

# Non-linear Fractional $q$ -Gaussian Diffusion Model of Stock Market

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Bitcoin time series is analyzed based on a fractional, non-linear  $q$ -Gaussian diffusion model. The analysis uses high-frequency intraday data from 2019 to 2022. We observe several stylized facts of the Bitcoin market, such as heavy tails, anomalous diffusion, short-time autocorrelation, and anomalous diffusion of the price return, which can be described as a  $q$ -Gaussian diffusion process.

This process is described using a fractional, non-linear diffusion equation with two exponents: the Hurst exponent ( $H$ ) for the fractionality and defines the anomalous behavior and autocorrelation, and the  $q$ -exponents that define the non-linearity of the diffusion and account for the heavy-tailed distribution. The temporal evolution of these coefficients is imposed by the Central Limit Theorem, which dictates the convergence of the diffusion process to a Gaussian diffusion over time. The diffusion process with time-varying exponents provides a more robust model for real-world financial systems.