

Mathematical models to explain the origin of urban scaling laws

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Empirical evidence reveals urban scaling patterns. Socioeconomic factors (patents, wages, GDP) often scale super-linearly with city population size, while infrastructure (gas stations, street length) scales sub-linearly, indicating economies of scale. We will present a review of theoretical models explaining these findings. Our goal is to uncover model connections and scenarios yielding similar outcomes.

These models stem from diverse premises, including gravity-based concepts, urban densification, and social network properties. Notably, we emphasize the role of mobility and accessibility in fostering a scaling economy. Furthermore, we explore scenarios where seemingly distinct fundamental ideas may share underlying similarities. Within the gravity model, we propose a comprehensive framework that encompasses all analyzed models as particular instances. In conclusion, we contemplate the future of this field and envision the development of a more comprehensive theory of cities.