Multifractal detrended fluctuation analysis approach using generalized logarithmic and exponential functions

Suzielli Martins Mendonça [(a)], Brenno C. Troca Cabella [(a)], Alexandre Souto Martinez [(a,b)]
[(a)]Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP), Universidade de São Paulo (USP), Ribeirão Preto, SP, Brazil [(b)]National Institute of Science and Technology for Complex Systems, Rio de Janeiro, Brazil

Detrended Fluctuation Analysis (DFA) and its generalization for multifractal signals, Multifractal Detrended Fluctuation Analysis (MFDFA) use a fluctuation function to estimate the Hurst exponent (H) of a time series. Here, we write the fluctuation functions as generalized means, using the generalized logarithm and exponential functions from the nonextensive statistical mechanics. This approach enables a unified formulation and interpretation of DFA and MFDFA, providing a compact algorithm applicable to time series. This generalized formulation is interpreted as the Box-Cox transformation of the dataset and establishes a relationship between entropic and multifractal indexes. Our formulation is validated estimating H of commonly used signals such as: the fractional Ornstein-Uhlenbeck process, the symmetric Lévy distribution, pink, white, and Brownian noises. In addition, we estimate the Hurst exponent (H) from a real-world electrocardiogram (ECG) signal.