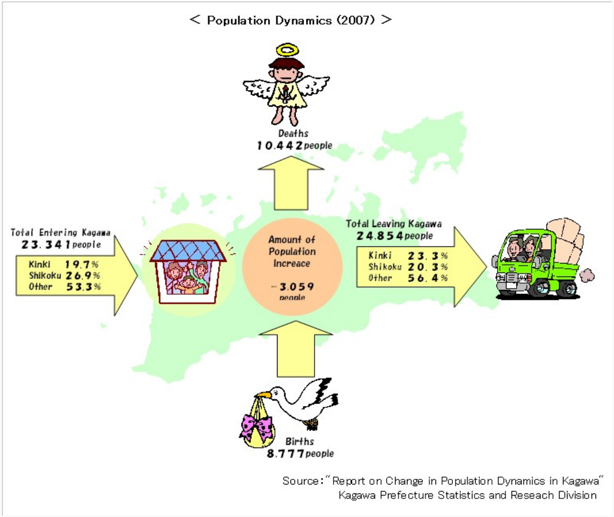
$$\frac{\ln_{-\tilde{q}} p(t)}{\ln_{-\tilde{q}} p(0)} = e^{-kt}$$

Scale function

$$s_{\tilde{q}}(t) = \ln_{-\tilde{q}}[p(t)]$$

B. C. T. Cabella, A. S. Marttinez and F. Ribeiro, Phys. Rev. E **83** 061902 (2011).

Richards-Schaefer's model

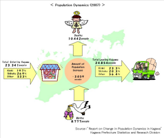


exogenous factors.

Richards-Schaefer's model

$$\frac{d \ln p}{d\tau} = \ln_{\tilde{q}} p + \epsilon(\tau)$$

$$\begin{aligned}
\tau &= kt \\
p(t) &= \frac{e_{\tilde{q}}[\epsilon(\tau)]}{e_{\tilde{q}}\left\{\ln_{\tilde{q}}\left[\frac{e_{\tilde{q}}(\epsilon)}{p_0}\right]\frac{e_{\tilde{q}}[\epsilon(\tau)]}{e_{\tilde{q}}[\epsilon(0)]}e^{-[1+\tilde{q}\bar{\epsilon}(\tau)]\tau}\right\}} \\
\bar{\epsilon}(\tau) &= \frac{1}{\tau}\int_0^{\tau} d\tau' \epsilon(\tau')
\end{aligned}$$



Extinction and survival transition ($\tau \to \infty$)

$$p^* = e_{\tilde{q}}(\bar{\epsilon})$$

Extinction: $\tilde{q}\bar{\epsilon} < -1$.
critical value: $\epsilon_c = -1/\tilde{q}$.
susceptibility: $\chi = \partial_{\bar{\epsilon}} p^* = (\bar{\epsilon} - \bar{\epsilon}_c)^{1/\tilde{q}}$.

B. C. T. Cabella, A. S. Martinez and F. Ribeiro, Phys. Rev. E **83** 061902 (2011).

Discretizinf Richards' model

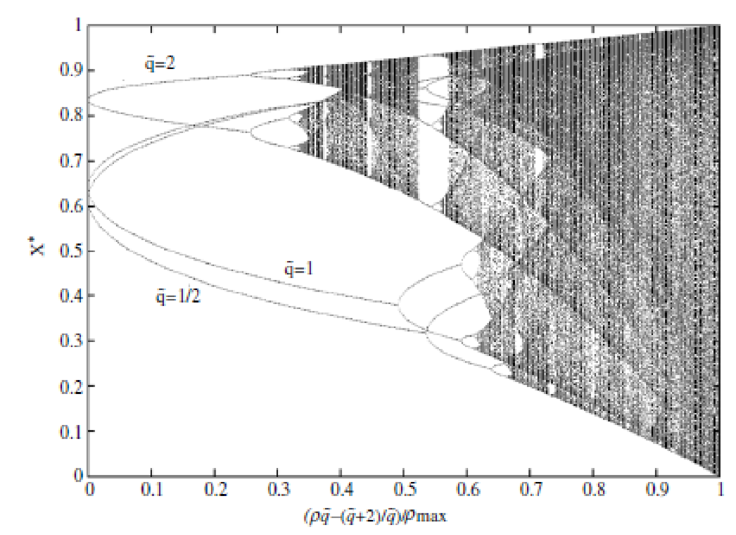
$$\frac{d \ln p}{dt} = -k \ln_{\tilde{q}}[p(t)] \Longrightarrow x_{i+1} = \rho_{\tilde{q}} x_i (1 - x_i^{\tilde{q}})$$

