Nonlinear dynamical systems: Time reversibility versus sensitivity to the initial conditions

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Time reversal of vast classes of phenomena has direct implications with predictability, causality and the second principle of thermodynamics. We propose a new method of investigating reversibility in time series data through the instance of a paradigmatic dissipative nonlinear dynamical system, namely the logistic map $x_{t+1}=1-ax_t^2$. The method splits the original data into two series, one symmetrical and the other antisymmetrical with respect to time. A close relation is revealed between time reversibility and the sensitivity to the initial conditions. Indeed, depending on the initial condition and the size of the time series, time reversal can enable the recovery, within a small error bar, of past information when the Lyapunov exponent is non-positive, notably at the Feigenbaum point (edge of chaos), where weak chaos is known to exist. Past information is gradually lost for increasingly large Lyapunov exponent (strong chaos), notably at $a=2$ where it attains a large value.