A Lorentz invariant velocity distribution for a relativistic gas

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In 1911, Jüttner proposed a generalization of the Maxwell-Boltzmann velocity distribution for a relativistic gas. But the velocity space and, consequently, momentum space are not flat in special relativity, they correspond to the Lobachevsky space, which has a negative curvature. This curvature induces a specific power for the Lorentz factor in the PDF, affecting the Jüttner normalization constant in one, two and three dimensions. In addition, the Jüttner distribution written in terms of a convenient variable, the rapidity, shows a change in curvature at the origin at sufficiently high energies, which is not in agreement with computational dynamics simulations of a relativistic gas. However, in one dimension, rapidity satisfies a simple additivity law. This allows us to obtain, via the central limit theorem, a new Lorentz invariant PDF whose curvature at the origin does not change for any energy value and which agrees with the data from our computational dynamics simulations.