For:

- $\tilde{q} = 0$, linear map (trivial);
- $\tilde{q} = 1$, logistico (Verhulst) map and
- $\tilde{q} = 2$, cubic map.

A. S. Martinez, R. S. Gonzalez and A. L. Espindola, Physica A **388** 2922 (2009).

Richards' map: $x_{i+1} = \rho_{\tilde{q}} x_i (1 - x_i^{\tilde{q}})$



A. S. Martinez, R. S. Gonzalez and A. L. Espindola, Physica A **388** 2922 (2009).

Microscopic Mombach *et al.* model

Cells grow in a fractal structure D_f.

Microscopic Mombach *et al.* model

Cells grow in a fractal structure D_f.

Repulsive potential among cells $r^{-\gamma}$.

Modelo Microscópico Mombach *et al.*

J. C. M. Mombach, N. Lemke, B. E. J. Bodmann and M. A. P. Idiart, Europhys. Lett. **59** 923 (2002).

Células crescem em uma estrutura fractal de dimensionalidade D_f.

Potencial repulsivo entre células $r^{-\gamma}$.

The final result of the integral is then written as

$$I(n) = \begin{cases} \frac{\omega}{D_\mathrm{f}\left(1-\frac{\gamma}{D_\mathrm{f}}\right)} \left[\left(\frac{D_\mathrm{f}}{\omega}n\right)^{1-\frac{\gamma}{D_\mathrm{f}}} - 1\right] , & \text{if } \gamma \neq D_\mathrm{f} , \\ \frac{\omega}{D_\mathrm{f}} \ln\left(\frac{D_\mathrm{f}}{\omega}n\right) , & \text{if } \gamma = D_\mathrm{f} . \end{cases}$$

Returning to eq. (3), we average to obtain the general differential equation for the growth of the cellular system:

$$\dot{n} = n\left[\langle G\rangle - JI(n)\right] . \qquad (7)$$

In the next section we present the differential equations and their solutions from the relation between γ and D_f.

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Richards-Schaefer's model:

$$\frac{d \ln p}{d\tau} = \ln_{\tilde{q}} p + \epsilon \quad \longrightarrow \quad p(t) = \frac{e_{\tilde{q}}(\epsilon)}{e_{\tilde{q}}\{\ln_{\tilde{q}}[e_{\tilde{q}}(\epsilon)/p_0]e^{-[1+\tilde{q}\epsilon]\tau}\}}$$

Modelo Microscópico Mombach *et al.*

J. C. M. Mombach, N. Lemke, B. E. J. Bodmann and M. A. P. Idiart, Europhys. Lett. **59** 923 (2002).

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926 EUROPHYSICS LETTERS

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$$\tilde{q} = 1 - \frac{\gamma}{D_f} .$$

Non-normal data

Non-normal data

Non-normal data vector: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_R \end{bmatrix}$.

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Box-Cox transformation: $\vec{y} = \ln_\lambda(\vec{x})$

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Box-Cox transformation: $\vec{y} = \ln_\lambda(\vec{x})$

Generalized mean: $M_f(x) = f^{-1}\left[\sum_{i=1}^{R} p_i f(x_i)\right]$ with normalized weighs $p_i \geq 0$: $\sum_{i=1}^{R} p_i = 1$.

