Title: Entropic Extensivity and Large Deviations in the Presence of Strong Correlations

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Abstract:

The standard Large Deviation Theory (LDT) mirrors the Boltzmann–Gibbs (BG) factor which describes the thermal equilibrium of short-range Hamiltonian systems, the velocity distribution of which is Maxwellian. It is generically applicable to systems satisfying the Central Limit Theorem (CLT), among others. When we focus instead on stationary states of typical complex systems (e.g., classical longrange Hamiltonian systems), both the CLT and LDT need to be generalized. We focus here on a scale-invariant stochastic process involving strongly-correlated exchangeable random variables which, through the Laplace-de Finetti theorem, is known to yield a long-tailed Q-Gaussian $N \to \infty$ attractor in the space of distributions $(1 < Q < 3)$. We present strong numerical indications that the corresponding LDT probability distribution is given by $P(N,z) = P_0 e_q^{-r_q(z)N} = P_0[1-(1-q)r_q(z)N]^{1/(1-q)}$ with $q = 2 - 1/Q \in (1, 5/3)$. The rate function $r_q(z)$ seemingly equals the relative nonadditive q_r–entropy per particle, with $q_r \simeq \frac{7}{10} + \frac{6}{10}\frac{1}{Q-1}$, thus exhibiting a singularity at $Q = 1$ and recovering the BG value $q_r = 1$ in the $Q \to 3$ limit. Let us emphasize that the extensivity of $r_q(z)N$ appears to be verified, consistently with what is expected, from the Legendre structure of thermodynamics, for a total entropy. The present analysis of a relatively simple model somewhat mirroring spin-$1/2$ long-range-interacting ferromagnets (e.g., with strongly anisotropic XY coupling) might be helpful for a deeper understanding of nonequilibrium systems with global correlations and other complex systems.