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Hölder mean: $f(x) = x^\lambda$, with $\lambda \in \Re$:
$\bar{M}_\lambda(x) = \left(\sum_{i=1}^{R} p_i x_i^\lambda\right)^{1/\lambda} = \langle x^\lambda \rangle_R^{1/\lambda}$. and arithmetic mean:
$\langle \ldots \rangle_R = \sum_{i=1}^{R} p_i \ldots_i$.

$$\bar{M}_\lambda(x) = e_\lambda(\langle \ln_\lambda(x) \rangle_R)$$

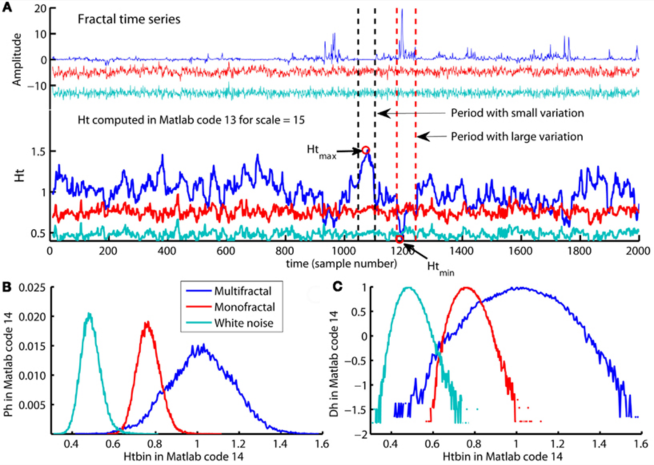
Hölder mean: $\bar{M}_\lambda(x) = e_\lambda(\langle \ln_\lambda(x)\rangle_R)$

For $x_i \geq 0$, $i = 1, 2, \ldots, R$ e $p_i = 1/R$:

- $\lambda \to -\infty$, $\bar{M}_{-\infty}(x) = \langle\langle x\rangle\rangle_{R,-\infty} = \min(x)$ lowest value;
- $\lambda = -1$, $\bar{M}_{-1}(x) = \langle\langle x\rangle\rangle_{R,-1} = \frac{1}{R} = \left(\sum_{i=1}^{R} \frac{1}{x_i}\right)^{-1}$ harmonic mean;
- $\lambda = 0$, $\bar{M}_0(x) = \left(\prod_{i=1}^{R} x_i\right)^{1/R}$, geometric mean;
- $\lambda = 1$, $\bar{M}_1(x) = \langle\langle x\rangle\rangle_{R,1} = \langle x\rangle_R = \frac{1}{R}\sum_{i=1}^{R} x_i = \bar{x}$, arithmetic mean;
- $\lambda = 2$, $\bar{M}_2(x) = \langle\langle x\rangle\rangle_{R,2} = (\frac{1}{R}\sum_{i=1}^{R} x_i^2)^{1/2} = \sqrt{\langle x^2\rangle_R}$.
  the variance is: $\sigma^2 = \bar{M}_2^1(x) - \bar{M}_1^2(x) = \langle x^2\rangle_R - \langle x\rangle_R^2$ and
- $\lambda \to \infty$, $\bar{M}_\infty(x) = \langle\langle x\rangle\rangle_{R,\infty} = \max(x)$, largest value.

MFDFA

Multi Fractal (MF) – Detrended Fractal Analysis (DFA)



Jan W. Kantelhardt, Stephan A. Zschiegner, Eva Koscielny-Bunde, Armin Bunde, Shlomo Havlin, H. Eugene Stanley, Physica A **316** , 87 (2002).

MFDFA

1. The standard formulation can be written as an arbitrary norm fluctuation function. This fluctuation function can be written as a Hölder mean, so that the mean of the Box-Cox transformed data and inverse of the Box-Cox transformation is applied

$$F_\lambda(s) = \bar{M}_\lambda(x) = e_\lambda(\langle \ln_\lambda(x) \rangle_R) \ .$$

   - notation concision;
   - automatic $\lambda = 0$ particular case
2. long correlation length, for $s \gg 1$: $F_\lambda(s) \sim s^{h(\lambda)}$. If the series is stationary $h(2)$ is the *Hurst exponente*. $h(\lambda)$ is *generalized Hurst exponente*.

Conclusions

Conclusion

- with the break and colapse method, since my undergraduatuion Constatino has illuminated my research;
- introducing me to Roger Maynard, which a did a nice PhD;
- introducing me how to use physical methods to address subjectivity in decision theory and other systems as population dynamics, epidemiological models and data statistical methods and
- to invite me to participate of INCT-SC
- healthy life and very happy birthday!!!!